HEAT AND MASS TRANSFER

(For IV Sem B.E. Mechanical Engineering Students)

(As per Anna University New Syllabus)

Dr. S.Ramachandran, B.E., M.E., Ph.D.,
Professor and Head

A. Anderson, M.E., (Ph.D)
G. Senthil Kumar, M.E., (Ph.D)
Associate Professors
Department of Mechanical Engineering

SATHYABAMA UNIVERSITY
Jeppiaar Nagar, Chennai - 600 119

AIR WALK PUBLICATIONS
(Near All India Radio)
80, Karneeshwarar Koil Street
Mylapore, Chennai - 600 004.
Ph.: 2466 1909, 94440 81904
Email: aishram2006@gmail.com,
airwalk800@gmail.com
www.airwalkpublications.com

Downloaded from Ktunotes.in

First Edition: September 2001
Second Edition: October 2007
Third Edition: Novermber-16-2012

© All Rights Reserved by the Publisher

This book or part thereof should not be reproduced in any form without the written permission of the author and publisher.

ISBN: 978-81-924031-1-3

Online purchasing can be done through our website:

www.airwalkpublications.com

(or) through

amazon.com

Books will be door delivered after payment into **AIR WALK PUBLICATIONS A/c No. 801620100001454 (IFSC: BKID0008016)** Bank of India, Santhome branch, Mylapore, Chennai - 4 (or)

S.Ramachandran, A/c.No.482894441 (IFSC:IDIB000S201), Indian Bank, Sathyabama University Branch, Chennai - 600119.

Typeset by: aksharaa muthra aalayam, Chennai - 18. Ph.: 044-2436 4303

Downloaded from Ktunotes.in

ME6502 Heat and Mass Transfer

Syllabus

UNIT I Conduction 9

General Differential equation of Heat Conduction– Cartesian and Polar Coordinates – One Dimensional Steady State Heat Conduction — plane and Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis –Semi Infinite and Infinite Solids –Use of Heisler's charts.

UNIT II Convection 9

Free and Forced Convection - Hydrodynamic and Thermal Boundary Layer. Free and Forced Convection during external flow over Plates and Cylinders and Internal flow through tubes .

UNIT III Phase Change Heat Transfer and Heat Exchangers 9

Nusselt's theory of condensation - Regimes of Pool boiling and Flow boiling. Correlations in boiling and condensation. Heat Exchanger Types - Overall Heat Transfer Coefficient – Fouling Factors - Analysis – LMTD method - NTU method.

UNIT IV Radiation 9

Black Body Radiation – Grey body radiation - Shape Factor – Electrical Analogy – Radiation Shields. Radiation through gases.

UNIT V Mass Transfer 9

Basic Concepts – Diffusion Mass Transfer – Fick?s Law of Diffusion – Steady state Molecular Diffusion – Convective Mass Transfer – Momentum, Heat and Mass Transfer Analogy –Convective Mass Transfer Correlations.

Total : 45

Downloaded from Ktunotes.in

CONTENTS

Chapter - I
CONDUCTION

Downloaded from Ktunotes.in

Downloaded from Ktunotes.in

Chapter - II
CONVECTION

Downloaded from Ktunotes.in

Downloaded from Ktunotes.in

Chapter - III
PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

Downloaded from Ktunotes.in

Chapter IV
RADIATION

Downloaded from Ktunotes.in

Downloaded from Ktunotes.in

Chapter - V
MASS TRANSFER

Downloaded from Ktunotes.in

Downloaded from Ktunotes.in

Chapter - I

CONDUCTION

1.1 INTRODUCTION

Heat transfer is defined as the transfer of heat from one region to another by virtue of the temperature difference between them. The devices for transfer of heat are called heat exchangers. The concept of heat transfer is necessary for designing heat exchangers like boilers, evaporators, condensers, heaters and many other cooling and heating systems.

1.2 MODES OF HEAT TRANSFER

There are three modes of Heat transfer as follows:

1. Conduction
2. Convection
3. Radiation.

1.3 CONDUCTION

Heat is always transferred by conduction from high temperature region to low temperature region. The conduction heat transfer is due to the property of matter

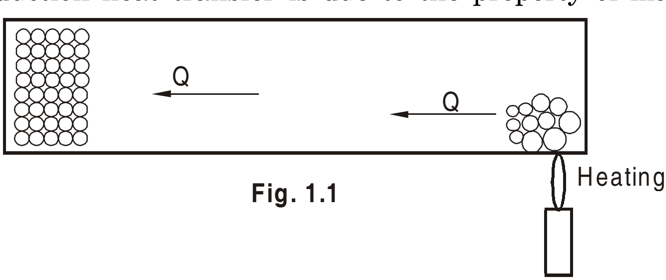


Fig. 1.1

Downloaded from Ktunotes.in

and molecular transport of heat between two regions due to temperature difference.

When one end of a rod gets heated, the atoms in that end get enlarged and vibrated due to heating. The enlarged, vibrated atoms touch the adjacent atom and heat is transferred. Similarly, all the atoms are heated, thereby the heat is transferred to the other end. This type of heat transfer is called as conduction heat transfer.

In solids, heat is conducted by

1. Atomic vibration – The faster moving, vibrating atoms in the hot area transfer heat to the adjacent atoms.
2. By transport of free electrons.

Heat is also conducted in liquid and gases by the following mechanism.

1. The kinetic energy (K.E) of a molecule is a function of temperature. When these molecules temperature increases, the K.E. increases.

2. The molecule from the high temperature region collides with a molecule from the low temperature region and thus heat is transferred.

1.4 CONVECTION

The heat transfer between a surface and the surrounding fluid which are at different temperatures, is called convection heat transfer. Convection heat transfer is defined as a process of heat transfer by the combined action of **heat conduction** and **mixing motion**.

Downloaded from Ktunotes.in

(i) First of all, heat is transferred from hot surface to adjacent fluid purely by **conduction**.

(ii) Then, the hot fluid's density decreases by increase in temperature. This hot fluid particles move to low temperature region and mix with cold fluid and thus transfer heat by **mixing motion**.

If the mixing motion of fluid particles takes place due to density difference caused by temperature difference, then this convection heat transfer is called **free convection (or) natural convection**.

If the motion of fluid particles is due to fan (or) pump (or) blower (or) any external means, then this convection heat transfer is called **forced convection**.

1.5 RADIATION

Conduction and convection needs a medium for heat transfer, but radiation heat transfer takes place even in vacuum.

Radiation heat transfer occurs when the hot body and cold body are separated in space. The space may be filled up by a medium (or) vacuum.

Energy, emitted in the form of electromagnetic waves, by all bodies due to their temperatures is called thermal radiation.

1.6 CONDUCTION

Most of the heat transfer problems involve a combination of all the three modes of heat transfer. But it will be useful, if we study each mode of heat transfer one by one. Hence, in the forth coming section, we can

Downloaded from Ktunotes.in

study conduction, convection and radiation separately and in some cases we can study with combination.

1.7 FOURIER'S LAW OF HEAT CONDUCTION

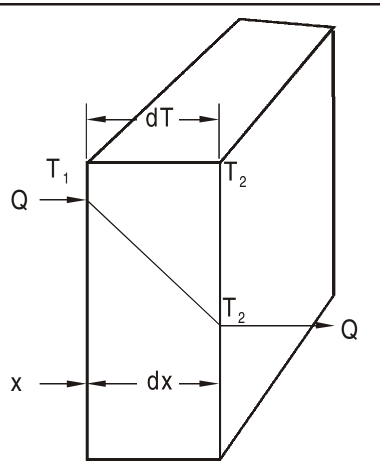
Fourier's law states that the Conduction heat transfer through a solid is **directly proportional to**

1. The area of section (A) at right angle to the direction of heat flow.
2. The change in temperature (dT) in between the two faces of the slab and
3. **Indirectly proportional to** the thickness of the slab (dx).

$$Q \cdot \propto A \frac{dT}{dx}$$

where Q = heat conducted in (Watts) W.

A = surface area of heat flow in m^2. (perpendicular to the direction of heat flow)



Plane wall or Slab **Fig. 1.2**

Downloaded from Ktunotes.in

dT = temperature difference between the faces of the slab in °C or K

dx = thickness of the slab in m.

$$\text{So, } Q = -kA\frac{dT}{dx}$$

Here dT is negative. Because $dT = T_2 - T_1$. (Change in temp.) Since T_2 is less than T_1, dT is negative.

So we get the equation

$$Q = -kA\frac{(T_2 - T_1)}{dx} = kA\frac{(T_1 - T_2)}{dx}$$

Here k = Constant of proportionality and is called **thermal conductivity of the material.**

$$Q = kA\frac{(T_1 - T_2)}{dx}$$

$$\text{So, } k = \frac{Q \times dx}{A\,(T_1 - T_2)} = \frac{Q}{\left(\frac{A\,dT}{dx}\right)} = \frac{W}{\left(\frac{m^2 \times °C}{m}\right)} = \text{W/m°C}$$

So the unit of k is W/m°C (or) W/mk

Assumptions

The Fourier's law is based on the following assumptions.

1. Conduction heat transfer takes place under steady state condition.
2. The heat flow is one direction only.
3. The temperature gradient 'dT' is constant.
4. The temperature profile in linear.

Downloaded from Ktunotes.in

5. No internal heat generation.
6. The thermal conductivity k is constant in all directions.

1.8 GENERAL DIFFERENTIAL EQUATION OF HEAT CONDUCTION - CARTESIAN COORDINATES

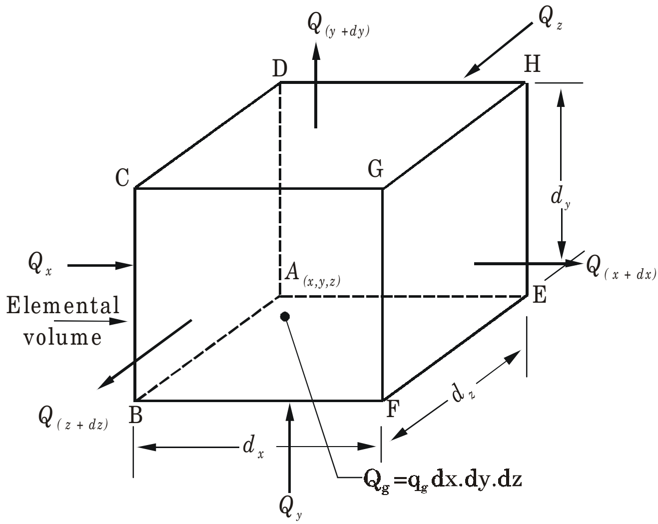


Fig. 1.3 Elemental volume for three-dimensional heat conduction analysis - Cartesian coordinates

Consider an infinitesimal rectangular element of sides dx, dy and dz as shown in **Fig. 1.3**.

Q_x = Rate of heat flow in x direction through the face $ABCD$

Q_{x+dx} = Rate of heat flow in x direction through the face $EFGH$

Downloaded from Ktunotes.in

$$q_x = \text{Heat flux}\left(\frac{Q_x}{A}\right) \text{in } x \text{ direction}$$

through face $ABCD$

$$q_{x+dx} = \text{Heat flux}\left(\frac{Q_x + dx}{A}\right) \text{in } x \text{ direction}$$

through face $EFGH$

kx, ky, kz = Thermal conductivities along x, y and z axes

$$\frac{\partial T}{\partial x} = \text{Temperature gradient in } x \text{ direction}$$

The differential equation of conduction can be derived based on the law of conservation of energy (or) the first law of Thermodynamics. Let us apply the first law of thermodynamics to the control volume of **Fig. 1.3**.

$$\begin{bmatrix} \text{Quantity of} \\ \text{heat conducted} \\ \text{to the} \\ \text{elementary} \\ \text{volume in} \\ \text{face } ABCD \\ Q_x \end{bmatrix} + \begin{bmatrix} \text{Heat} \\ \text{generated} \\ \text{from inner} \\ \text{heat source} \\ \text{with in the} \\ \text{element} \\ Q_g \end{bmatrix} = \begin{bmatrix} \text{Change in} \\ \text{enthalpy} \\ \text{of element} \\ \dfrac{dh}{dt} \end{bmatrix} + \begin{bmatrix} \text{Work done} \\ \text{by} \\ \text{element} \\ W \end{bmatrix} \quad \text{...(1.1)}$$

The work done by an element is small and can be neglected in the above equation.

Hence, the above equation can be written as

$$Q_x + Q_g = \frac{dh}{dt} + Q_x + dx \quad \text{...(1.2)}$$

Now let us see one by one.

