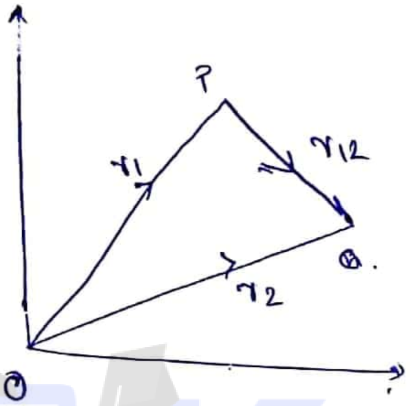
MODULE - II

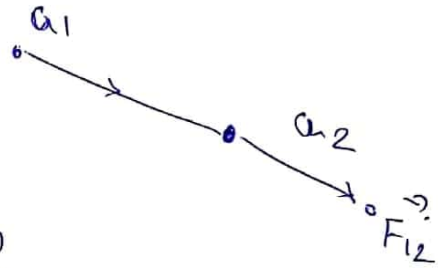
Electrostatic is the branch of electromagnetics dealing with the effects of electric charges at rest.

The fundamental law of electrostatic is coulumb law.

coulumbs law

$$F \propto \frac{q_1 q_2}{r^2}$$





$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

$$r_2 = \vec{r_1} + \vec{r_{12}}$$

$$r_{12} = \vec{r_2} - \vec{r_1}$$

$$a_{12} = \frac{r_{12}}{|r_{12}|}$$

$$= \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$$

$$F_{12} = a_{R12} \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 r_{12}^2}$$

Expression for force in vector form

$$F_{12} = \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q_1 \cdot Q_2}{r_{12}^2}$$

$$= \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q_1 \cdot Q_2}{(r_2 - \vec{r_1})^2} \cdot \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - r_1|}$$

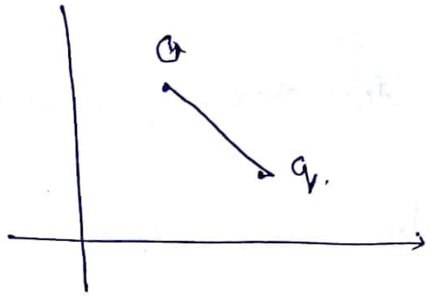
$$= \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q_1 \cdot Q_2}{|\vec{r_2} - \vec{r_1}|^3} \; N$$

$$\vec{F_{21}} = -\vec{F_{12}}$$

Electric field intensity [E]

Force experienced ~~by~~ on unit postive test charge placed in a electric field is called electric field intensity

$$\vec{E} = \frac{1}{4\pi\epsilon_0 \cdot \epsilon_r} \frac{Q}{r^2}$$



Electric field:

~~Electric field:~~

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

concept of test charge :→ Small and postive

Does not affect charge distribution

(*) Direction of electricfield is that of the force that acting on a unit - postive test charge

Continoues distribution of charge

(1) point charges (C)

(2) volume charge (C/m^3) ⇐ most general

(3) Surface charges (C/m^2)

(4) Line charges (C/m)

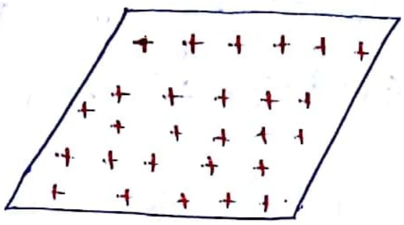
Linear charge density

$$\frac{\text{charge}}{\text{unit length}} = \rho_L = \rho_l$$

charge = $\rho_l \times$ total length

Surface charge density

$$\rho_s = \frac{\text{charge}}{\text{unit area}} = C/m^2$$

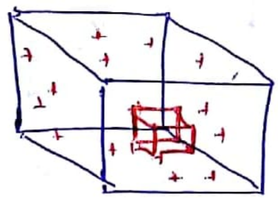


total charge = $\rho_s \times$ total area

$$E = \int \frac{\rho_s \, ds}{4\pi\varepsilon_0 \cdot R^2} a_R$$

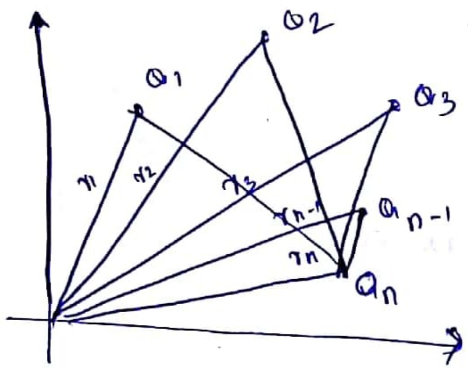
Volume charge density

$$\rho_v = \frac{\text{charge}}{\text{unit volume}} = C/m^3$$



total charge = $\rho_v \times$ total volume

$$\bar{E} = \int \frac{\rho_v \cdot dv}{4\pi\varepsilon_0 R^2} a_R$$



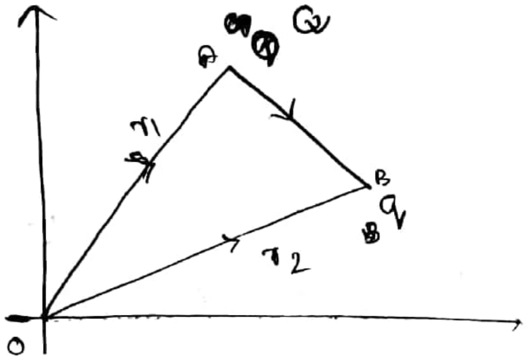
$$\vec{F}_n = \vec{F}_{1n} + \vec{F}_{2n} \cdots \quad \vec{F}_{n-1}$$

$$\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_1 Q_n}{|\vec{r}_n - \vec{r}_1|^3} \times (\vec{r}_n - \vec{r}_1)$$

$$\vec{F}_{2n} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_2 Q_n}{|\vec{r}_n - \vec{r}_2|} (\vec{r}_n - \vec{r}_2)$$

$$\vec{F}_{n-1} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{Q_{n-1} \cdot Q_n}{|\vec{r}_n - \vec{r}_{n-1}|} (\vec{r}_n - \vec{r}_{n-1})$$

(a)



$$\vec{r_1} + \vec{AB} = \vec{r}$$

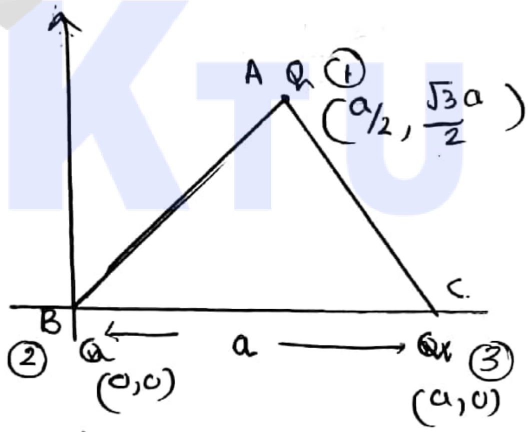
$$\vec{AB} = \vec{r} - \vec{r_1}$$

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q}{|\vec{r}-\vec{r_1}|^2} \frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|}$$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q}{|\vec{r}-\vec{r_1}|^3} (\vec{r}-\vec{r_1})$$

Problems

(1) Find the force at point A



$\cancel{F = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}}$

$\cancel{F = \frac{Q_1 \cdot Q_2}{|\vec{r_2}-\vec{r_1}|^3}(\vec{r_2}-\vec{r_1})}$

$$\vec{F} = \vec{F_A} + \vec{F_B} + \vec{F_C}$$

General eq^n

$$F = \sum_{i=2}^{n} \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_1 \cdot Q_i}{|r_i - \vec{r_1}|^3} (\vec{r_i} - \vec{r_1})$$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_1 \cdot Q_2}{|\vec{r_2}-\vec{r_1}|^3} (\vec{r_2}-\vec{r_1}) + \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q_1 \cdot Q_3}{|\vec{r_3}-\vec{r_1}|^3} (\vec{r_3}-\vec{r_1})$$

Position vector of first charge . $\vec{r_1} = a/2\, \hat{a_x} + \frac{\sqrt{3}a}{2} \hat{a_y}$