Downloaded from Ktunotes.in

Qx: Quantity of heat conducted to the elementary volume

The rate of heat flow in to the element in x direction through the face $ABCD$ is

$$\boxed{Q_x = q_x\, dy\, dz = -\, k_x \frac{\partial T}{\partial x}\, dy\, dz} \qquad ...(1.3)$$

The rate of heat flow out of the element in x direction through the face $EFGH$ is

$$Q_{x\,+\,dx} = Q_x + \frac{\partial}{\partial x}\,(Q_x)\, dx$$

$$= -\, k_x \frac{\partial T}{\partial x}\, dy\, dz + \frac{\partial}{\partial x}\left[-\, k_x \frac{\partial T}{\partial x}\, dy\, dz\right] dx$$

$$\boxed{Q_{x\,+\,dx} = -\, k_x \frac{\partial T}{\partial x}\, dy\, dz - \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] dx\, dy\, dz} \qquad ...(1.4)$$

$Q_x - Q_{x\,+\,dx}$ gives

$$Q_x - Q_{(x\,+\,dx)} = -\, k_x \frac{\partial T}{\partial x}\, dy\, dz - \left[-\, k_x \frac{\partial T}{\partial x}\, dy\, dz - \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] dx\, dy\, dz\right]$$

$$= -\, k_x \frac{\partial T}{\partial x}\, dy\, dz + k_x \frac{\partial T}{\partial x}\, dy\, dz + \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] dx\, dy\, dz$$

$$\Rightarrow Q_x - Q_{(x\,+\,dx)} = \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] dx\, dy\, dz \qquad ...(1.5)$$

Similarly

Downloaded from Ktunotes.in

$$Q_y - Q_{(y+dy)} = \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] dx\, dy\, dz \qquad ...(1.6)$$

$$Q_z - Q_{(z+dz)} = \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] dx\, dy\, dz \qquad ...(1.7)$$

Add (1.5) + (1.6) + (1.7)

$$\left.\begin{array}{r}\text{Total heat conducted}\\ \text{in all direction}\end{array}\right\} = \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] dx\, dy\, dz +$$

$$\frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] dx\, dy\, dz + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] dx\, dy\, dz$$

$$= \left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right] dx\, dy\, dz$$

Total heat conducted into the element from all directions

$$= \left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right] dx\, dy\, dz \qquad ...(1.8)$$

Change in enthalpy of the element $\left(\frac{dh}{dt}\right)$

We know that,

$$\left\{\begin{array}{l}\text{Change in}\\ \text{enthalpy}\\ \text{of the}\\ \text{element}\end{array}\right\} = \left\{\begin{array}{l}\text{Mass}\\ \text{of the}\\ \text{element}\end{array}\right\} \times \left\{\begin{array}{l}\text{Specific}\\ \text{heat}\\ \text{of the}\\ \text{element}\end{array}\right\} \times \left\{\begin{array}{l}\text{Rise in}\\ \text{temperature}\\ \text{of element}\end{array}\right\}$$

$$= m \times C_p \times \frac{\partial T}{\partial t}$$

$$= (\rho \times dx\, dy\, dz) \times C_p \times \frac{\partial T}{\partial t}$$

[$\because$ Mass = Density × Volume]

Downloaded from Ktunotes.in

$$\boxed{\left\{\begin{array}{l}\textbf{Change in enthalpy of}\\ \textbf{the element}\end{array}\right\} = \rho\, C_p \frac{\partial T}{\partial t}\, dx\, dy\, dz} \quad ...(1.9)$$

Heat generated from inner heat source with in the element Q_g

Heat generated within the element is given by

$$Q_g = q_g\, dx\, dy\, dz \quad ...(1.10)$$

Substituting equation (1.8), (1.9) and (1.10) in equation (1.2)

$$\left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right] dx\, dy\, dz$$

$$+ q_g\, dx\, dy\, dz = \rho\, C_p \frac{\partial T}{\partial t}\, dx\, dy\, dz$$

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] + q_g = \rho\, C_p \frac{\partial T}{\partial t}$$

When the material is isotropic,

$k_x = k_y = k_z = k =$ constant

$$\Rightarrow k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + q_g = \rho\, C_p \frac{\partial T}{\partial t}$$

Divided by k,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{\rho\, C_p}{k} \frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}} \quad ...(1.11)$$

Downloaded from Ktunotes.in

$$\nabla^2 T + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{...(1.11)}$$

It is a general three dimensional heat conduction equation in cartesian coordinates

where, $\alpha =$ Thermal diffusivity $= \dfrac{k}{\rho\, C_p}$

Qx

Case (i)

When no internal heat generation is present

ie when $q_g = 0$, then the equation 1.11 becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{(Fourier equation)} \quad \text{...(1.12)}$$

Case (ii)

In steady state conditions, the temperature does not change with respect to time. Then the conduction takes place in the steady state ie $\dfrac{\partial T}{\partial t} = 0$. Hence the equation 1.11 becomes

$$\nabla^2 T + \frac{q_g}{k} = 0 \quad \text{(Poisson's equation)} \quad \text{...(1.13)}$$

Case (iii)

No heat generation; steady state conditions. Then the equation 1.11 becomes,

Downloaded from Ktunotes.in

$$\nabla^2 T = 0 \quad \text{(Laplace equation)} \qquad ...(1.14)$$

Case (iv)

Steady state, one-dimensional heat transfer,

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = 0 \qquad ...(1.15)$$

Case (v)

Steady state, one dimensional, without internal heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0 \qquad ...(1.16)$$

1.9 GENERAL DIFFERENTIAL EQUATION OF HEAT CONDUCTION - CYLINDRICAL COORDINATES

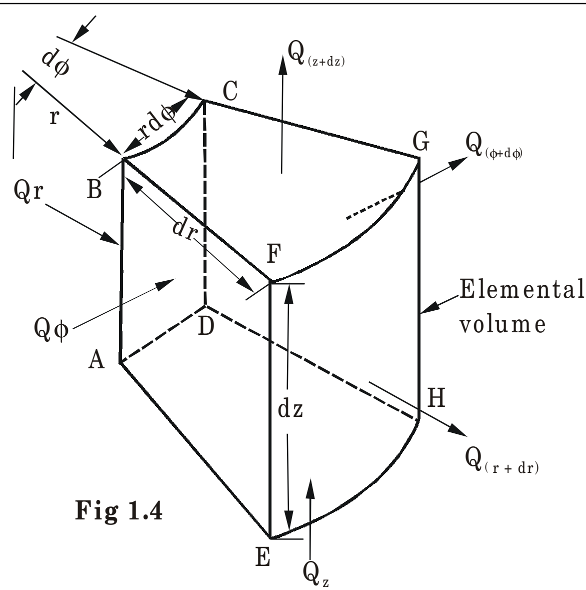


Fig 1.4

Downloaded from Ktunotes.in

The heat conduction equation in cartesian coordinates can be used for rectangular solids like slabs, cubes, etc. But for cylindrical shapes like rods and pipes, it is convenient to use cylindrical coordinates. **Fig. 1.4** shows a cylindrical coordinate system for general conduction equation.

Q_r = Heat conducted to the element in the 'r' direction through left face $ABCD$

Q_g = Heat generated with in the element

$\frac{dh}{dt}$ = Change in ethalpy per unit time

Q_{r+dr} = Heat conducted out of the element in 'r' direction through the right face $EFGH$

By applying I law of thermodynamics,

$$Q_r + Q_g = \frac{dh}{dt} + Q_{r+dr} \quad ...(1.17)$$

$$Q_r = q_r A = -k A \frac{\partial T}{\partial r} - k (r \cdot d\phi \cdot dz) \frac{\partial T}{\partial r} \quad ...(1.18)$$

Where A = area of element = $r \cdot d\phi \cdot dz$

$$Q_g = q_g (dr \cdot r\, d\phi \cdot dz) \quad ...(1.19)$$

$\frac{dh}{dt}$ = mass of the element × specific heat × change in temperature of the element in time dt

$$= [\rho (dr \cdot rd\, \phi \cdot dz)] \times c_p \times \frac{\partial T}{\partial t} \quad ...(1.20)$$

Downloaded from Ktunotes.in

$$Q_{r+dr} = q_r\,A + \left[\frac{\partial}{\partial r}(q_r\,A)\right]dr = -\,kA\,\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(-\,kA\,\frac{\partial T}{\partial r}\right)dr \qquad \text{...(1.21)}$$

where

k = thermal conductivity of the material in the r – direction

$\partial T/\partial r$ = temperature gradient in the r – direction

q_r = heat flux in the r – direction at r, i.e. at left face, i.e. at $ABCD$ (W/m^2)

q_g = internal energy generated per unit time and per unit volume (W/m^3)

ρ = density of the material (kg/m^3)

$(\partial T/\partial r)\,dr$ = change in temperature through distance dr

Substituting Eqs. (1.18), (1.9), (1.20) and (1.21) in Eq. (1.17) we get

$$-\,kA\,\frac{\partial T}{\partial r} + q_g\,Adr = \rho\,c_p\,Adr\,\frac{\partial T}{\partial t} - \left[kA\,\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(kA\,\frac{\partial T}{\partial r}\right)dr\right]$$

$$k\,(d\phi \cdot dr \cdot dz)\,\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + q_g\,(r \cdot d\phi \cdot dz \cdot dr) = \rho \cdot c_p\,r \cdot dz \cdot dr \cdot d\phi\,\frac{\partial T}{\partial t}$$

$$k\left[r\,\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right] + q_g\,r = \rho \cdot c_p\,r\,\frac{\partial T}{\partial t}$$

Downloaded from Ktunotes.in

$$\left.\begin{array}{l}\left[\dfrac{\partial^2 T}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial T}{\partial r}\right]+\dfrac{q_g}{k}=\dfrac{\rho\, c_p}{k}\dfrac{\partial T}{\partial t}\\ \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right)+\dfrac{q_g}{k}=\dfrac{\rho\, c_p}{k}\dfrac{\partial T}{\partial t}\end{array}\right\} \quad ...(1.22)$$

Equation (1.22) is the one-dimensional cylindrical coordinate time-dependent equation for heat conduction with internal heat generation.

This Equation (1.22) can be reduced to different cases as follows:

Case 1: Steady state, one-dimensional heat transfer with internal heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q_g}{k}=0 \quad ...(1.23)$$

Case 2: Steady state, one-dimensional, without internal heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0 \quad ...(1.24)$$

Case 3: Unsteady state, one-dimensional, without heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad ...(1.25)$$

The three dimensional general heat conduction equation in cylindrical coordinates is given as

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial t^2}\right]+\frac{q_g}{k}=\frac{1}{\alpha}\cdot\frac{\partial T}{\partial t} \quad ...(1.26)$$

Downloaded from Ktunotes.in

1.20 Heat Conduction Through Plane Walls

Figure 1.85 shows the simplest heat transfer problem. It is one-dimensional, steady state conduction in a plane wall of homogeneous material having constant thermal conductivity and with each face held at a constant uniform temperature and without heat generation. The *y* and *z* directions heat transfers are neglected. Here, the temperature is only a function of *x*.

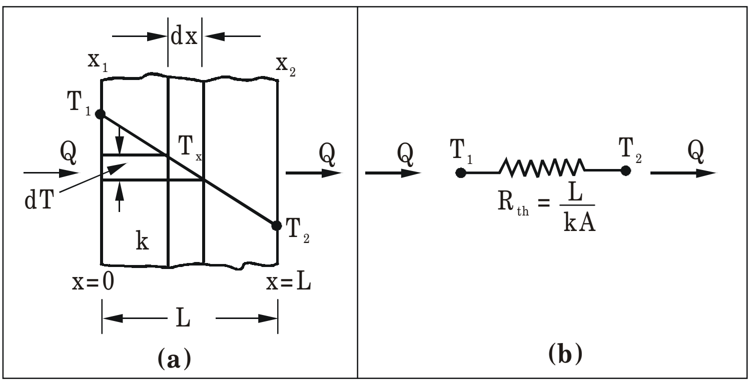


Fig. 1.5 Heat conduction through a plane wall: (a) mechanical system, (b) equivalent thermal resistance

Here,

L = thickness of the plane wall (m)

A = cross–sectional area perpendicular to the rate of heat transfer (m^2)

k = thermal conductivity of plane wall material (W/m – K)

T_1, T_2 = constant uniform temperature at $x = 0$, $x = L$, respectively (°C or K)

Downloaded from Ktunotes.in

The following equation gives the general heat conduction in Cartesian coordinates, i.e.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{...(1.27)}$$

Assumptions

Steady state, $\therefore \frac{\partial T}{\partial t} = 0$

One-dimensional, $\therefore \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial z^2} = 0$

No heat generation, $\therefore q_g = 0$

Substituting the above assumptions in Eq. (1.27),

$$\frac{\partial^2 T}{\partial x^2} = 0 \text{ or } \frac{d^2 T}{dx^2} = 0 \quad \text{...(1.28)}$$

Integrating Eq. (1.27) twice, we get

$$\frac{dT}{dx} = C_1 \text{ and } T = C_1 x + C_2 \quad \text{...(1.29)}$$

where, C_1 and C_2 are arbitrary constants.

The values of these constants can be obtained from the boundary conditions.

$$\text{At } x = 0, T = T_1; \text{ At } x = L, T = T_2$$

Substituting these conditions in Eq. (1.29), we get

$$T_1 = 0 + C_2; \therefore C_2 = T_1$$

$$T_2 = C_1 L + C_2 = C_1 L + T_1$$

$$\therefore C_1 = (T_2 - T_1)/L$$

Downloaded from Ktunotes.in

Substituting C_1 and C_2 in Eq. 1.29,

$$T = \frac{(T_2 - T_1)\,x}{L} + T_1 \qquad (1.30)$$

Equation (1.30) is the temperature distribution equation for one-dimensional, steady state, no heat generation in Cartesian coordinates. This equation shows that the temperature distribution is linear and is independent of thermal conductivity.

Differentiating Eq. (1.30) with respect to x,

$$\frac{dT}{dx} = \frac{d}{dx}\left[\frac{(T_2 - T_1)\,x}{L} + T_1\right] = \frac{T_2 - T_1}{L} \qquad ...(1.31)$$

Substituting Eq. (1.31) in Fourier equation, i.e.

$$Q = -\,kA\,\frac{dT}{dx} = -\,kA\,\frac{(T_2 - T_1)}{L} = \frac{kA\,(T_1 - T_2)}{L} \qquad ...(1.32)$$

or $$Q = \frac{T_1 - T_2}{\dfrac{L}{kA}} = \frac{T_1 - T_2}{R_{th}} \qquad ...(1.33)$$

$\therefore \quad R_{th} = \dfrac{L}{kA}$ = thermal resistance to heat conduction

Alternative method

Fourier equation can be written as

$$Q\,dx = -\,kA\,dT$$

Integrating the above equation between the boundaries of the plane, i.e. $0 \le x \le L$, $T_1 \le T \le T_2$,

Downloaded from Ktunotes.in

we have

$$Q \int_0^L dx = -kA \int_{T_1}^{T_2} dT$$

$$QL = -kA\,(T_2 - T_1)$$

or $$Q = -\frac{kA\,(T_2 - T_1)}{L} = \frac{kA\,(T_1 - T_2)}{L} \qquad ...(1.34)$$

Equations (1.32) and (1.34) are same.

The temperature distribution $T(x)$ can be obtained by integrating Fourier equation between 0 an x and T_1 and T.

$$Q \int_0^{\infty} dx = -kA \int_{T_1}^{T} dT$$

$$Q\,(x - 0) = -kA\,(T - T_1)$$

$$Qx = -kA\,(T - T_1)$$

$$Q = -\frac{kA}{x}(T - T_1) \qquad ...(1.35)$$

Comparing Eqs. (1.32) and (1.35), we have

$$\frac{T_2 - T_1}{L} = \frac{T - T_1}{x}$$

$$T(x) = (T_2 - T_1)\frac{x}{L} + T_1 \qquad ...(1.36)$$

Equations (1.30) and (1.36) are same.

Downloaded from Ktunotes.in

1.21 THERMAL CONDUCTIVITY

Thermal conductivity of a material is defined as the heat conducted through a body of unit area and unit thickness in unit time with unit temperature difference.

1.21.1 Thermal Resistance

The heat transfer process is analogous to the flow of electricity.

According to Ohm's law,

$$\text{Current } (I) = \frac{\text{Voltage difference } (dv)}{\text{Electrical resistance } (R)}$$

According to Fourier law,

$$\text{Heat flow } (Q) = \frac{\text{Temperature difference } (\Delta T)}{\text{Thermal resistance} \left(\dfrac{L}{kA}\right)}$$

$$Q = \frac{kA\,(T_1 - T_2)}{L}$$

It can be rewritten as

$$Q = \frac{T_1 - T_2}{L/kA}$$

where $\frac{L}{kA}$ is called thermal resistance R_{th}.

So, $R_{th} = \frac{L}{kA}$

The reciprocal of thermal resistance is thermal conductance.

Downloaded from Ktunotes.in

So, $Q = \dfrac{\Delta T}{R}$

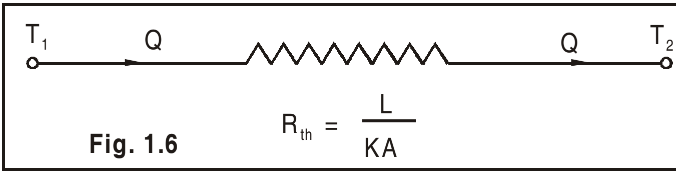


Fig. 1.6

R for different figures are given in page No: 44, 45, 46 and 47 of HMT data book by CPK.

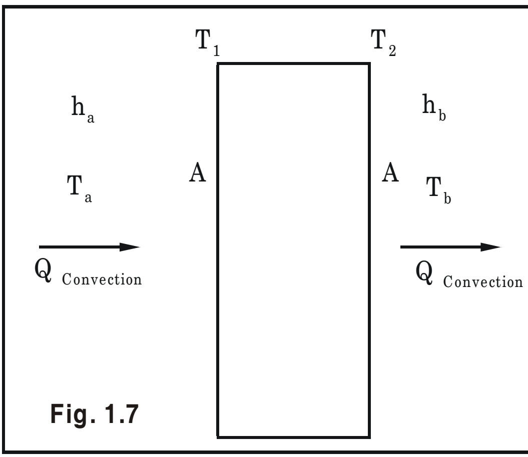


Fig. 1.7

According to Newton's law of cooling,

Heat transfer through left face by convection is given by

$$Q = h_a A (T_a - T_1)$$

where

h_a = Convective heat transfer coefficient in W/m^2 K in left side

Downloaded from Ktunotes.in

A = Area exposed to heat transfer in m^2
(surface area of heat transfer)

T_a - Surrounding fluid temperature in left side in K

T_1 - Temperature of surface (extreme left surface)

Similarly, heat transfer through right face by convection is given by,

$$Q = h_b\, A\, (T_2 - T_b)$$

where h_b = Convective heat transfer coefficient in rightside

T_2 = Temperature of surface (extreme right surface)

T_b = Surrounding fluid temp in right face

1.22 Heat conduction through composite walls with fluid on both sides (with inside and outside convection)

A composite walk is composed of several different layers, each having a different thermal conductivity.

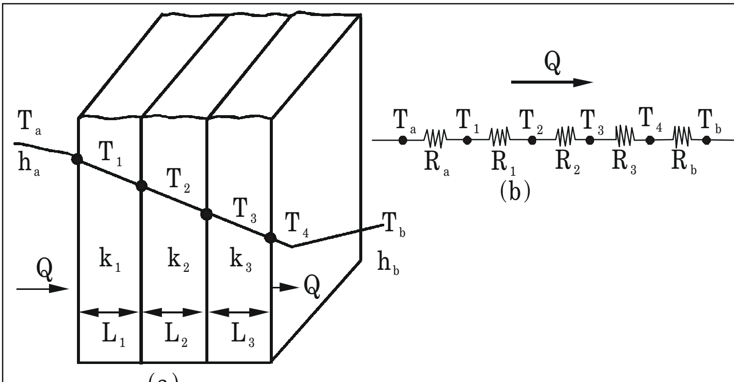


Fig. 1.8 (a) conduction through a composite slab with fluid on both sides and (b) equivalent thermal resistance circuit

Downloaded from Ktunotes.in

Consider a composite wall made up of three parallel layers as shown in **Figure 1.8**.

Since the rate of heat transfer through each layer (slab) is same, we have

$$Q = \frac{k_1 A (T_1 - T_2)}{L_1} = \frac{k_2 A (T_2 - T_3)}{L_2} = \frac{k_3 A (T_3 - T_4)}{L_3} \quad ...(1.37)$$

In most of the science and engineering applications, fluid flows on both sides of the composite walls.

Hence, we should consider convection on both sides. Then,

$$Q = h_a A (T_a - T_1) = \frac{k_1 A (T_1 - T_2)}{L_1} = \frac{k_2 A (T_2 - T_3)}{L_2}$$

$$= \frac{k_3 A (T_3 - T_4)}{L_3} = A h_b (T_4 - T_b)$$

Equation (1.38) can be written as ...(1.38)

$$(T_a - T_1) = \frac{Q}{h_a A} = Q R_a \quad ...(1.39)$$

$$(T_1 - T_2) = \frac{Q L_1}{k_1 A} = Q R_1 \quad ...(1.40)$$

$$(T_2 - T_3) = \frac{Q L_2}{k_2 A} = Q R_2 \quad ...(1.40)$$

$$(T_3 - T_4) = \frac{Q L_3}{k_3 A} = Q R_3 \quad ...(1.41)$$

$$(T_4 - T_b) = \frac{Q}{h_b A} = Q R_b \quad ...(1.42)$$

Downloaded from Ktunotes.in

where, R_a and R_b are the thermal resistance of convection. Adding Eqs. from 1.38 to 1.42, we get

$$T_a - T_b = Q\ (R_a + R_1 + R_2 + R_3 + R_b)$$

or $$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_3 + R_b} \quad ...(1.43)$$

For n number of slabs,

$$Q = \text{heat flow} = \frac{T_a - T_b}{R_a + R_b + \sum_{i=1}^{n} R_i}$$

$$= \frac{\text{overall temperature difference}}{\text{thermal resistance}} = \frac{\Delta T_0}{\sum R} \quad ...(1.44)$$

[Refer HMT Data book Page 45 for formula]

$$\therefore \quad Q = \frac{A\ (T_a - T_b)}{\left(\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right)} = \frac{A\ (T_a - T_b)}{\left(\frac{1}{h_a} + \frac{1}{h_b} + \sum_{i=1}^{n} \frac{L_i}{k_i}\right)}$$

$$= UA\ (T_a - T_b) \quad ...(1.45)$$

where U = overall heat transfer coefficient

$$\text{i.e.} U = \frac{1}{\left(\frac{1}{h_a} + \frac{1}{h_b} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} = \frac{1}{\left(\frac{1}{h_a} + \frac{1}{h_b} + \sum_{i=1}^{n} \frac{L_i}{k_i}\right)} \quad ...(1.46)$$

An electrical analogy is used to solve complex problems involving both series and parallel thermal resistance. **Figure 1.9** shows a complex problem.

Downloaded from Ktunotes.in

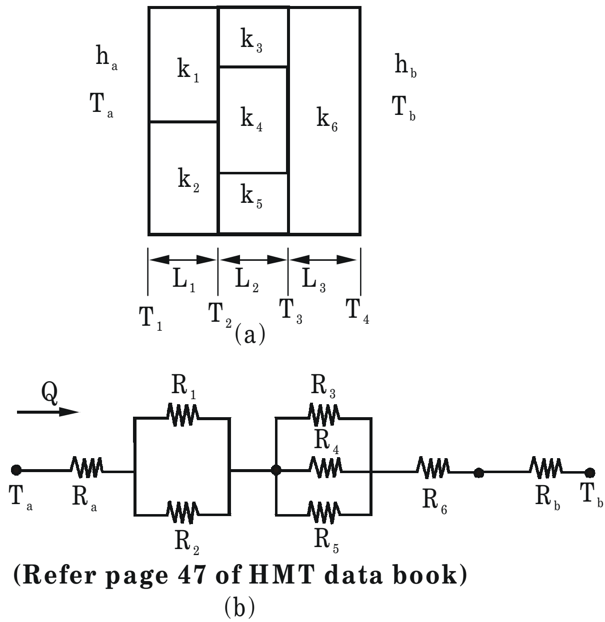


Fig. 1.9. (a) A mix of series and parallel composite walls and (b) the equivalent thermal resistance circuit

$$\Sigma R = R_a + \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} + \frac{1}{\dfrac{1}{R_3} + \dfrac{1}{R_4} + \dfrac{1}{R_5}} + R_b = \frac{1}{UA} \quad ...(1.47)$$

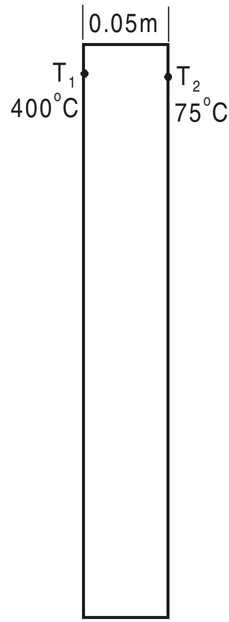
$$Q = \frac{T_a - T_b}{\Sigma R} \quad ...(1.48)$$

1.23 SOLVED PROBLEMS

Problem 1.1: *Find the rate of heat transfer per unit area through a copper plate 50 mm thick, whose one face is maintained at* 400°C *and otherface at* 75°C. *Take thermal conductivity of copper as* 370 W/m°C.