$\vec{r_2} = 0$

$\vec{r_3} = a \cdot \hat{a_x}$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{Q_1 \cdot Q_2}{\left|\frac{a}{2} + \frac{\sqrt{3}a}{2}\right|} \left(\frac{a}{2}\hat{a_x} + \frac{\sqrt{3}a}{2}\hat{a_y}\right) + \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r} \cdot \frac{Q_1 \cdot Q_3}{\left|a\cdot\hat{a_x} - \frac{a}{2} + \frac{\sqrt{3}a}{2}\right|} \left(a\cdot\hat{a_x} - \frac{a}{2}\right.$$

$$\therefore F_{21} = \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r} \frac{Q^2}{\left(\frac{a^2}{4} + \frac{3a^2}{4}\right)^3} \quad -\frac{a}{2}\hat{a_x} - \frac{\sqrt{3}a}{2}$$

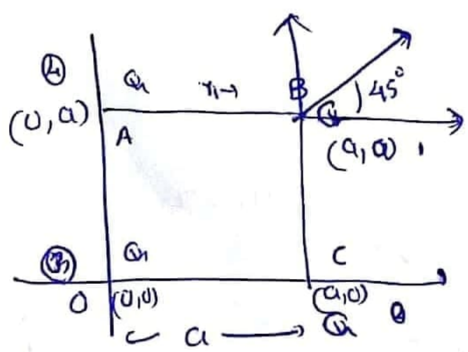
$$= \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r} \frac{Q^2}{a^3}\left(-\frac{a}{2}\hat{a_x} - \frac{\sqrt{3}a}{2}\hat{a_y}\right)$$

$$\vec{F_{31}} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{Q^2}{a^3}\left(\frac{a}{2}\hat{a_x} - \frac{\sqrt{3}a}{2}\hat{a_y}\right)$$

$$\vec{F_1} = \vec{F_{21}} + \vec{F_{31}}$$

$$= \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r} \frac{Q^2}{a^3}\left(-\sqrt{3}a\,\hat{a_y}\right)$$

Find the Force at B



$$\boxed{F = \sum_{i=1}^{3} \frac{1}{4\pi \varepsilon_0 \varepsilon_r} \frac{Q_1 \cdot Q_i}{|r_i - r_1|^3} (\vec{r}_i - \vec{r}_1)}$$

$\vec{r}_1 = a \cdot \hat{a_x} + a \cdot \hat{a_y}$

$\vec{r}_2 = a \cdot \hat{a_x}$

$\vec{r}_3 = 0$

$r_4 = a \cdot \hat{a_y}$

$\cancel{F_{21} = \frac{1}{4\pi \varepsilon_0 \cdot \varepsilon_r} \cdot \frac{Q^3}{a^2 - a^2 + a^2}}$

$\vec{r}_2 - \vec{r}_1 = -a \cdot \hat{a_y}$

$r_3 - \vec{r}_1 = -a \cdot \hat{a_x} - a \cdot \hat{a_y}$

$\vec{r}_4 - \vec{r}_1 = -a \cdot \hat{a_x}$

$|\vec{r}_2 - \vec{r}_1| = a$

$|\vec{r}_3 - \vec{r}_1| = \sqrt{a^2 + a^2} = \sqrt{2}a$

$|\vec{r}_4 - \vec{r}_1| = a$

$F_{21} = \frac{1}{4\pi \varepsilon_0 \cdot \varepsilon_r} \cdot \frac{Q^2}{a^3} \times (-a \cdot \hat{a_y})$

$F_{31} = \frac{1}{4\pi \varepsilon_0 \cdot \varepsilon_r} \frac{Q^2}{(\sqrt{2}a)^3} \times (-a \cdot \hat{a_x} - a \cdot \hat{y})$

$F_{41} = \frac{1}{4\pi \varepsilon_0 \cdot \varepsilon_r} \frac{Q^2}{a^3} \times (-a \cdot \hat{a_x})$

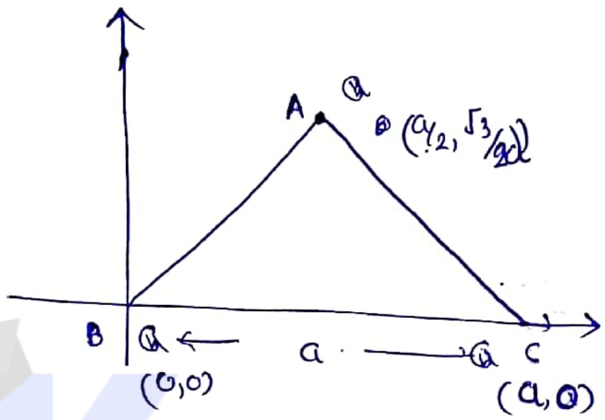
$F = F_{21} + F_{31} + F_{41}$

$= \frac{1}{4\pi \varepsilon_0 \cdot \varepsilon_r} \frac{Q^2}{a^3} \left[-a \cdot a_y - \frac{a \cdot \hat{a_x} - a \cdot \hat{a_y}}{(\sqrt{2})^3} - a \, \hat{a_x} \right]$

$$= -\frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\,\frac{Q^2}{a^3}\left(a\cdot\hat{a_y} + a\cdot\hat{a_x}\right)\left[1+\frac{1}{2\sqrt{2}}\right]$$

$$= -\left(\frac{1}{2\sqrt{2}}\right)\left[\left(\frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\cdot\frac{Q^2}{a^2}\,\hat{a_x} + \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\,\frac{Q^2\cdot\hat{a_y}}{a^2}\right)\right]$$

(*) Find the electric field intensity vector at point A



$$\frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\,\frac{Q_i}{|r_i - r_1|^3}\,(\vec{r_i} - \vec{r})$$

$$\vec{r} = \frac{a}{2}\hat{a_x} + \frac{\sqrt{3}a}{2}\hat{a_y}$$

$$\vec{r_1} = 0$$

$$\vec{r_2} = a\cdot\hat{a_x}$$

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{Q}{(r-r_1)^3}\times(r-r_1)$$

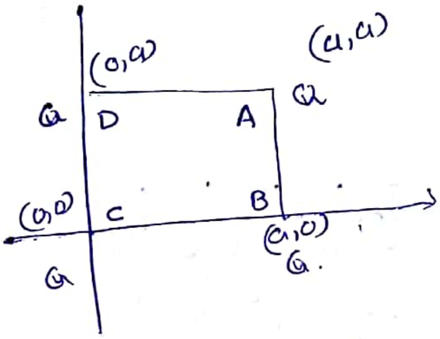
$$\cancel{F_1 = \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\times\frac{Q}{a^2/4}} \qquad E_{BA} = \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\,\frac{Q}{a^3}\left(a/2\,\hat{a_x} + \sqrt{3}/2\,a\,\hat{a_y}\right)$$

$$E_{CA} = \frac{1}{4\pi\varepsilon_0\cdot\varepsilon_r}\times\frac{Q}{a^3}\left(-a/2\,\hat{a_x} + \sqrt{3}/2\,a\cdot\hat{a_y}\right)$$

$$E_A = \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q}{a^2} \left(\tfrac{1}{2} a_x^{\wedge} + \tfrac{\sqrt{3}}{2} a_y^{\wedge} - \tfrac{a}{2} a_x^{\wedge} + \tfrac{\sqrt{3}}{2} a_y^{\wedge} \right)$$

$$= \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q}{a^2} \times \sqrt{3}\, a_y^{\wedge}$$

(2)



$$E_{CA} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{Q}{(\sqrt{2}a)^3} \; a \cdot a_x^{\wedge} + a \cdot a_y^{\wedge}$$

$$E_{DA} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{Q}{a^3} \cdot a \cdot a_x^{\wedge}$$

$$E_{BA} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{Q}{a^3} \cdot a \cdot a_y^{\wedge}$$

$$E_A = E_{CA} + E_{BA} + E_{BA}$$

$$E_A = \frac{1}{4\pi\varepsilon_0 \cdot \varepsilon_r} \frac{Q}{a^2} \times a \left[\frac{1}{2\sqrt{2}} a_x^{\wedge} + \frac{1}{2\sqrt{2}} a_y^{\wedge} + a_x^{\wedge} + a_y^{\wedge} \right]$$

$$E_A = \frac{Q}{4\pi\varepsilon_0 \cdot \varepsilon_r} \times \left[1 + \frac{1}{2\sqrt{2}} \right] \left[a_x^{\wedge} + a_y^{\wedge} \right]$$