Downloaded from Ktunotes.in

Solution



Given: $L = 0.05$ m ; $A = 1$ m^2

$k = 370$ W/m°C

$T_1 = 400$ °C

$T_2 = 75$ °C

$$\frac{Q}{A} = q = \frac{\Delta T}{R}$$

Refer Pg. 44 for formula.

$$R = \frac{L}{kA}$$

$$= \frac{0.05}{370 \times 1}$$

$$R = 1.351 \times 10^{-4} \text{ K/W}$$

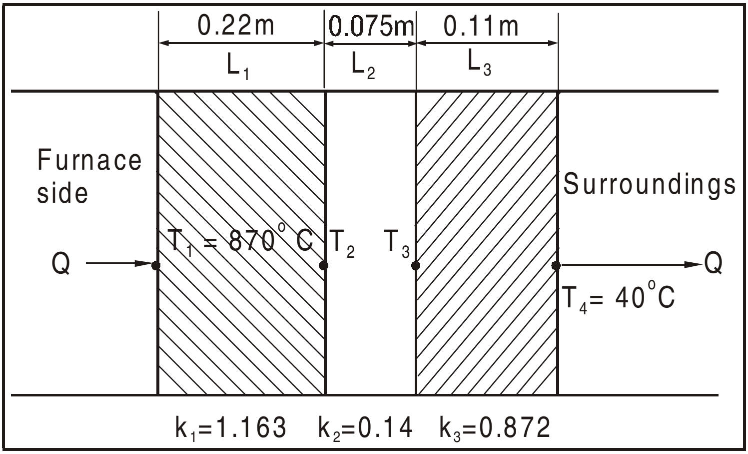
$$q = \frac{\Delta T}{R} = \frac{T_1 - T_2}{R} = \frac{400 - 75}{1.351 \times 10^{-4}}$$

$$q = 2405.63 \text{ kW/m}^2$$

Problem 1.2: *A furnace wall is made up of three layers, one is fire brick, one is insulating layer and one is red brick. The inner and outer surfaces temperature are at* 870°C and 40°C *respectively. The respective conductive heat transfer coefficients of the layers are 1.163, 0.14 and 0.872* W/m°C *and the thicknesses are 22 cm, 7.5 cm and 11 cm. Find the rate of heat loss per sq. meter and the interface temperatures.*

Downloaded from Ktunotes.in

Solution



From Pg. 45-CPK-Data book

$$Q = \frac{\Delta T}{R}$$

Thermal Resistance *R*

$$R \text{ for composite wall } = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$$

Since we are not considering convective heat transfer, we can ignore $\frac{1}{h_a}$ and $\frac{1}{h_b}$ (i.e., $\frac{1}{h_a} = 0$ and $\frac{1}{h_b} = 0$)

Also $A = 1 \text{ m}^2$

$$\text{So } R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right]$$

$$= \frac{1}{1}\left[\frac{0.22}{1.163} + \frac{0.075}{0.14} + \frac{0.11}{0.872}\right]$$

$$= 0.8510 \text{ K/W}$$

Downloaded from Ktunotes.in

$$Q = \frac{T_1 - T_4}{R} = \frac{(870 - 40)}{0.8510} = \frac{830}{0.8510} = \mathbf{975.3}\ \text{W}$$

$$\boldsymbol{q = \frac{Q}{A} = \frac{Q}{1} = 975.3\ \text{W/m}^2}$$

For Interface Temperature

From Pg. 45, Use $\Delta T_1 = Q \times R_1 = 975.3 \times R_1$

$$(T_1 - T_2) = 975.3 \times R_1$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.22}{1.163 \times 1}$$

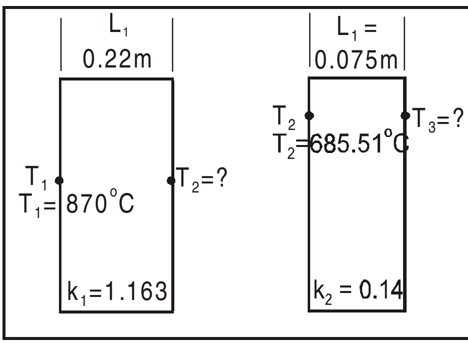
$$= 0.1892$$

$$870 - T_2 = 975.3 \times 0.1892 = 184.5$$

$$T_2 = 870 - 184.5$$

$$= 685.5°\ \text{C}$$

$$T_2 = 685.5\ °\ \text{C}$$



Similarly,

$$\Delta T_2 = T_2 - T_3$$

$$= Q \times R_2$$

$$R_2 = \frac{L_2}{k_2 A_2} = \frac{0.075}{0.14 \times 1} = 0.5357$$

$$T_2 - T_3 = Q \times R_2$$

$$685.51 - T_3 = 975.3 \times 0.5357$$

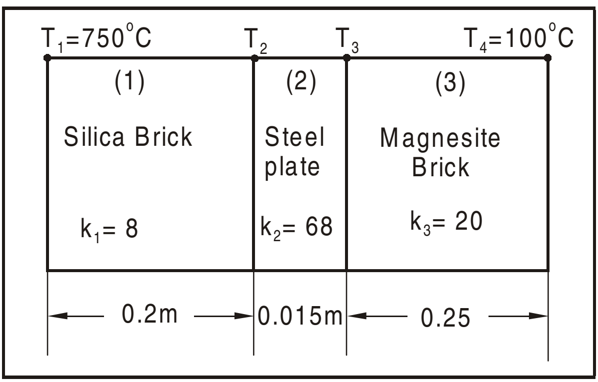
$$T_3 = 685.51 - (975.3 \times 0.5357)$$

$$\boxed{\boldsymbol{T_3 = 163.03°\ \text{C.}}}$$

Downloaded from Ktunotes.in

Problem 1.3: *A composite wall is made of 15 mm thick of steel plate lined inside with Silica brick-200 mm thick and on the outside magnesite brick-250 mm thick. The inner and outer surface temperature are* 750°C and 100°C *respectively. The k for silica, steelplate and magnesite brick are* $8\ \dfrac{\text{W}}{\text{m°C}}$, $68\ \dfrac{\text{W}}{\text{m°C}}$ and $20\ \dfrac{\text{W}}{\text{m°C}}$ *respectively. Determine heat flux, interface temperatures.*

Solution



$$\text{Heat flux} = \frac{Q}{A} = q$$

From Pg 45 of CPK

$$R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right] \qquad \left[\because \frac{1}{h_a} = 0 \text{ and } \frac{1}{h_b} = 0\right]$$

$$= \frac{1}{1}\left[\frac{0.2}{8} + \frac{0.015}{68} + \frac{0.25}{20}\right]$$

$$= 0.03772 \text{ K/W}$$

Downloaded from Ktunotes.in

$$Q = \frac{(\Delta T)_{overall}}{R}$$

$$= \frac{(T_1 - T_4)}{R} = \frac{750 - 100}{0.03772} = 17232 \text{ W}$$

$$\boldsymbol{Q = 17232 \text{ W}}$$

To Find Interface Temperatures

To find $\boldsymbol{T_2}$

$$\Delta T_1 = Q \times R_1$$

$$T_1 - T_2 = 17232 \times R_1$$

$$R_1 = \frac{1}{A}\left[\frac{L_1}{k_1}\right] \qquad [\because A = 1 \text{ m}^2]$$

$$= \frac{1}{1}\left[\frac{0.2}{8}\right]$$

$$= 0.025 = \text{K/W}$$

$$T_1 - T_2 = 750 - T_2 = Q \times R_1$$

$$= 17232 \times 0.025$$

$$T_2 = 750 - (17232 \times 0.025) = 319.2°\text{C}$$

$$\boxed{\boldsymbol{T_2 = 319.2\,°\text{C.}}}$$

To find $\boldsymbol{T_3}$

$$T_2 - T_3 = Q \times R_2$$

$$R_2 = \frac{L_2}{Ak_2} = \frac{0.015}{1 \times 68} = 2.205 \times 10^{-4} \text{ K/W}$$

Downloaded from Ktunotes.in

$$319.2 - T_3 = 17232 \times 2.205 \times 10^{-4}$$

$$T_3 = 319.2 - (17232 \times 2.205 \times 10^{-4}) = \; = 315.4\,°\text{C}$$

$$\boxed{\boldsymbol{T_3 = 315.4\,°\text{C}}}$$

Alternate Method : To find T_3

$$Q = \frac{T_1 - T_3}{R_1 + R_2}$$

$$17232 = \frac{750 - T_3}{0.025 + 2.205 \times 10^{-4}}$$

$$17232\,(0.025 + 2.205 \times 10^{-4}) = 750 - T_3$$

$$T_3 = 750 - [\,17232\,(0.025 + 2.205 \times 10^{-4})\,]$$

$$= 315.4\,°\text{C}.$$

To find T_2

$$\left[R_3 = \frac{0.25}{1 \times 20} = 0.0125 \right]$$

$$Q = \frac{T_2 - T_4}{R_2 + R_3}$$

$$17232 = \frac{T_2 - 100}{2.205 \times 10^{-4} + 0.0125}$$

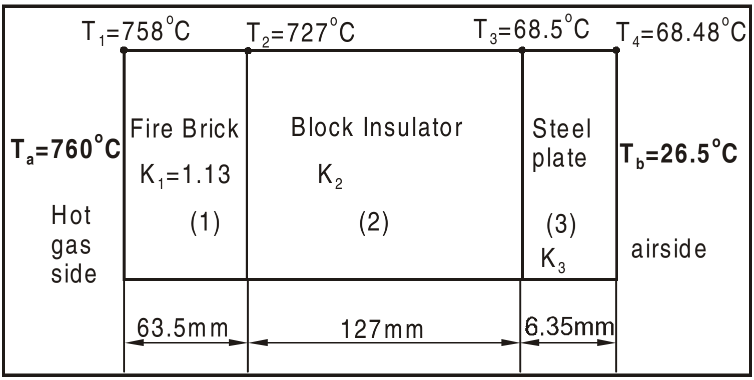
$$17232\,(2.205 \times 10^{-4} + 0.0125) = T_2 - 100$$

$$T_2 = 17232\,(2.205 \times 10^{-4} + 0.0125) + 100$$

$$= 319.19\,°\text{C}.$$

Downloaded from Ktunotes.in

Problem 1.4: *The temperature distribution through a furnace wall consisting of fire brick, block insulation and steel plate is given below. Determine heat flux, thermal conductivity of block insulation and steel plate, heat transfer coefficient for gas side and air side.*



Solution

Note

From hot gas side to fire brick, heat is transferred by convection. So

$$Q_{\text{convection}} = h_a A\,(T_a - T_1)$$

where h_a = convective heat transfer coefficient for gas side.

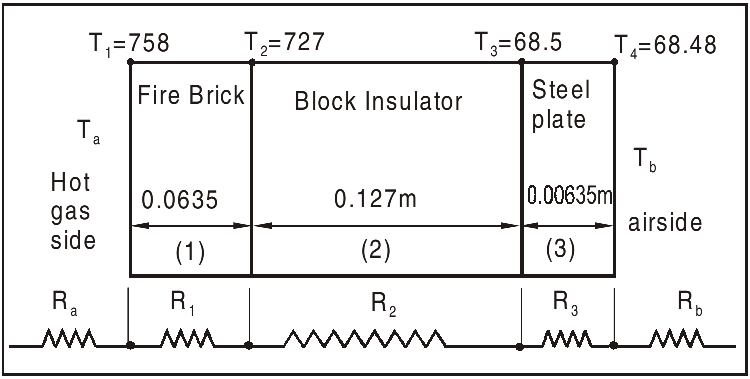
Similarly, from steel plate to air side, heat is transferred by convection.

$$\text{So, } Q_{\text{convection}} = h_b A\,(T_3 - T_b)$$

where h_b = convective heat transfer coefficient for air side.

From HMT Table, Pg No. 45,

Downloaded from Ktunotes.in



$$Q = \frac{\Delta T_{\text{overall}}}{R}$$

where R for composite wall $= \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$

$A = 1 \text{ m}^2$ (not given)

To find Q

$$Q = \frac{T_1 - T_2}{R_1}$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.0635}{1.13 \times 1} = 0.0562 \text{ K/W}$$

$$Q = \frac{T_1 - T_2}{R_1} = \frac{758 - 727}{0.0562}$$

$$= 551.6535 \text{ W/m}^2$$

$$\boldsymbol{Q = 551.65 \text{ W/m}^2}$$

Downloaded from Ktunotes.in

To find k_2

$$Q = \frac{T_2 - T_3}{R_2}$$

$$551.65 = \frac{727 - 68.5}{R_2}$$

$$R_2 = 1.1937 \text{ K/W}$$

$$R_2 = \frac{L_2}{k_2 A_2}$$

$$k_2 = \frac{L_2}{R_2 A_2} = \frac{0.127}{1.1937 \times 1}$$

$$= 0.1064 \text{ W/m K}$$

To find k_3

$$Q = \frac{T_3 - T_4}{R_3}$$

$$551.65 = \frac{68.5 - 68.48}{R_3}$$

$$R_3 = 3.625 \times 10^{-4}$$

$$R_3 = \frac{L_3}{k_3 A_3}$$

$$k_3 = \frac{L_3}{R_3 A_3}$$

$$= \frac{0.00635}{3.625 \times 10^{-4} \times 1} = 175.15 \text{ W/m K}$$

Downloaded from Ktunotes.in

So $k_3 = k$ for steel $= 175.15$ W/m K.

To find h_a

$$Q_{\text{conduction}} = Q_{\text{convection from gas side to fire brick}}$$

$$= 551.65 \text{ W}$$

$$Q_{\text{convection}} = h_a A\,(T_a - T_1)$$

$$551.65 = h_a \times 1 \times (760 - 758)$$

$$h_a = \frac{551.65}{760 - 758}$$

$$h_a = 275.83 \text{ W/m}^2{}^\circ\text{C}.$$

To find h_b

$Q_{\text{convection}}$ from -steel plate to air $= h_b \times A \times (T_4 - T_b)$

$$551.65 = h_b \times 1 \times (68.48 - 26.5)$$

$$h_b = 13.141 \text{ W/m}^2{}^\circ\text{C}.$$

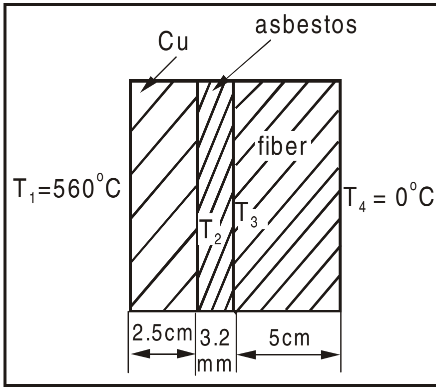
Problem 1.5: *A composite wall is for med of a 2.5 cm copper plate* $k = 355$ W/mK) *a 3.2 mm layer of asbestors* ($k = 110$ W/mK) *and a 5 cm layer of fiber plate* ($k = 0.049$ W/mK). *The wall is subjected to an overall temperature difference of* 560°C *on the Cu plate side and* 0°C *on the fiber plate side. Estimate the heat flux through this composite wall and interface temperature between asbestos and fiber plate.* *(Apr/May - 2008 - AU)*

Given:

Thickness of Cu plate $L_1 = 2.5$ cm $= 0.025$ m

Thickness of asbestos $L_2 = 3.2$ mm $= 0.0032$ m

Downloaded from Ktunotes.in



Thickness of fiber plate

$L_3 = 5 \text{ cm} = 0.05 \text{ m}$

Thermal conductivity of Cu plate

$k_1 = 355 \text{ W/mK}$

Thermal conductivity of asbestos

$k_2 = 0.110 \text{ W/mK}$

Thermal conductivity of fiber plate $k_3 = 0.49 \text{ W/mK}$

Overall temperature difference, $\Delta T = 560°\text{C}$

From HMT DB page 44.

$$\text{Heat flux } \frac{Q}{A} = \frac{\Delta T}{\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}}$$

$$= \frac{560}{\dfrac{0.025}{355} + \dfrac{0.0032}{0.110} + \dfrac{0.05}{0.049}}$$

$$= 533.55 \text{ W/m}^2$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{\dfrac{L_1}{k_1}} = \frac{T_2 - T_3}{\dfrac{L_2}{k_2}} = \frac{T_3 - T_4}{\dfrac{L_3}{k_3}}$$

$$533.55 = \frac{T_3 - 0}{\left(\dfrac{0.05}{0.049}\right)}$$

Interface temperature between asbestos and fiber plate $T_3 = 544.43°\text{C}$

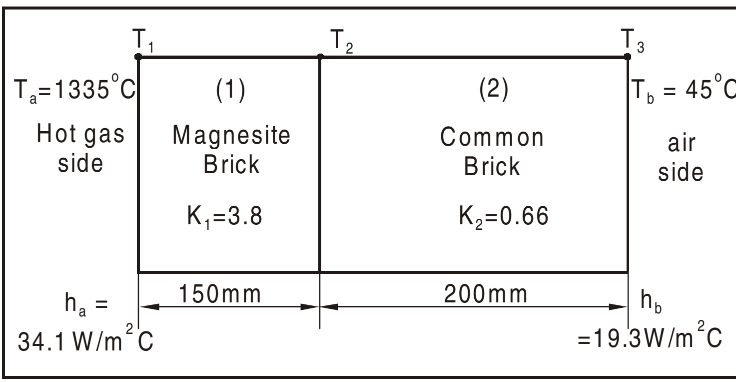
Downloaded from Ktunotes.in

Problem 1.6: *A 200 mm common brick* ($k_2 = 0.66$ W/m°C) *with 150 mm magnesite brick* ($k_1 = 3.8$ W/m°C) *lined inside the common brick. The combustion gases are at* 1335°C *and surrounding air temperature is at* 45°C. *Heat transfer coefficient of gas side and air side are* 34.1 W/m^2°C *and* 19.3 W/m^2°C *respectively. Determine*

(a) Heat transfer / sq. meter (Heat flux)

(b) Max. temperature to which common brick is subjected to

Solution



$$\theta_{\text{convection from gas to MB}} = h_a A\,(T_a - T_1)$$

$$Q_{\text{convection from CB to air}} = h_b A\,(T_3 - T_b)$$

To find q – heat flux

$$Q = \frac{(\Delta T)_{\text{overall}}}{R}$$

$$R = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_b}\right]$$

$$= \frac{1}{1}\left[\frac{1}{34.1} + \frac{0.15}{3.8} + \frac{0.2}{0.66} + \frac{1}{19.3}\right]$$

Downloaded from Ktunotes.in

$$= 0.4236 \text{ K/W}$$

$$Q = \frac{1335 - 45}{0.4236} = 3045.02$$

$$\mathbf{Q = 3045.02 \ W/m^2.}$$

To find max temperature of common brick

Max. temperature of common brick is T_2. To find T_2, first of all, we have to find T_1.

To find T_1

$$Q_{\text{conduction}} = Q_{\text{convection}}$$

$$Q = 3045.02 = h_a A(T_a - T_1)$$

$$= 34.1 \times 1(1335 - T_1)$$

$$(1335 - T_1) = \frac{3045.02}{34.1}$$

$$T_1 = 1335 - \left[\frac{3045.02}{34.1}\right] = 1245.69\,°\text{C}$$

$$\mathbf{T_1 = 1245.69\,°C.}$$

$$Q = \frac{T_1 - T_2}{R_1}$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.15}{3.8 \times 1} = 0.03947 \text{ K/W}$$

$$Q = 3045.02 = \frac{1245.69 - T_2}{0.03947}$$

$$1245.69 - T_2 = 3045.02 \times 0.03947$$

$$= 120.198$$

Downloaded from Ktunotes.in

$$T_2 = 1245.69 - 120.198 = 1125.492 \ ^\circ\text{C}$$

$$\boldsymbol{T_2 = 1125.492\ ^\circ\mathbf{C}}$$

To find T_3

$$Q = 3045.02 = h_b A\,(T_3 - T_b)$$

$$3045.02 = 19.3 \times 1(T_3 - 45)$$

$$T_3 - 45 = \frac{3045.02}{19.3} = 157.77$$

$$T_3 = 157.77 + 45 = 202.77\,^\circ\text{C}$$

$$\boldsymbol{T_3 = 202.77\ ^\circ\mathbf{C}}$$

Problem 1.7.: *Air at* 20°C *flows over a plate of* 50 cm × 60 cm × 2 cm *maintained at* 220°C. *The convection heat transfer coefficient is* 25 W/m^2°C. *Calculate heat loss/hr if the heat loss by radiation from the plate is 200 W in addition to the above heat loss. Determine inside plate temperature. The thermal conductivity of plate is* 50 W/m°C. *(Oct-2000, Madras University)*

Solution

$$Q_{\text{convection}} = hA\,(T_p - T_a)$$

$$= 25 \times (0.5 \times 0.6)\,(220 - 20)$$

$$= 1500 \text{ W} = 1500 \text{ J/sec}$$

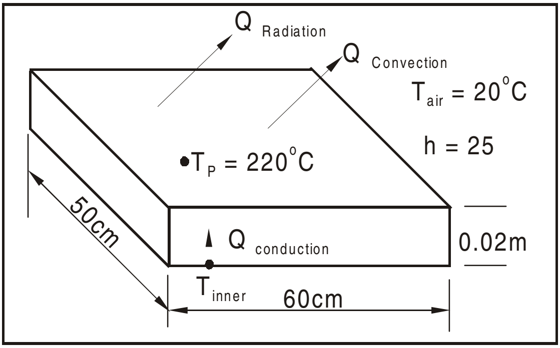
$$Q_{\text{convection/hr}} = 1500 \times 3600$$

$$= 5.4 \times 10^6 \text{ J/hr}$$

Total heat transfer from the plate = $Q_{\text{convection}} + Q_{\text{radiation}}$

$$= 1500 + 200$$

Downloaded from Ktunotes.in



$$= 1700 \text{ W} = 6.12 \times 10^6 \text{ J}/hr$$

1700 W of heat is conducted from the inner side of plate to outer side of plate.

$$Q_{conducted} = 1700 \text{ Watts}$$

$$= \frac{(T_{inner} - T_{plate\ outer})}{R}$$

$$R = \frac{L}{kA}$$

$$= \frac{0.02}{50 \times (0.5 \times 0.6)} = 1.333 \times 10^{-3} \text{ K/W}$$

$$1700 = \frac{(T_{inner} - 220)}{1.333 \times 10^{-3}}$$

$$T_{inner} = (1700 \times 1.333 \times 10^{-3}) + 220$$

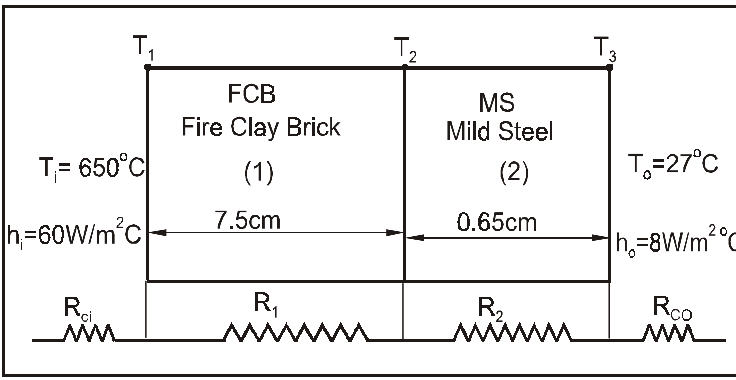
$$= 222.26666 \text{ °C}$$

$$\boldsymbol{T_{inner} = 222.27 \text{ °C.}}$$

Problem 1.8: *A furnace wall is made up of 7.5 cm thick Fire clay brick and 0.65 cm mild steel plate. The inner surface is exposed to hot gases at* 650°C. *Outside air*

Downloaded from Ktunotes.in

temperature is **27°C.** *The convection and radiation heat transfer coefficient for the gas side is* 60 W/m^2°C. *The convection heat transfer coefficient for the air side is* 8 W/m^2°C. *Determine heat loss/* m^2*-hr surface temperature of steel plate.* *(Apr-2000, Madras University)*



Solution

h = convective and radiation heat transfer coefficient.