Stokes theorem

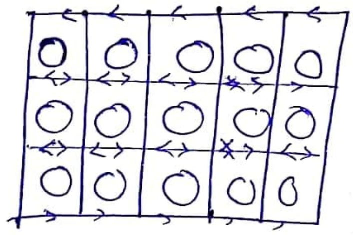
$\hat{a_n}$ - normal unit vector

Δs - area bound by curve

$$\text{curl } A = \nabla \times A = \lim_{S \to 0} \frac{\oint_L A \cdot dL}{\Delta S} \, \hat{a_n}$$

Stokes theorem states that

$$\boxed{\oint_L A \cdot dl = \int_S \nabla \times A \cdot ds}$$



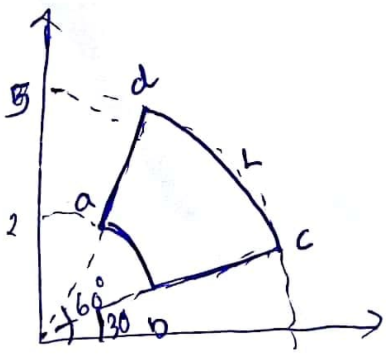
consider a cell ΔS_K, length L_K

$$\oint A \cdot dl = \sum_K \oint_{L_K} A \cdot dl$$

$$= \sum_K \oint_{L_K} \frac{A \cdot dl}{\Delta S_K} \Delta S_K$$

$$\oint_L A \, dL = \sum_K \lim_{\Delta S_K \to 0} \oint \frac{A \, dl}{\Delta S_K} \cdot \Delta S_K.$$

(a) If $A = \rho \cos\phi \, \hat{a_\rho} + \sin\phi \, a_\phi$



Evaluate $\oint A \cdot dl$ around the path shown in figure, confirm this by stoke theorem.

a to b $\qquad d\vec{l} = d\rho\, \hat{a_\rho} + \rho\, d\phi \cdot \hat{a_\phi} + dz \cdot \hat{a_z}$

$$\int_{60}^{30} \rho \sin\phi\, d\phi + \int_{2}^{5} \rho \cos\phi \cdot d\rho + \int_{30}^{60} \rho \sin\phi \cdot d\phi + \int_{5}^{2} \rho \cos\phi \cdot d\rho$$

$$= 2\left(\cancel{\cos 30 - \cos 60}\right) + \cos\phi^{30} \cancel{\frac{\rho^2}{2}\Big|_2^5} + \cancel{\rho\left[\cos 30 - \cos 60\right)}$$

$$+ \cancel{\cos\phi^{60}\left[\frac{\rho^2}{2}\Big|_5^2\right]}$$

$$2 \times \left[\cos 60 - \cos 30\right] + \cos 30 \times \frac{\rho^2}{2}\Big|_2^5 + 2 \times \left[\cos 30 - 60\right]$$

$$+ \cos 60 \times \left[\frac{\rho^2}{2}\right]_5^2$$

$$= \underline{\underline{4.941}}$$

$$\int_{30}^{60}\int_{2}^{5} \nabla \times A = \int_{30}^{60}\int_{2}^{5} \begin{vmatrix} a_\rho & \rho\, \hat{a_\phi} & \hat{a_z} \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ \rho\cos\phi & \rho\sin\phi & 0 \end{vmatrix} \rho \cdot d\phi \cdot d\rho = \int_{30}^{60}\int_{2}^{5}$$

$\cancel{a_\rho}\left[\cancel{\frac{\partial}{\partial z} \rho \sin\phi}\right.$

$$\int_{30}^{60}\int_{2}^{5} (\sin\phi + \rho\sin\phi)\, \hat{a_z} \cdot \rho\, d\phi\, ds$$

Gauss's Law

- The total flux through a closed surface is equal to the total charge enclosed by that surface.

$$\boxed{\iint_S D \cdot ds = Q} \qquad \boxed{\oint D \cdot ds = Q_{encl}}$$

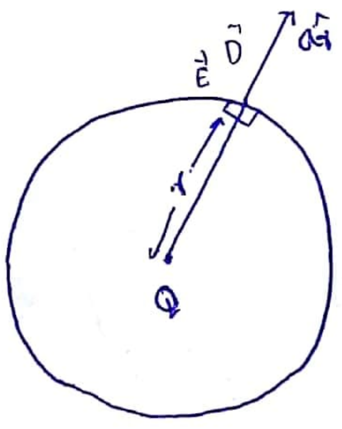
$$= \int_l \rho_L \cdot dl$$

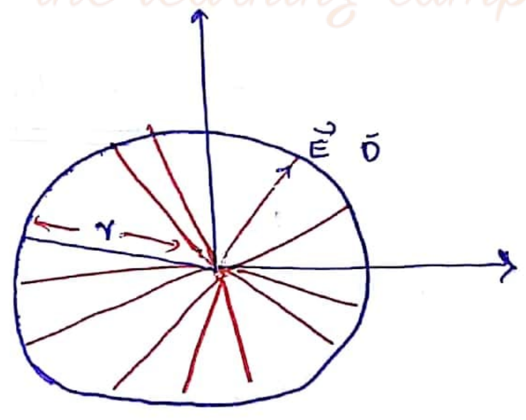
$$= \iint_S \rho_s \cdot ds$$

$$= \iiint \rho_v \cdot dv.$$

$\boxed{\nabla \cdot D = \rho_v}$ maxwells Eqn

Proof:





$$\vec{dl} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$$

$$ds = r^2 \sin\theta\, d\theta\, d\phi \qquad \vec{D} = \varepsilon_0 \cdot \vec{E}$$

let $D = \varepsilon_0 \cdot \vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{a_r}$$

$$\vec{D} = \frac{1}{4\pi} \frac{Q}{r^2} \hat{a_r}$$

$$\iint_s \frac{Q}{4\pi r^2} \hat{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$\theta \to 0$ to π

$\phi \to 0 - 2\pi$

$$= \iint_s \frac{Q \sin\theta \cdot d\theta \cdot d\phi}{4\pi}$$

$$= \int_0^{2\pi} \left[\int_0^{\pi} \frac{Q}{4\pi} \sin\theta \cdot d\theta \right] \cdot d\phi$$

$$= \int_0^{2\pi} \left[\frac{Q}{4\pi} - \cos\theta \Big|_0^{\pi} \right] d\phi$$

$$= \int_0^{2\pi} \frac{Q}{4\pi} \left[\cos 0 - \cos\pi \right] \cdot \phi$$

$$= \frac{2Q}{4\pi}$$

$$\int_0^{2\pi} \frac{Q}{2\pi} \cdot d\phi$$

$$= \frac{Q}{2\pi} \times \phi \Big|_0^{2\pi}$$

$$= \underline{\underline{Q}}$$

$$D = \frac{\psi}{A} = \frac{Q}{4\pi r^2} \hat{a_r} \quad (\psi = Q)$$

(sphere)

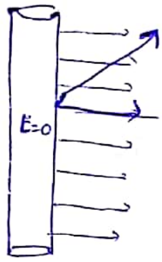
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{a}_r$$

$$\boxed{D = \varepsilon_0 \vec{E}}$$

Facts about a conductor

(1) $E = 0$, inside the conductor

(2) The net charge resides on the surface of the conductor

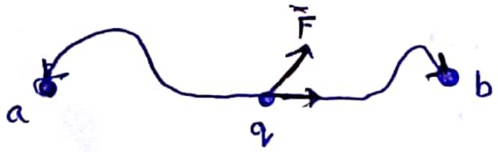


(3) 'E' is perpendicular to the conductor on the surface.

Electrostatic potential

It is the work done in moving a test charge from one point to another in a region of electric field.

$$W_{a\to b} = -\int_a^b F \cdot dl = -q\int_a^b E \cdot dl$$



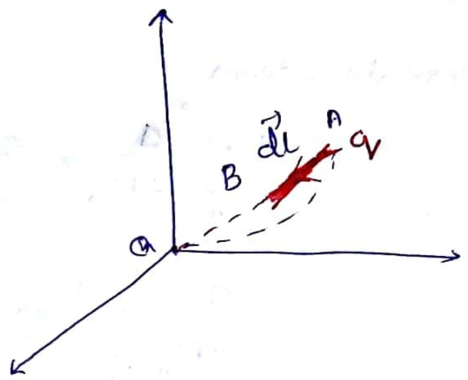
Potential due to a point charge

$$V = \frac{\text{work done}}{\text{unit charge}}$$

$$= \frac{F \cdot dl}{q}$$

$$= \frac{-(q \cdot \vec{E}) \cdot dl}{q}$$

$$= \vec{E} \cdot dl$$



$$V_{AB} = -\int_A^B \vec{E} \cdot \vec{dl}$$

In spherical co-ordinate system

$$\vec{dl} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$$

$$\vec{E} = E_r\hat{a_r} + E_\theta\hat{a_\theta} + E_\phi\hat{a_\phi}$$

$$= -\int_A^B (E_r \cdot \hat{a_r}) \cdot (dr\,\hat{a_r})$$

$$= -\int_A^B E_r \cdot dr$$

$$V_{AB} = -\int_A^B \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot dr$$

$$\frac{r^{-2+1}}{-2+1} = \frac{r^{-1}}{-1} = -1/r,$$

$$= -\frac{Q}{4\pi\varepsilon_0} \int_{R_A}^{R_B} r^{-2} \cdot dr$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot 1/r \Big|_A^B$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]}$$

Absolute potential

$$V_A = -\int_{\infty}^{A} \frac{Q}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{A} \frac{dr}{r^2}$$

$$\boxed{V_A = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r_A}}$$

$$V_B = -\int_{\infty}^{B} \frac{1}{4\pi\varepsilon_0}\cdot\frac{dr}{r^2}$$

$$\boxed{V_B = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r_B}}$$

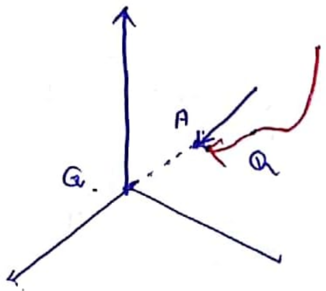
$$\therefore \boxed{V_{AB} = V_B - V_A}$$

NOTE:

Potential is path independant, it depends only on the starting and ending point of positions.

Another Method:

$$V_A = -\int_{\infty}^{r_A} \vec{E}\cdot\vec{dl}$$



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\,\hat{a_r}$$

$$\vec{dl} = dr\,\hat{a_r}$$

$$\vec{dl} = dr\,\hat{a_r} + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a_\phi}$$

Here also the path is independant.

$$V_A = \int_{\infty}^{r_A} (E_r \cdot \hat{a}_r) \cdot dr \cdot \hat{a}_r$$

$$= \int_{\infty}^{r_A} E_r \cdot dr$$

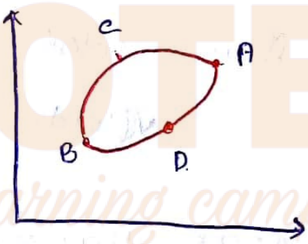
$$= -\int_{\infty}^{r_A} \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \cdot dr$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r_A}$$

∴ Here also the path is independant

Potential - conservative

$$\oint E \cdot dl = 0$$



$$V_{AB} = -\int_{A\,(ACB)}^{B} \vec{E} \cdot \vec{dl}$$

$$V_{AB} = -\int_{A\,(ADB)}^{B} \vec{E} \cdot dl$$

$$\therefore \int_{ACB} E \cdot dl = \int_{ADB} E \cdot dl$$

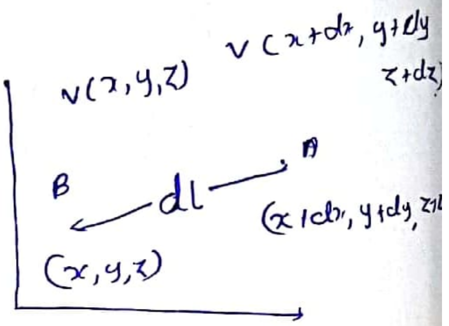
$$\Rightarrow \int_{ACB} E\,dl - \int_{ADB} E\,dl = 0$$

$$\int_{ACB} E\,dl + \int_{BDA} E\,dl = 0$$

$\therefore \oint E \cdot dl = 0$ this is called potential - conservative.

Derivation to proove $\boxed{E = -\nabla V}$

let $V = -\int E \cdot dl$



$$dV = -E \cdot dl \quad \text{——} (1)$$

$$\vec{E} = E_x \cdot \hat{a_x} + E_y \hat{a_y} + E_z \cdot \hat{a_z}$$

$$\vec{dl} = dx \cdot \hat{a_x} + dy \cdot \hat{a_y} + dz \cdot \hat{a_z}$$

$$\vec{E} \cdot dl = E_x \cdot dx + E_y \cdot dy + E_z \cdot dz \quad \text{——} (2)$$

difference in potential

$$V(x+dx, y+dy, z+dz) - V(x,y,z) = dV$$

$$V(x+dx, y+dy, z+dz) = V(x,y,z) + \frac{\partial V}{\partial x} \cdot dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$\therefore \frac{\partial V}{\partial x} \cdot dx + \frac{\partial V}{\partial y} \cdot dy + \frac{\partial V}{\partial z} \cdot dz = dV \quad \text{——} (3)$$

Put these (2) and (3) in (1)

$$\frac{\partial V}{\partial x} \cdot dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz = -\left[E_x \cdot dx + E_y dy + E_z dz\right]$$

$$\frac{\partial V}{\partial x} \cdot dx = -E_x \cdot dx \qquad \frac{\partial V}{\partial y} dy = -E_y \cdot dy$$

$$\frac{\partial V}{\partial x} \cdot dz = -E_z dz.$$

$$\frac{\partial V}{\partial x} = -E_x \qquad \frac{\partial V}{\partial y} = -E_y \qquad \frac{\partial V}{\partial z} = -E_z.$$

$$E = E_x \cdot \hat{a_x} + E_y \hat{a_y} + E_z \cdot \hat{a_z}$$

put above in the equation in E

$$\vec{E} = -\left(\frac{\partial V}{\partial x} \hat{a_x} + \frac{\partial V}{\partial y} \hat{a_y} + \frac{\partial V}{\partial z} \hat{a_z} \right)$$

$$\therefore \boxed{\vec{E} = -\nabla V}$$ – Gradient of V

NOTE:

From maxwells equation

$$\nabla \cdot D = \rho_v$$

we know $D = \varepsilon_0 \cdot E$

and $E = -\nabla V$.

Then equa..

$$\nabla \cdot \varepsilon_0 \cdot E = \rho_v$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon_0}.$$

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\varepsilon_0}$$

$$\nabla(\nabla V) = -\frac{\rho_v}{\varepsilon_0}.$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}}$$ — Poisonss equation

if $\rho_v = 0$

$$\boxed{\nabla^2 V = 0}$$ is called Laplace eqn

(4) Given that $D = \rho z \cos^2\phi \hat{a_z}$ C/m^2, calculate the charge density at $(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1m with $-2 \leq z \leq 2$ m

Ans:

$$D = \rho z \cos^2\phi \hat{a_z}$$

$$\nabla \cdot D = \rho_v$$

$$\left(\frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z} \right) \cdot \rho z \cos^2\phi \cdot \hat{a_z}$$

$$= \frac{\partial}{\partial z} \rho z \cos^2\phi$$

$$= \underline{\underline{\rho \cos^2\phi}}$$

$$\cos^2 x = \frac{1 + 2\sin x}{2}$$

$$\rho_v = \rho \cos^2\phi$$

$\rho_v(1, \pi/4, 3)$

$$= 1 \cos^2(\pi/4)$$

$$= \underline{\underline{1/2}} \ C/m^3$$

total charge enclosed by the surface

$$Q = \iiint \rho_v \, dv$$

$$dv = d\rho \times \rho d\phi \times dz$$

$$= \int_0^1 \int_0^{2\pi} \int_{-2}^{2} [(\rho \cos^2\phi) \times \rho \cdot dz] \cdot d\phi \cdot d\rho$$

$$\int_0 \left[\int_0^{2\pi} \left[\int_{-2}^{2} (\rho^2 \cos^2\phi) . dz \right] . d\phi \right] . dz$$

$\cos^2\phi = \frac{1 + 2\sin\phi}{2}$

$$= \int_0^1 \int_0^{\pi} (\rho^2 \cos^2\phi) . dz \; \cancel{d\phi} \; z \Big|_{-2}^{2}$$

$$\int_0^1 \int_0^{2\pi} 4\rho^2 \left(\frac{1 + \cos 2\phi}{2} \right) . d\phi . d\rho$$

$$= 4\rho^2 \; \tfrac{1}{2} \phi \Big|_0^{\pi} - \; - \frac{\sin 2\phi}{4} \Big|_0^{\pi}$$

$$= 4\rho^2 \; \pi/2$$

$$\Rightarrow \int_0^1 \frac{4\pi \rho^2}{2} . d\rho$$

$$= \int_0^1 2\pi \rho^2$$

$$= 2 \times 2\pi \frac{\rho^3}{3} = \underline{\underline{4\pi/3}}$$

$$= 2\pi \times 1/3$$

$$= \underline{\underline{2\pi/3}}$$

Given the potential $V = \frac{10}{r^2} \sin\theta . \cos\phi$

(i) find the electric flux density (D) at $(2, \pi/2, 0)$

(ii) calculate the workdone in moving a ~~ton~~ $10\mu C$ charge from point A $(1, 30^\circ, 120^\circ)$ to be $(4, 90^\circ, 60^\circ)$