From Pg 1 HMT Data book - CPK

Steel (Carbon steel) for 0.5 C

$k = 53.6$ W/m°C, So $k_2 = 53.6$

From Pg 11 HMT Data book - For Fire clay Brick

$k = 1.5$ W/m°C, So $k_1 = 1.5$ W/m°C

To find Q

$$Q = \frac{(\Delta T)_{\text{overall}}}{R}$$

Downloaded from Ktunotes.in

$$R = \frac{1}{A}\left[\frac{1}{h_i} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_0}\right]$$

$$= \frac{1}{1}\left[\frac{1}{60} + \frac{0.075}{1.5} + \frac{0.65 \times 10^{-2}}{53.6} + \frac{1}{8}\right]$$

$$= 0.192 \text{ K/W}$$

$$Q = \frac{T_1 - T_0}{R}$$

$$= \frac{650 - 27}{0.192} = 3245 \text{ W/m}^2$$

$$\boxed{Q = 3245 \text{ W/m}^2}$$

$$\text{Q} - \text{Heat transfer} = 3245 \frac{\text{J}}{\text{sec.m}^2}$$

$$\text{Heat transfer Q/hr} = 3245 \times 3600 \frac{\text{J}}{\text{hr} - \text{m}^2}$$

$$= 11.68 \times 10^6 \text{ J/hr} - \text{m}^2$$

To find surface temperature of mild steel (T_3)

$$Q_{\text{convected}} = h_0 A\,(T_3 - T_0)$$

$$Q_{\text{conducted}} = Q_{\text{convected}} = 3245 = 8 \times 1 \times (T_3 - 27)$$

$$(T_3 - 27) = \frac{3245}{8} = 405.62$$

$$\boxed{T_3 = 432.63\,°\text{C}}$$

Problem 1.9.: *A composite wall is made up of three layers 15 cm, 10 cm and 12 cm of thickness. The first layer is made up of material with* $k = 1.458$ W/m°C *for 60% of area*

Downloaded from Ktunotes.in

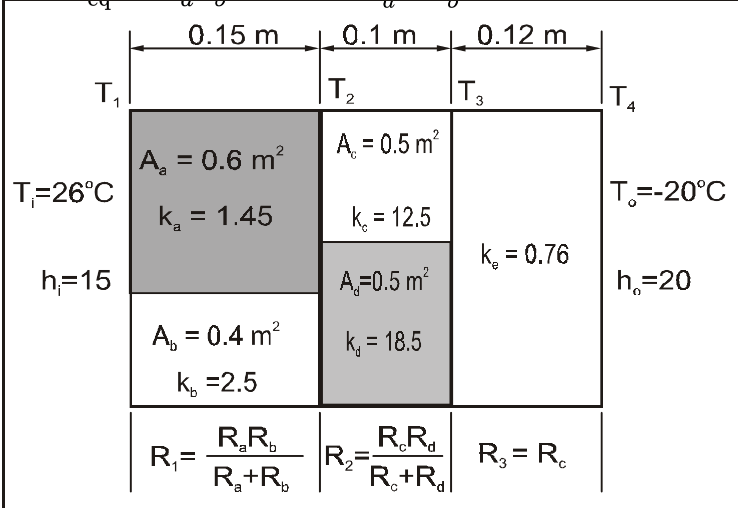
and rest of the material with $k = 2.5\ \dfrac{\text{W}}{\text{m°C}}$. *The second layer is made with material of* $k = 12.5$ W/m°C *for 50% of the area and the rest of the material with* $k = 18.5$ W/m°C. *The third layer is of single material with* $k = 0.76$ W/m°C. *The composite slab is exposed to warm air at* 26°C *and cold air of* – 20°C *on the other side. The inner and outer heat transfer coefficients are* 15 W/m^2°C and 20 W/m^2°C. *Determine heat flux rate and interface temperatures. (Oct-96, Madras University)*

Solution

$$R_{\text{eq}} = \text{Equivalent } R$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_a} + \frac{1}{R_b}$$

$$\frac{1}{R_{\text{eq}}} = \frac{R_a + R_b}{R_a R_b} \text{ or } R_{\text{eq}} = \frac{R_a R_b}{R_a + R_b}$$



Downloaded from Ktunotes.in

Assume $A = 1\ m^2$ (Since surface area is not given).

Refer Pg 47 of CPK - Data book

To find R_1

$$R_a = \frac{L_a}{k_a A_a} = \frac{0.15}{1.45 \times 0.6} = 0.17241 \text{ K/W}$$

$$R_b = \frac{L_b}{k_b A_b} = \frac{0.15}{2.5 \times 0.4} = 0.15 \text{ K/W.}$$

$$R_1 = \frac{R_a R_b}{R_a + R_b} = \frac{0.17241 \times 0.15}{0.17241 + 0.15} = 0.080213 \text{ K/W}$$

$$R_1 = 0.080213 \text{ K/W.}$$

To find R_2

$$R_c = \frac{L_c}{k_c A_c} = \frac{0.1}{12.5 \times 0.5} = 0.016 \text{ K/W}$$

$$R_d = \frac{L_d}{k_d A_d} = \frac{0.1}{18.5 \times 0.5} = 0.01081 \text{ K/W}$$

$$R_2 = \frac{R_c R_d}{R_c + R_d} = \frac{0.016 \times 0.01081}{0.016 + 0.01081}$$

$$= 6.4513 \times 10^{-3}$$

To find R_3

$$R_3 = \frac{L_e}{k_e A_e} = \frac{0.12}{0.76 \times 1} = 0.15789 \text{ K/W}$$

Downloaded from Ktunotes.in

$$R_{\text{for convection inner}} = R_{ci} = \frac{1}{h_i A_i}$$

$$= \frac{1}{15 \times 1} = 0.06667 \text{ K/W}$$

$$R_{\text{for convection outer}} = R_{co} = \frac{1}{h_0 A_0}$$

$$= \frac{1}{20 \times 1} = 0.05 \text{ K/W}$$

To find Q

$$R = R_{ci} + R_1 + R_2 + R_3 + R_{co}$$

$$= 0.06667 + 0.080213 + 6.4513 \times 10^{-3} + 0.15789 + 0.05$$

$$= 0.36122 \text{ K/W.}$$

$$\frac{Q}{A} = q = \frac{(\Delta T)_{\text{overall}}}{R} = \frac{T_i - T_0}{R}$$

$$= \frac{26 - (-20)}{0.36122} = \frac{46}{0.36122} = 127.345$$

$$\boxed{q = 127.345 \text{ W/m}^2.}$$

To find Interface Temperatures

To find T_1

$Q_{\text{conducted}} = Q_{\text{convected}}$ under steady state condition.

So, $Q_{\text{convection}} = 127.345 = h_i A\,(T_i - T_1)$

$$127.345 = 15 \times 1 \times (26 - T_1)$$

Downloaded from Ktunotes.in

$$26 - T_1 = \frac{127.345}{15} = 8.4896$$

$$T_1 = 26 - 8.4896 = 17.51\,°C$$

$$\boxed{\boldsymbol{T_1 = 17.51\,°C.}}$$

To find T_2

$$Q = \frac{T_1 - T_2}{R_1}$$

$$T_1 - T_2 = QR_1$$

$$T_2 = T_1 - QR_1$$

$$T_2 = 17.51 - (127.345 \times 0.080213) = 7.295\,°C$$

$$\boxed{\boldsymbol{T_2 = 7.295\,°C.}}$$

To find T_3

$$Q = \frac{T_2 - T_3}{R_2}$$

$$T_2 - T_3 = QR_2$$

$$T_3 = T_2 - QR_2$$

$$T_3 = 7.295 - (127.345 \times 6.4513 \times 10^{-3}) = 6.4737\,°C$$

$$\boxed{\boldsymbol{T_3 = 6.4737\,°C.}}$$

To find T_4

$$Q_{\text{convected}} = 127.345 = h_0 A\,(T_4 - T_0)$$

$$127.345 = 20 \times 1 \times (T_4 - (-20))$$

Downloaded from Ktunotes.in

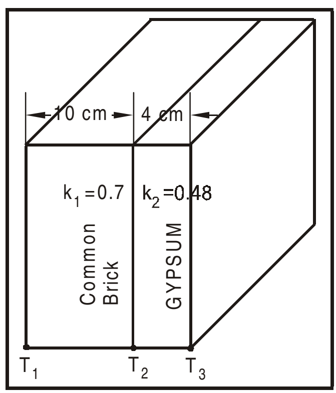
$$T_4 + 20 = \frac{127.345}{20} = 6.36725$$

$$\boxed{T_4 = -13.6327\ °C.}$$

Problem 1.10: *An interior wall of a house is made up of 10 cm common brick* (k = 0.7 W/m°C) *followed by 4 cm layer of gypsum plaster* (k = 0.48 W/m°C). *What thickness of loosely packed rock wool insulation* (k = 0.065 W/m°C) *should be added to reduce the heat loss (or gain) through the wall by 80%. (Oct-99, Madras University)*

Solution

Q_A through interior wall without insulation.



$$R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2}\right]$$

$$= \frac{1}{1}\left[\frac{0.1}{0.7} + \frac{0.04}{0.48}\right]$$

$$= 0.2262 \text{ K/W}$$

$$\boxed{Q_A = \frac{\Delta T}{R_A}}$$

Q_B through interior wall with insulation

$$R_B = \frac{1}{A}\left[\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{L_3}{0.065}\right]$$

$$= 0.2262 + \frac{L_3}{0.065}$$

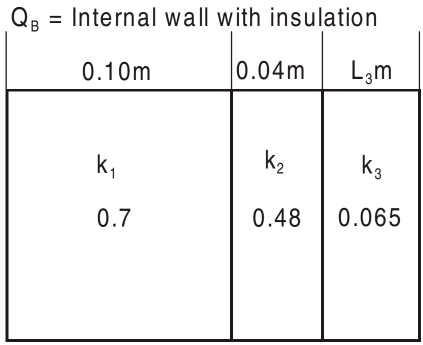
$$Q_B = \frac{(\Delta T)}{R_B}$$

Downloaded from Ktunotes.in

Insulation prevents heat loss by 80%

So initially, without insulation $Q_A = 100\%$

$$= \frac{(\Delta T)}{0.2262} \quad \ldots(1)$$



After insulation,

$$Q_B = 20\% = \frac{\Delta\, T}{0.2262 + \dfrac{L_3}{0.065}} \quad \ldots\ (2)$$

Divide (1) by (2)

$$\frac{100}{20} = \frac{\Delta\, T}{0.2262} \qquad \frac{\Delta\, T}{0.2262\ + \dfrac{L_3}{0.065}}$$

$$5 = \frac{1}{0.2262} \times \left(0.2262\ + \frac{L_3}{0.065} \right)$$

$$= \left(1 + \frac{L_3}{0.065 \times 0.2262} \right)$$

$$5 = 1 + \frac{L_3}{0.014703}$$

$$4 = \frac{L_3}{0.014703}$$

$$L_3 = 0.0588\ \text{m} \quad \text{(or)}\ L_3 = 5.88\ \text{cm}$$

i.e., 5.88 cm thick insulation will reduce heat loss by 80%.

Downloaded from Ktunotes.in

1.24 Heat conduction through a hollow cylinder

Figure 1.10 shows a long hollow cylinder made of a material having constant thermal conductivity and insulated at both ends. The inner and outer radii are r_1 and r_2, respectively. The length of the cylinder is L.

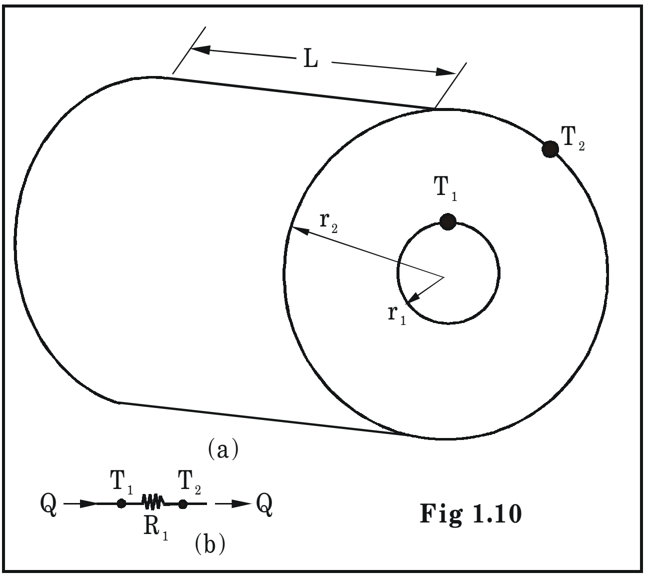


Fig. 1.10 (a) conduction through a hollow cylinder without fluid flowing inside and outside the cylinder (b) equivalent thermal resistance circuit

The general heat conduction equation in cylindrical coordinates is given by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{\partial T}{\alpha \partial t} \qquad \text{...(1.49)}$$

Assumptions:

Steady state: $\partial T / \partial t = 0$

Downloaded from Ktunotes.in

No heat generation: $q_g = 0$

One dimension: $\frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} = \frac{\partial^2 T}{\partial x^2} = 0$

Substitute these assumptions in Eq. (1.49), we have

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = 0; \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0$$

$$\therefore \qquad \frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0, \text{ since } \frac{1}{r} \neq 0 \qquad ...(1.50)$$

Integrating Eq. (1.50) twice we get

$$r \frac{dT}{dr} = C_1 \text{ or } \frac{dT}{dr} = \frac{C_1}{r}$$

$$T = In\ r\ C_1 + C_2 \qquad ...(1.51)$$

where C_1 and C_2 are arbitrary constants

Boundary conditions

At $r = r_1, T = T_1$

At $r = r_2, T = T_2$

Substituting these boundary conditions in Eq. (1.51)

$$T_1 = ln\ r_1\ C_1 + C_2$$

$$T_2 = ln\ r_2\ C_2 + C_2$$

Solving the above two equations, we have

$$C_1 = \frac{T_1 - T_2}{ln \frac{r_1}{r_2}} = \frac{T_2 - T_1}{ln \frac{r_2}{r_1}}$$

Downloaded from Ktunotes.in

$$C_2 = T_1 - C_1 \ln r_1 = T_1 - \frac{(T_2 - T_1) \, In \, r_1}{\ln \dfrac{r_2}{r_1}}$$

Substituting C_1 and C_2 in Eq. (1.51), we get

$$T = \ln r \left[\frac{T_2 - T_1}{\ln \dfrac{r_2}{r_1}} \right] + T_1 - (T_2 - T_1) \left[\frac{\ln r_1}{\ln \dfrac{r_2}{r_1}} \right]$$

$$T = \frac{T_2 - T_1}{\ln \dfrac{r_2}{r_1}} (\ln r - \ln r_1) + T_1$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln \dfrac{r}{r_1}}{\ln \dfrac{r_2}{r_1}} \qquad \text{...(1.52)}$$

Equation (1.52) gives the temperature distribution in a hollow cylinder. The heat flow rate through the cylinder over the surface area A is given by Fourier's conduction equation.

$$Q = -kA \left| \frac{dT}{dr} \right| \text{ when } (r = r_1)$$

Substituting dT/dr from Eq. (1.51) into the above equation (when $r = r_1$)

$$Q = -kA \frac{(T_2 - T_1)}{\ln \dfrac{r_2}{r_1}} \times \frac{1}{r_1} = \frac{k 2 \pi r_1 L \, (T_1 - T_2)}{\ln \dfrac{r_2}{r_1}} \times \frac{1}{r_1}$$

Downloaded from Ktunotes.in

$$= \frac{2\pi\, k\, L\, (T_1 - T_2)}{ln\, \frac{r_2}{r_1}} \quad ...(1.53) \quad [A = 2\pi\, r\, L]$$

$$\text{or} \quad Q = \frac{T_1 - T_2}{\left(\frac{ln\, \frac{r_2}{r_1}}{2\pi\, k\, L}\right)} = \frac{T_1 - T_2}{R_{th}} \quad ...(1.54)$$

$\therefore R_{th}$ = thermal resistance for conduction heat transfer

1.25 HEAT CONDUCTION THROUGH COMPOSITE (COAXIAL) CYLINDERS WITH CONVECTION

Consider the rate of heat transfer through a composite cylinder as shown in **Figure 1.11 (a)** and its equivalent thermal resistance in **Figure 1.11 (b)**

Let

T_1, T_2, T_3 = temperature at inlet surface, between first and second cylinders and outer surface, respectively

L = length of the cylinder

h_a, h_b = convective heat transfer coefficients at inside and outside the composite cylinder respectively

T_a, T_b = temperature of the fluid flowing inside and outside the composite cylinder

k_1, k_2 = thermal conductivity of the first and second material, respectively

The rate of heat transfer is given by Eq. (1.53)

Downloaded from Ktunotes.in

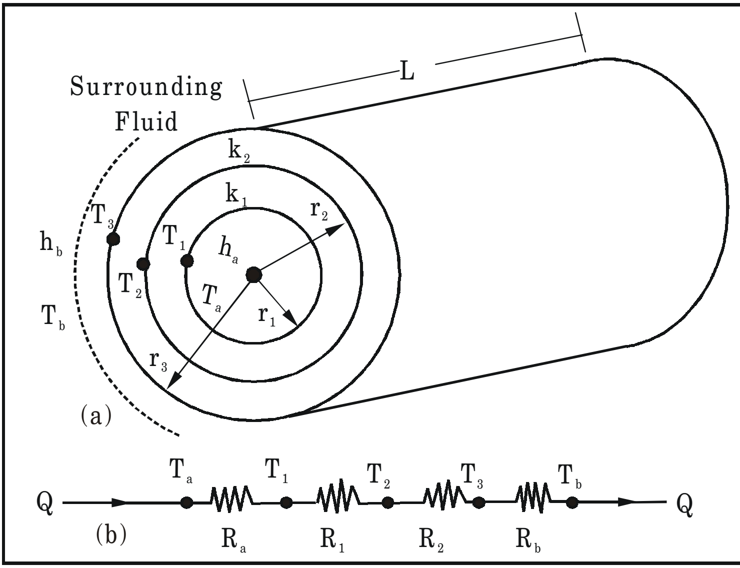


Fig. 1.11 (a) conduction through a composite cylinder with fluid flowing inside and outside the cylinder (b) equivalent thermal resistance circuit

$$Q = h_a \, 2\pi \, r_1 \, L \, (T_a - T_1) = \frac{k_1 \, 2\pi \, L \, (T_1 - T_2)}{ln \, \dfrac{r_2}{r_1}}$$

$$= \frac{k_2 \, 2\pi \, L \, (T_2 - T_3)}{ln \, \dfrac{r_3}{r_2}} = h_b \, 2\pi \, r_3 \, L \, (T_3 - T_b) \qquad \text{...(1.55)}$$

Arranging Eq. (1.55), we get

$$(T_a - T_1) = \frac{Q}{h_a \, 2\pi \, r_1 \, L} = Q \times R_a \qquad \text{...(1.56)}$$

Downloaded from Ktunotes.in

$$(T_1 - T_2) = \frac{Q}{\dfrac{k_1\, 2\pi\, L}{ln\ (r_2/r_1)}} = Q \times R_1 \quad ...(1.57)$$

$$(T_2 - T_3) = \frac{Q}{\dfrac{k_2\, 2\pi\, L}{ln\ (r_3/r_2)}} = Q \times R_2 \quad ...(1.58)$$

$$(T_3 - T_b) = \frac{Q}{h_b\, 2\pi\, r_3\, L} = Q \times R_b \quad ...(1.59)$$

Adding Eqs. from 1.56 to 1.59, we get

$$(T_a - T_b) = Q\ (R_a + R_1 + R_2 + R_b) \quad ...(1.60)$$

$$(T_a - T_b) = Q \left(\frac{1}{h_a\, 2\pi\, L\, r_1} + \frac{ln\, \dfrac{r_2}{r_1}}{k_1\, 2\pi\, L} + \frac{ln\, \dfrac{r_3}{r_2}}{L\, k_2\, 2\pi} + \frac{1}{h_b\, 2\pi\, L\, r_3} \right) \quad ...(1.61)$$

$$\text{or } Q = \frac{(T_a - T_b)\, 2\pi\, L}{\dfrac{1}{h_o\, f_1} + \dfrac{ln\ r_2/r_1}{k_1} + \dfrac{ln\ r_3/r_2}{k_2} + \dfrac{1}{h_b\, r_3}} \quad ...(1.62)$$

1.25.1 Summary - Composite Cylinder

Refer from Pg 46 of HMT Data book - CPK.

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3} ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_b r_4} \right]$$

Downloaded from Ktunotes.in

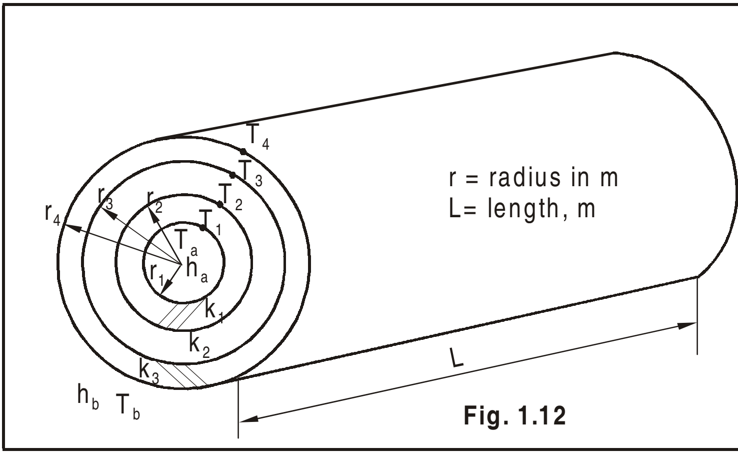


Fig. 1.12

$$Q = \frac{(\Delta T)_{\text{overall}}}{R}$$

T_i or T_a = inner temperature; T_0 or T_b = outer temperature

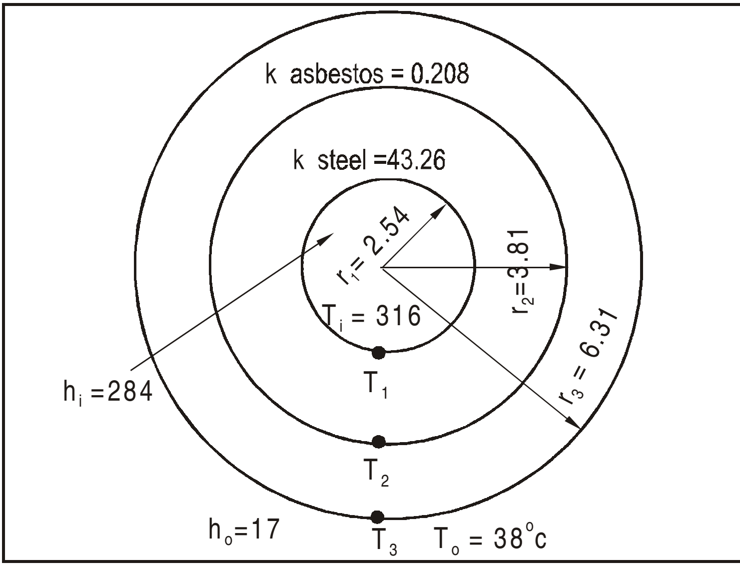
Problem 1.11: *A steel tube of 5.08 cm ID and 7.62 cm OD is covered with 2.5 cm thick of asbestos.* $k_{\text{steel}} = 43.26$ W/m°K $k_{\text{asbestos}} = 0.208$ W/m°C. *The inside surface receives heat from hot gases at* 316°C *with heat transfer coefficient of* 284 W/m^2°C *whereas outer surface is exposed to air at* 38°C *with heat transfer coefficient of* 17 W/m^{2}C. *Determine (1) heat loss for 3 m length.*

(Oct-2001, Madras University)

Solution

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_i r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_o r_3}\right]$$

Downloaded from Ktunotes.in



$$= \frac{1}{2\pi \times 3}\left[\frac{1}{284 \times 0.0254} + \frac{1}{43.26}\, ln\left(\frac{0.0381}{0.0254}\right) + \frac{1}{0.208}\, ln\left[\frac{0.0631}{0.0381}\right] + \frac{1}{17 \times 0.0631}\right]$$

$$= 0.05305\ [0.1386 + 9.37 \times 10^{-3} + 2.426 + 0.9322]$$

$$= 0.18599 \text{ K/W}$$

$$Q = \frac{(\Delta T)_{\text{overall}}}{R} = \frac{316 - 38}{0.18599} = 1494.665 \text{ W.}$$

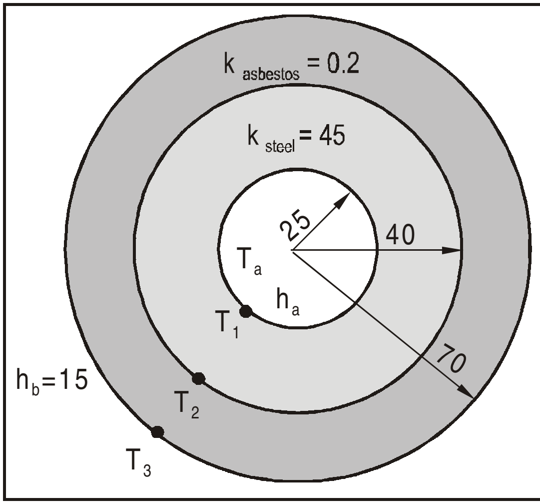
$$\boldsymbol{Q = 1494.665 \text{ W}}$$

Problem 1.12: *A steel tube of 50 mm ID and 80 mm OD is covered by 30 mm thick of asbestos. The thermal conductivity of steel, asbestos are* 45 W/m K, 0.2 W/m K. *The tube receives heat from hot gases at* 400°C *with heat transfer coefficient of* 300 W/m^2°C. *The outer surface is exposed to air at* 30°C *with heat transfer coefficient of* 15 W/m^2 K.