$\vec{E} = -\nabla V$, $D = \varepsilon_0 \vec{E}$

$=$

$$= \frac{\partial}{\partial r} \hat{a_r} + \frac{1}{r} \cdot \frac{\partial}{\partial \theta} \cdot \hat{a_\theta} + \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial \phi} \cdot \hat{a_\phi}$$

$$-\nabla V = -\left[\frac{\partial V}{\partial r} \hat{a_r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a_\theta} + \frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi} \hat{a_\phi} \right]$$

$= [$

$=$

$$= \left[-10 \sin\theta \cos\phi \times \frac{2}{r^3} + \frac{10\cos\phi}{r^3} \cos\theta - \frac{10}{r^3} \sin\phi \right.$$

$$E = \frac{20\sin\theta\cos\phi}{r^3} \hat{a_r} + \frac{10\cos\phi\cos\theta}{r^3} - \frac{10}{r^3}\sin\phi$$

$D = \varepsilon_0 \cdot E$

$r = 2$

$\theta = \pi/2$

$\phi = 0$

$$D(2, \pi/2, 0) = \frac{20\sin\pi/2}{2^3} - \frac{10 \times \cos\pi/2}{2^3} - 0 \Big]$$

$$= 2.5\,\varepsilon_0$$

$$V_{AB} = \frac{W_{AB}}{Q}$$

$$W_{AB} = Q(V_{AB})$$

$$= Q(V_B - V_A)$$

$$V_B = \frac{10}{4^2} \times \sin 90 \times \cos(\cancel{60})$$

$$= -\cancel{20}\ \cancel{16}.\ 0.3125$$

$$V_A = 10/4 \times \sin 30 \ \cos(120)$$

$$= -2.5$$

$$W_{AB} = Q(V_B - V_A)$$

$$= \cancel{11.4 - 2.5)} \quad 10\mu c \times (2.8125)$$

$$= \cancel{39 \times 10} \quad = \underline{28.125\, \mu\, \text{Joule}}$$

A charge distribution with spherical symmetry has density -

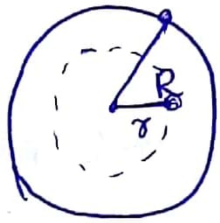
$$\rho_v = \begin{cases} \frac{\rho_0 r}{R} & , 0 \leq r \leq R \\ 0 & r \geq R \end{cases}$$

, find ~~...~~ determine E every where.

$$\rho_v = \begin{cases} \frac{\rho_0 r}{R} & , 0 \leq r \leq R \\ 0 & r > R \end{cases}$$

case-I

$0 \leq r \leq R$



Gaussets law

$$\iint_S D \cdot ds = Q = \iiint \rho_v \cdot dv$$

$$\iint_S D \cdot ds = \iint_S D \cdot ds = D \iint_S ds$$

$$= \underline{\underline{D 4\pi r^2}}$$

take RHS $\iiint_V \frac{\rho_0 r}{R} \cdot dv$

$$= \frac{\rho_0 r}{R} \iiint dv = \frac{\rho_0 r}{R} \times \frac{4}{3}\pi r^3$$

$$= \frac{\rho_0 r}{R} \times \frac{4}{3}\pi r^3$$

$$D 4\pi r^2 = \frac{\rho_0 r}{R} \frac{4}{3}\pi r^3$$

$$\boxed{D = \frac{\rho_0 r^2}{3R}}$$

$$D = \varepsilon_0 \cdot E$$

$$E = D/\varepsilon_0 = \underline{\underline{\frac{\rho_0 r^2}{3R\varepsilon_0}}}$$

case-(2) point on the spherical symmetry

$$\iint_S D \cdot ds = Q = \iiint \rho_v \cdot dv$$

$$\iint_S D \cdot ds = \iint_S D \cdot ds = D \iint_S ds$$

$$= D\, 4\pi r^2$$

$$\iiint_v \frac{\rho_0 r}{R} \cdot dv \qquad r = R$$

$$\cancel{\iiint_v \rho_0 R}$$ then on the surface $r = R$.

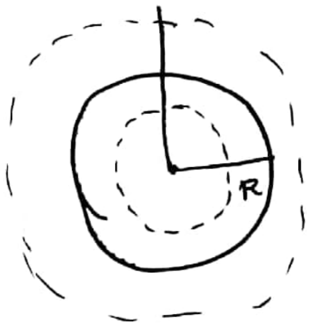
$$D = \frac{\rho_0 R^2}{3R}$$

$$E = \frac{\rho_0 R}{3\varepsilon_0}$$

case-(3)

outside the sphere.

$$\rho_v = \frac{\rho_0 r}{R}$$



$$\iint_S D \cdot ds = Q = \iiint_v \rho_v \cdot dv$$

$$\iint_S D \cdot ds = Q$$

$$D \iint_S ds = Q$$

$$D\, 4\pi r^2 = \frac{\rho_0 r}{R} \times \frac{4}{3}\pi R^3 \qquad \text{for } r = R.$$

$$D = \frac{\rho_0 R^2}{\;} = \rho_0\, \frac{4}{3}\pi R^3$$

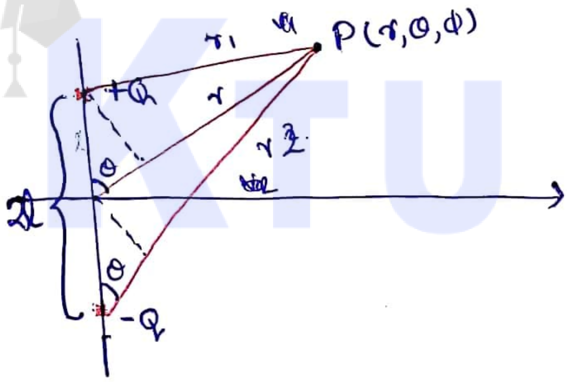
$$D = \frac{1}{4\pi r^2} \rho_0 \, \tfrac{4}{3}\pi R^3$$

$$= \frac{\rho_0 R^3}{3r^2} \qquad \text{then } D = \varepsilon_0 \cdot E$$

$$E = \frac{\rho_0 R^3}{3\varepsilon_0 r^2} \qquad \text{outside the sphere}$$

$$\vec{E} = \frac{\rho_0 R^3}{3\varepsilon_0 r^2}\, \hat{a_r}$$

Potential due to a Electric Dipole



Total potential at point 'P' can be determined by

$$V_P = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{+Q}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{-Q}{r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 \cdot r_2}\right]$$

let r_1 can be written as

$$r_1 = r - AB$$
$$= r - l\cos\theta$$

$$r_2 = r + l\cos\theta$$

$$x^n = \frac{x^{n+1}}{n+1}$$

$$r^{-2} \quad \frac{d}{dx} x^n = nx^{n-1}$$

$$-2r^{-2-1}$$
$$-2r^{-3}$$

then

$$V_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$\boxed{V_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{2l\cos\theta}{r^2}\right]}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{P\cos\theta}{r^2}\right)$$

$$\vec{E} = -\nabla V$$

$$= \frac{\partial V}{\partial r}\hat{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a_\theta} + \frac{1}{r\sin\theta}\hat{a_\phi}$$

$$= -\left[\frac{\partial}{\partial r}\frac{Q}{4\pi\varepsilon_0}\left[\frac{2l\cos\theta}{r^2}\right] + \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{2l\cos\theta}{r^2}\right]\hat{a_\theta} + \cancel{\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\frac{Q}{4\pi\varepsilon_0}\frac{2l\cos\theta}{r^2}}\right]$$

$$= \cancel{\frac{Q}{4\pi\varepsilon_0}}\, 2 \times -2\cancel{r^{-3}}\, l\cos\theta$$

$$= \frac{P\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) + \frac{P}{4\pi\varepsilon_0 r^3}\frac{\partial}{\partial\theta}\cos\theta + \frac{P\cos\theta}{r^3 4\pi\varepsilon_0 \sin\theta}$$

$$\Rightarrow \cancel{\frac{P\cos\theta}{4\pi\varepsilon_0}} \Rightarrow \frac{-2lQ}{4\pi\varepsilon_0}\left[\cos\theta\,\frac{-2}{r^3}\hat{a_r} + \frac{1}{r}\times\frac{1}{r^2} - \sin\theta\,\hat{a_\theta} + 0\right]$$

$$= \frac{-P}{4\pi\varepsilon_0}\cdot\left[-\frac{2}{r^3}\cos\theta\,\hat{a_r} - \frac{1}{r^3}\sin\theta\,\hat{a_\theta}\right]$$

Equipotential Surface

Surface with same potential at every where is called equipotential surface

1. workdone to move charge in Equipotential surface becomes zero ($Q \times V_{AB} = W$), $V_{AB}=0$, then $Q=0$
2. Two equipotential surface never intersect each other
3. $\vec{E}$ is allway normal to the surface.

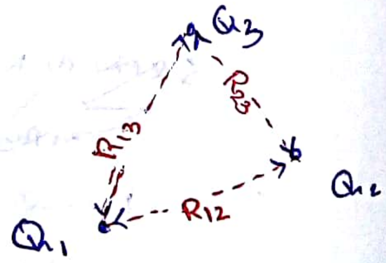
Electrostatic potential energy

let $V = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{r}$

then workdone $W = V \cdot Q_2$

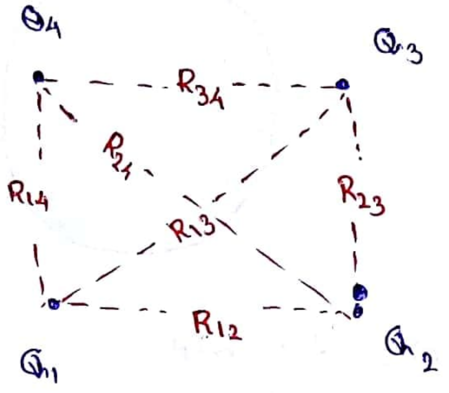
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{R_{12}}$$

If three charges then



$$W = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 \cdot Q_2}{R_{12}} + \frac{1}{4\pi\varepsilon_0} \frac{Q_1 \cdot Q_3}{R_{13}} + \frac{1}{4\pi\varepsilon_0} \frac{Q_2 \cdot Q_3}{R_{23}}$$

If we have four charges



$$W = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R_{12}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2 \cdot Q_3}{R_{23}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_3 Q_4}{R_{34}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_4 \cdot Q_1}{R_{14}}$$

$$+ \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_3 \cdot Q_1}{R_{13}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_4 \cdot Q_2}{R_{24}}$$

If we have n charges

$$\boxed{W_E = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{Q_i Q_j}{4\pi\varepsilon_0 \cdot R_{ij}}}$$

$i=3$

$$W_E = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0} \left[\sum_{j=1}^{n} \frac{Q_1 \cdot Q_j}{R_{1j}} + \sum_{j=1}^{n} \frac{Q_2 \cdot Q_j}{R_{2j}} + \sum_{j=1}^{3} \frac{Q_3 \cdot Q_j}{R_{3j}} \right.$$

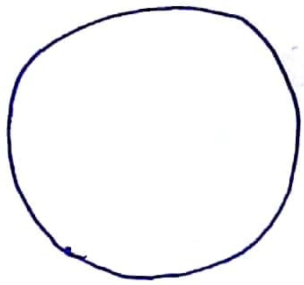
$$= \frac{Q_1 \cdot Q_2}{R_{12}} + \frac{Q_2 \cdot Q_3}{R_{23}} + \frac{Q_2 \cdot Q_1}{R_{21}} + \frac{Q_2 \cdot Q_3}{R_{23}} + \frac{Q_3 \cdot Q_1}{R_{31}}$$

$$+ \frac{Q_3 \cdot Q_2}{R_{32}}$$

(4) By applying gausses law the value of $\vec{E}$ by the following cases

Scanned by CamScanner

(1) Hollow sphere

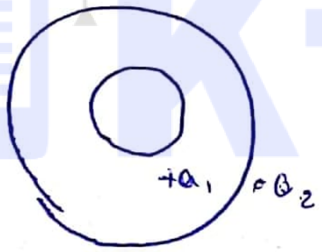


(2) Solid sphere. - uniformly charged

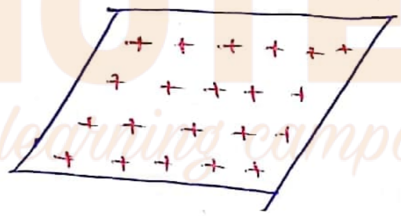
(3) Infinity long time.

(4) charged cylinder

(5) concentric shell of charge

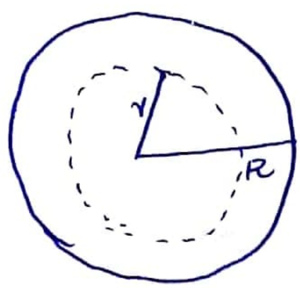


(6) plane sheet of charge



(1) Hollowsphere

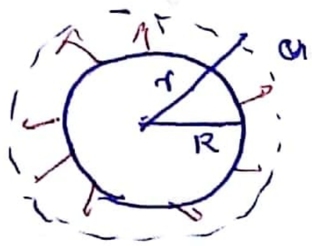
(I) point lies within the sphere (Q = 0)



$$\iint_s D \cdot ds = 0$$

$$\vec{E} = 0$$

II point lies outside the sphere



$$\iint_s D \cdot ds = Q$$

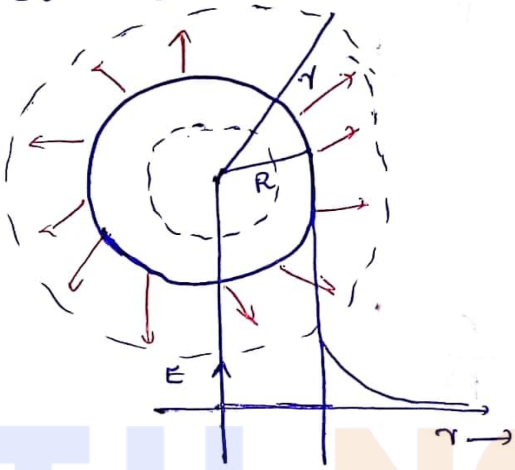
$$D \cdot 4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\hat{a_r}$$

(iii) point on the Surface

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}\,\hat{a_r}$$



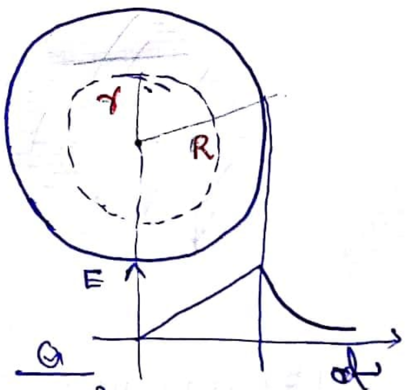
(2) uniformly charged solid sphere

(1) point lies within the sphere

$$\iint_s D\cdot ds = 0$$

$$\Rightarrow \cancel{D\,4\pi r^2 = Q}$$

$$E \cancel{= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}}\,\hat{a_r}$$



$$\rho_v = \frac{Q}{\frac{4}{3}\pi R^3}$$

charge = ρ_v × volume.

$$Q_{en} = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$\Rightarrow Q_{en} = \frac{Q r^3}{R^3}$$

(2) ~~point on the outside~~ the sphere

$$\cancel{\iint_s D\cdot ds = 0}$$

$$\cancel{E = 0}$$

(3) ~~point on~~ Surface.

$$\oint\!\!\oint_s D \cdot ds = Q_{en}$$

$$D \, 4\pi r^2 = \frac{Q r^3}{R^3}$$

$$D = \frac{Q}{4\pi} \frac{r}{R^3}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r}{R^3}}$$

(i) on the surface

$$r = R$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}}$$

(ii) point lies outside the sphere

$$D \cdot 4\pi r^2 = Q$$

$$D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

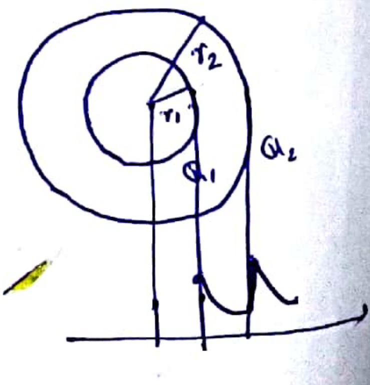
$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$

(5) concentric shell of charge

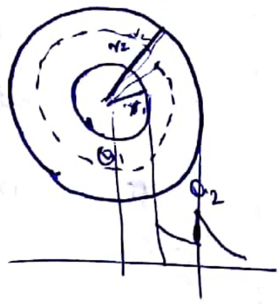
case (1) inside the shell

$$\oint\!\!\oint_s D \cdot ds = 0$$

$$\vec{E} = 0$$



Case (2) Between the shell



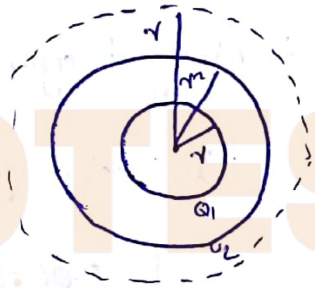
$$\iint_S D \cdot ds = Q_1$$

$$D \cdot 4\pi (r_2 - r_1)^2 = Q_1$$

$$D = \frac{Q_1}{4\pi (r_2 - r_1)^2}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{4\pi (r_2 - r_1)^2}}$$

Case (3) Outside the shell



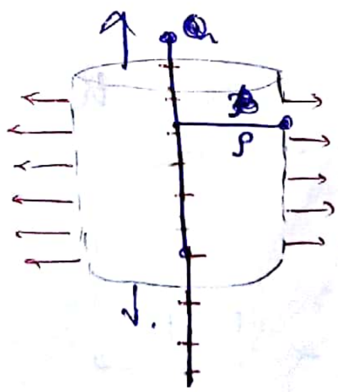
$$\iint_S D \cdot ds = Q_2 + Q_1$$

$$D \cdot 4\pi (r_2 + r_1)^2 = (Q_2 + Q_1)$$

$$D = \frac{(Q_2 + Q_1)}{4\pi (r_2 + r_1)^2}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Q_1 + Q_2)}{(r_2 + r_1)^2}} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 + Q_2}{r^2}}$$

(3) Infinitely long line.



$$\iint_S D \cdot ds = Q.$$

$$\cancel{\iint_{S\,(\text{upper})} D \cdot ds} + \cancel{\iint_{\text{lower}} D \cdot ds} + \iint_{\text{curved}} D \cdot ds$$

$$\iint_{\text{curved}} D \cdot \vec{ds} = 2\pi \rho l \times D = Q.$$

D =

$$\boxed{E = \frac{Q}{2\pi\varepsilon_0} \cdot \frac{1}{\rho l}}$$

$$D = \frac{Q}{2\pi\rho l}$$

Q in linear charge densit

$$Q = \rho_l \times l$$

$$D = \frac{\rho_l \times l}{2\pi\rho l}$$

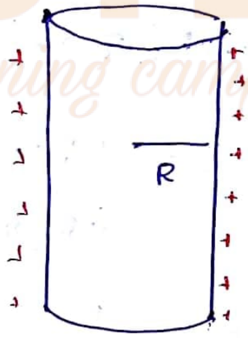
$$= \frac{\rho_l}{2\pi\rho}$$

$$\boxed{E = \frac{\rho_l}{2\pi\varepsilon_0 \cdot \rho}}$$

(4) Charged cylinder (hollow)

(i) Inside the cylinder

$$\iint_S D \cdot ds = 0$$



(ii) on the surface outside

$$\iint D \cdot ds = 2\pi\rho R \times D$$

$$D = \frac{Q}{2\pi R l}$$

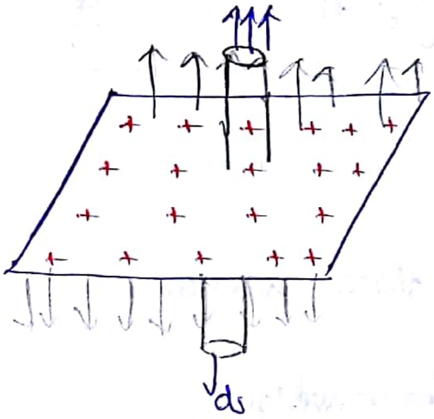
$$E = \frac{Q}{2\pi\varepsilon_0} \cdot \frac{1}{Rl}$$

$$\boxed{E = \frac{\rho_l}{2\pi\varepsilon_0 R}}$$ in the case of linear charge density

Surface - on the surface of cylinder take 'R' radius of cylinder

$$\boxed{E = \frac{\rho_l}{2\pi\varepsilon_0 \cdot R}}$$

(6) Plane sheet of charge



$$\iint_S D \cdot ds = Q$$

$$= \rho_s \cdot ds$$

$$\iint_{S\,(top)} D \cdot ds + \iint_{S\,(Bottom)} D \cdot ds = \rho_s \, ds$$

$$D \cdot ds + D \cdot ds = \rho_s \, ds$$

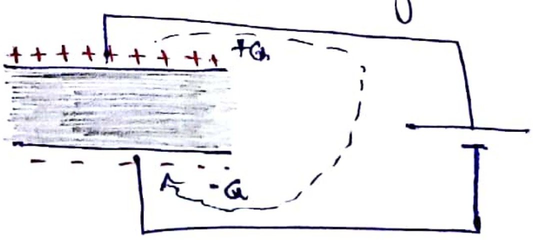
$$2 D \cdot ds = \rho_s \cdot ds$$

$$\boxed{D = \frac{\rho_s}{2}}$$

$$\boxed{E = \frac{\rho_s}{2\varepsilon_0}}$$

Capacitance

Two parallel plates separated by a dielectric is a capacitor.



The ability to store charge on plate

$$C = \frac{Q}{V}$$

let

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 \cdot Q_2}{R^2}$$

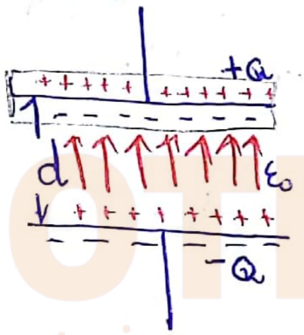
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^2}$$

$$V = -\int \vec{E} \cdot d\vec{l} \qquad C = \frac{Q}{V}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

(1) parallel plate capacitor

By applying gauss's law



$$\vec{E} = \frac{\cancel{1}}{4\pi\varepsilon_0}$$

$$\iint_S D \cdot ds = Q$$

$$D \iint_S ds =$$

$$DA = Q$$

$$D = \frac{Q}{A}$$

then

$$E = D/\varepsilon_0 \quad \text{then.}$$

$$= \frac{\cancel{\varepsilon_0} Q}{\cancel{A}} = \frac{Q}{A\varepsilon_0}.$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$V = E\,d$$

$$= \cancel{\frac{\varepsilon_0 \cdot Q}{A}} \times d \; :$$

$$= E\int dl$$

$$C = \frac{Q}{V}$$

$$\cancel{V = \varepsilon_0} \cdot =$$

$$= \frac{Q}{E.d.}$$

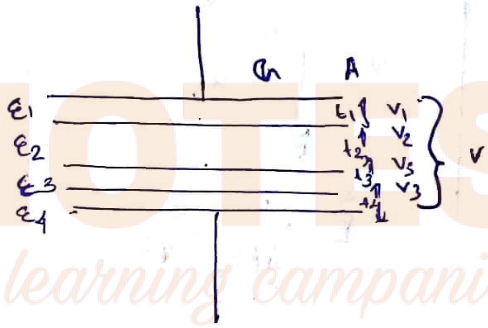
$$= \frac{Q}{\frac{Q}{A\varepsilon_0} \times d}$$

$$\boxed{C = \frac{A\varepsilon_0}{d}} \quad \text{or} \quad \boxed{C = \frac{\varepsilon_0 \cdot \varepsilon_r \cdot A}{d}}$$

(ii) for composite medium

$$DA = Q$$

$$D = \frac{Q}{A}$$



$$\Rightarrow V = V_1 + V_2 + V_3 + V_4$$

$$C = \frac{Q}{V} \qquad V = E \cdot d$$

$$V_1 = E_1 t_1 \qquad V_2 = E_2 t_2 \qquad V_3 = E_3 t_3$$

$$\Rightarrow V = E_1 t_1 + E_2 t_2 + E_3 t_3 + E_4 t_3$$

$$V = D/\varepsilon_1\, t_1 + D/\varepsilon_2\, t_2 + D/\varepsilon_3\, t_3 + D/\varepsilon_4\, t_4$$

$$= \frac{Q}{A\varepsilon_1} t_1 + \frac{Q}{A\varepsilon_2} t_2 + \frac{Q}{A\varepsilon_3} t_3 + \frac{Q}{A\varepsilon_4} t_4$$

$$= Q/A \left[t_1/\varepsilon_1 + t_2/\varepsilon_2 + \frac{t_3}{\varepsilon_2} + t_4/\varepsilon_4 \right]$$

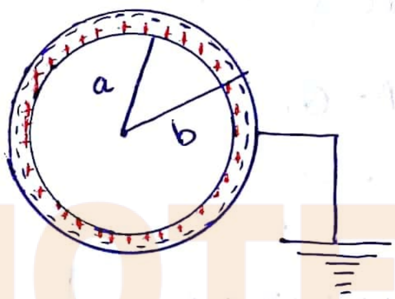
$$C = \frac{Q}{V}$$

$$C = \frac{A}{t_1/\varepsilon_1 + t_2/\varepsilon_2 + t_3/\varepsilon_3 + t_4/\varepsilon_4}$$

$$\boxed{C = \frac{A}{t_1/\varepsilon_1 + t_2/\varepsilon_2 + t_3/\varepsilon_3 + t_4/\varepsilon_4}}$$

(iii) Spherical capacitor

total charge on the sphere is Q



$$C = \frac{Q}{V}$$

$$V = -\int_b^a E \cdot dl$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon_0} \frac{dr}{r^2}$$

$$-\frac{1}{r}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{V} = \frac{Q}{Q/4\pi\varepsilon_0 \cancel{(a-b)} \left(\frac{1}{a} - 1/b\right)}$$

$$\cancel{C = 4\pi\varepsilon_0 (a-b)} \qquad 4\pi\varepsilon_0 [1/a - 1/b]$$

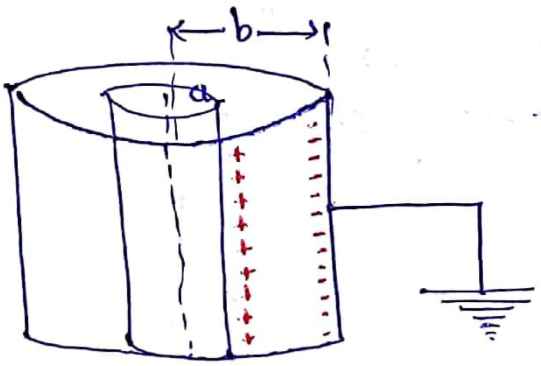
$$\boxed{C = 4\pi\varepsilon_0 / [1/a - 1/b]}$$

$$\Rightarrow \boxed{C = \frac{4\pi\varepsilon_0}{[1/a - 1/b]}}$$

Single spher capacitor

$$\boxed{C = 4\pi\varepsilon_0 a}$$

(v) co-axial cable.



$$C = \frac{Q}{V}$$

$$D \cdot 2\pi\rho l = Q.$$

$$E = \frac{Q}{2\pi\rho l\varepsilon_0} a_\rho$$

$$V = -\int_b^a E \cdot dl$$

$$= -\int_b^a \frac{Q}{2\pi\rho l\varepsilon_0} \cdot a_\rho \cdot d\rho$$

$$= \frac{-Q}{2\pi l\varepsilon_0} \int_b^a \frac{1}{\rho} \cdot dl$$

$$= \frac{Q}{2\pi l\varepsilon_0} \left[\ln \rho\right]_b^a$$

$$= \frac{Q}{2\pi\varepsilon_0 l} \cdot \ln(b/a)$$

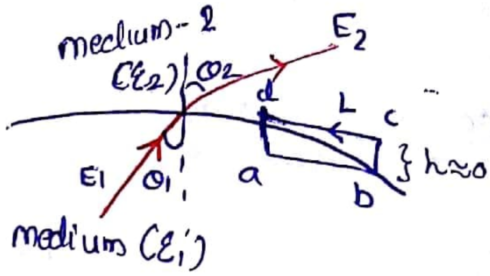
$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi l\varepsilon_0} \times \ln(b/a)}$$

$$C = \frac{2\pi l\varepsilon_0}{\ln(b/a)}$$

ELECTRIC BOUNDARY CONDITIONS

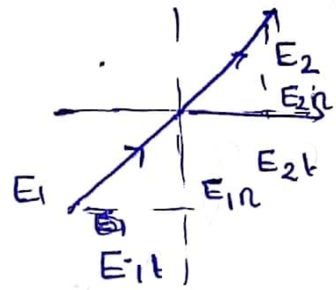
- Here analyse the parameters 'E' and 'D' By using conservative property

$$\oint E \cdot dl = 0$$



$$\int_a^b E \cdot dl + \int_b^c \cancel{E \cdot dl} + \int_c^d E \cdot dl + \int_d^a \cancel{E \cdot dl} = 0$$

$$= \int_a^b E_1 \cdot dl + \int_c^d E_2 \cdot dl$$



$$= \int_a^b (E_{1t} + E_{1n}) \cdot dl + \int_c^d (E_{2t} + E_{2n}) \cdot dl = 0$$

$$= \int_a^b E_{1t} \cdot dl - \int_c^d E_{2t} \cdot dl = 0$$

$$= \int_a^b E_{1t} \cdot dl - \int_c^d E_{2t} \cdot dl = 0$$

$$E_{1t} [l]_a^b - E_{2t} [l]_c^d = 0$$

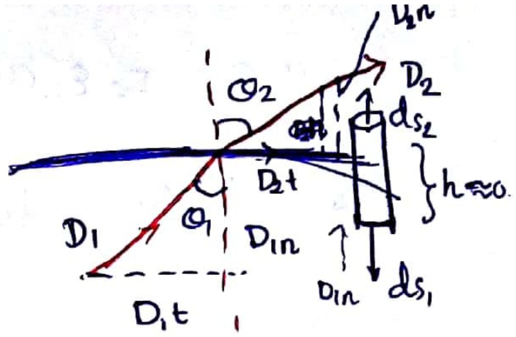
$$E_{1t}\, l - E_{2t}\, l = 0$$

$$E_{1t} - E_{2t} = 0$$

$$E_{1t} = E_{2t}$$

, these are tangential components and here tangential components of electric field intensity vectors are continuous at the surface.

$\iint D \cdot ds = Q$, By applying the gauss's law



$$\iint_{S\,(top)} D_2 \cdot ds + \iint_{S\,(bottom)} D_1 \cdot ds = Q$$

$$\iint_{S\,(top)} (D_2t + D_2n) \cdot ds + \iint_{S\,(Bottom)} (D_1t + D_1n) \cdot ds = Q$$

$$\iint_{(top)\,S} D_2n \, ds_2 - \iint_{S\,(bottom)} D_1n \cdot ds_1 = Q$$

Here Q will be

$Q = \rho_s \cdot ds$

$= D_2n\,ds - D_1n\,ds = \rho_s \cdot ds$

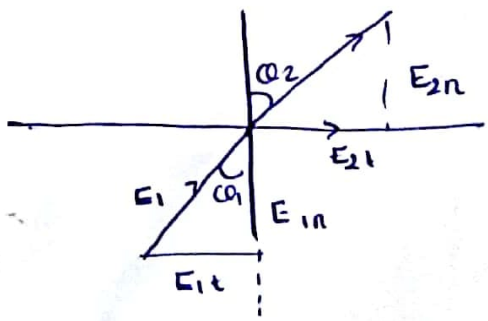
$D_2n - D_1n = \rho_s$

If the $\rho_s = 0$, then

$D_{1n} = D_{2n}$

at the interface the normal components of flux density ~~(D)~~ vectors are continuous if $\rho_s = 0$.

Law of Refraction:



consider $E_1t = E_2t$ —①

$D_1n = D_{2n}$ —②

here

① ⇒ $E_1 \sin\theta_1 = E_2 \sin\theta_2$ —④

② ⇒ $D_1 \cos\theta_1 = D_2 \cos\theta_2$ —⑤ ③

$$\varepsilon_1 E_1 \cos\theta_1 = E_2 \varepsilon_2 \cos\theta_2 \quad (4)$$

$$\frac{(3)}{(4)} = \frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$\therefore$ $\boxed{\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}}$ is called law of refraction.

There is no tangential components in the case of the medium becomes conductor and Dielectric. That is one of the medium is conductor $\therefore E_{1t} = 0$