Downloaded from Ktunotes.in

Determine (1) heat loss /m length (2) Interface temperature and surface temperature. (Apr-2000, Madras University)

Solution



$r_1 = 0.025$ m

$r_2 = 0.04$ m

$r_3 = 0.07$ m

$T_{air} = T_b = 30°C$

$h_b = 15$

$h_a = 300$

$T_a = 400°C$

To find Q

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{300 \times 0.025} + \frac{1}{45} ln\left(\frac{40}{25}\right) + \frac{1}{0.2} ln\left(\frac{70}{40}\right) + \frac{1}{15 \times 0.07}\right]$$

$$= \frac{1}{2\pi}[\ 3.894\ ] = 0.6197 \text{ K/W}$$

$$Q = \frac{(\Delta T)_{overall}}{R} = \frac{T_a - T_b}{R} = \frac{(400 - 30)}{0.6197} = 597 \text{ W}$$

Q = 597 Watts .

To find Interface temperatures

To find T_1

$$Q = h_a A\ (T_a - T_1)$$

$$= 300 \times (2\pi r_1 \times L)\ (400 - T_1)$$

Downloaded from Ktunotes.in

$$597 = 47.12 (400 - T_1)$$

$$T_1 = 400 - \left(\frac{597}{47.12} \right)$$

$$= 387.331 \, ^\circ\text{C}$$

$$\boxed{\boldsymbol{T_1 = 387.33 \, ^\circ\text{C.}}}$$

To find T_2

$$Q = \frac{(T_1 - T_2)}{R_1}$$

$$R_1 = \frac{1}{2\pi L} \left[\frac{1}{k_1} \, ln \left[\frac{r_2}{r_1} \right] \right]$$

$$= \frac{1}{2\pi \times 1} \left[\frac{1}{45} \, ln \left(\frac{40}{25} \right) \right]$$

$$= 1.66229 \times 10^{-3}$$

$$Q = \frac{387.33 - T_2}{1.66229 \times 10^{-3}} = 597$$

$$597 \times 1.66229 \times 10^{-3} = (387.33 - T_2)$$

$$T_2 = 387.33 - (597 \times 1.66229 \times 10^{-3})$$

$$= 386.34 \, ^\circ\text{C}$$

$$\boldsymbol{T_2 = 386.34 \, ^\circ\text{C.}}$$

To find T_3

$$Q = h_b A \, (T_3 - T_b)$$

$$597 = 15 \times (2\pi r_3 L) \, (T_3 - 30)$$

Downloaded from Ktunotes.in

$$597 = 15 \times (2\pi \times 0.07 \times 1)\ (T_3 - 30)$$

$$(T_3 - 30) = 90.49$$

$$T_3 = 90.49 + 30 = 120.491\ °C$$

$$\mathbf{T_3 = 120.491\ °C.}$$

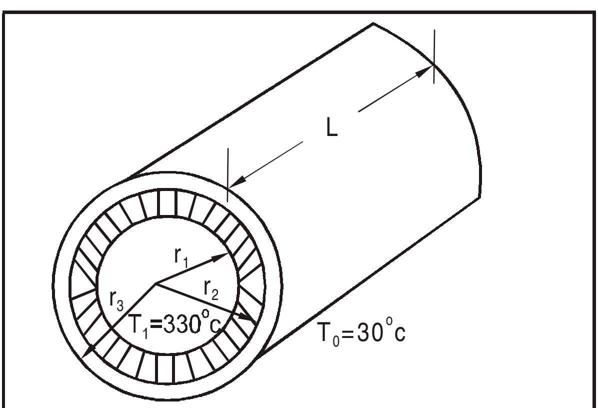
Problem 1.13 *A steel tube of 5 cm ID, 7.6 cm OD and* $k = 15$ W/mK *is covered with an insulation of thickness 2 cm and thermal conductivity. 0.2 W/m.K. A hot gas at* 330°C *and* $h = 400$ W/m^{2}K *flows inside the tube. The outer surface of the insulation is exposed to cold air at* 30°C *with* $h = 50$ W/m^2.K. *Assuming a tube lengbth of 10 m, find the heat loss from the tube to the air. Also find, across which layer the largest temperature drop occurs.*

(May/June 2009 - Anna University)

Given

$r_1 = 2.5$ cm= 0.025 m, $k_1 = 15$ W/mK

$r_2 = 3.8$ cm= 0.038 m, $k_2 = 0.2$ W/mK



Downloaded from Ktunotes.in

$$r_3 = 0.038 + 0.02 = 0.058 \text{ m}$$

Inside temperature, $T_i = 330°\text{C}$

$$h_i = 400 \text{ W/m}^2\text{K}$$

Outside temperature, $T_0 = 30°\text{C}$

$$h_0 = 60 \text{ W/m}^2\text{K}$$

tube length, $L = 10$ m

Heat loss from tube to air (HMT DB pg No. 46)

$$Q = \frac{2\,\pi\,L\,[T_i - T_0]}{\dfrac{1}{h_i\,r_1} + \dfrac{\ln\,(r_2/r_1)}{k_1} + \dfrac{\ln\,(r_3/r_2)}{k_2} + \dfrac{1}{h_0\,r_3}}$$

$$= \frac{2\,\pi \times 10\;(300)}{\dfrac{1}{400 \times 0.025} + \dfrac{\ln\,(0.038/0.025)}{15} + \dfrac{\ln\,(0.058/0.038)}{0.2} + \dfrac{1}{60 \times 0.058}}$$

$$= 7451.77 \text{ W}$$

To find largest temperature drop

$$Q = \frac{2\,\pi\,L\,(T_1 - T_2)}{\dfrac{\ln\,(r_2/r_1)}{k_1}} = \frac{2\,\pi\,L\,(T_2 - T_3)}{\dfrac{\ln\,(r_3/r_2)}{k_2}}$$

$$7451.77 = \frac{2\,\pi \times 10 \times \Delta\,T_1}{\dfrac{\ln\left(\dfrac{0.038}{0.025}\right)}{15}} \Rightarrow \Delta\,T_1 = 33°\text{C}$$

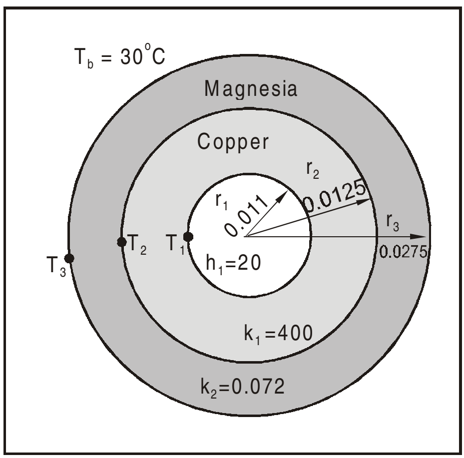
$$7451.77 = \frac{2\,\pi \times 10 \times \Delta\,T_2}{\dfrac{\ln\,(0.058/0.038)}{0.2}} \Rightarrow \Delta\,T_2 = 250.75\,°\text{C}$$

Largest temperature drop occurs in outer layer.

Downloaded from Ktunotes.in

Problem 1.14 *A copper pipe carries steam at* 100°C *for some processing purpose. The OD and ID of pipe are 25 mm and 22 mm respectively. The room temperature is* 30°C. *The pipe is insulated with standard 85% magnesia* $[k = 0.072 \text{ W/m°K}]$ *for a thickness of 15 mm. The cost of insulation per m length is Rs. 10. The cost of heating the steam is 20 paise/3000 KJ.* k *copper* $= 400$ W/m K. *The resistance of fluid film is roughly constant and is equal to* $0.05 \dfrac{\text{m}^2\text{K}}{\text{W}}$. *How long must pipe be in operation to save the insulation cost.* *(Oct-97, Madras University)*

Solution



$T_i = 100 = T_a$

$r_1 = 0.011$ m

$r_2 = 0.0125$ m

$r_3 = 0.0275$ m

$T_{\text{room}} = T_b = 30°\text{C}$

$k_2 = 0.072$ W/m°K

The resistance of fluid film

$$= \frac{1}{h_i} = 0.05 \; \frac{\text{m}^2\,\text{K}}{\text{W}}$$

$$\text{So, } h_i = 20 \; \frac{\text{W}}{\text{m}^2\,\text{K}}$$

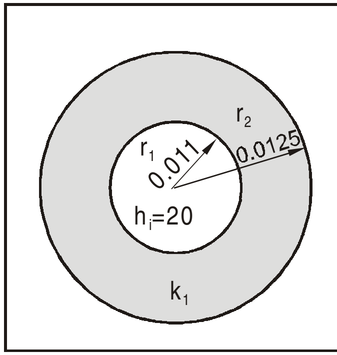
Downloaded from Ktunotes.in

Case A: Q_1 Without insulation

$$R_{ci} = \frac{1}{h_i A_i} = \frac{1}{h_i 2\pi r_i L}$$

$$= \frac{1}{20 \times 2 \times \pi \times 0.011 \times 1}$$

$$\mathbf{= 0.7234\ K/W.}$$



since R_{co} is not given,

we can assume $R_{co} = 0$.

$$R_1 = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right)\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{400} ln\left(\frac{0.0125}{0.011}\right)\right] = \mathbf{5.086 \times 10^{-5}}$$

$$Q_1 = \frac{T_i - T_0}{R}$$

$$Q_1 = R_{ci} + R_1$$

$$= 0.7234 + 5.086 \times 10^{-5}$$

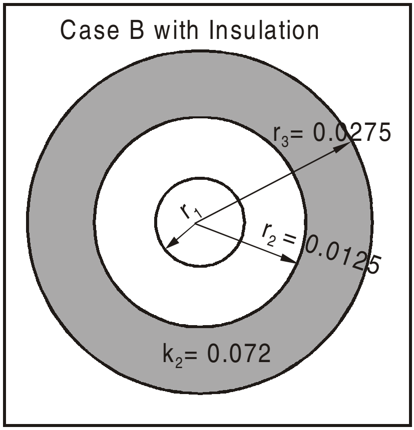
$$= 0.72345\ \text{K/W}$$

$$Q_1 = \frac{100 - 30}{0.72345} = 96.7584\ \text{W}$$

Heat loss without insulation $\boldsymbol{Q_1}$ **= 96.7584 W.**

Downloaded from Ktunotes.in

Case B: Q_2 – With Insulation



$$R = R_{ci} + R_1 + R_2$$

where

$$R_2 = \frac{1}{2\pi \times 1}\left[\frac{1}{k_2} ln\left(\frac{r_3}{r_2}\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{0.072} ln\left[\frac{0.0275}{0.0125}\right]\right]$$

= 1.743 K/W.

$$Q_2 = \frac{T_i - T_0}{0.7234 + 5.086 \times 10^{-5} + 1.743}$$

$$= \frac{100 - 30}{2.466} = 28.382 \text{ W}$$

Q_2 = Heat loss with insulation = **28.382 W.**

Drop in heat loss = Saving in steam

$$= Q_1 - Q_2$$

$$= 96.7584 - 28.382$$

$$= 68.3761 \text{ W.}$$

Let t = operating time in sec.

$$Q_{\text{saving}} \text{ in kJ} = \frac{68.3761}{1000} \times t$$

$$= 0.0683761 \; t \frac{\text{kJ}}{\text{sec}} \times \text{sec}$$

For 3000 kJ, operating cost is 0.20

Downloaded from Ktunotes.in

For 0.0683761 t kJ, operation cost is $\frac{0.2}{3000} \times 0.068371\ t$

$$= 4.55818 \times 10^{-6} t.$$

So, operating cost for 't' sec $= 4.55818 \times 10^{-6} t$

For breakeven, operating cost = Insulation cost

$$4.55818 \times 10^{-6} t = 10$$

$$t = \frac{10}{4.85 \times 10^{-6}} = 2193858\ sec$$

$$t = 609.41\ hrs.$$

After 609.41 hrs. of operation, the operation cost = insulation cost. So the insulation cost will be covered only after 609.41 hrs. i.e. after 25.39 days of operation.

Problem 1.15 *Derive the log mean area of a cylinder used to transform into an equivalent slab. (Nov/Dec 2007 AU)*

Consider a cylinder and slab, both made of same material. Let T_i and T_o be the temperatures maintained on the two sides of plane slab and also on inside and outside of cylinder. (Nov-Dec 2007 - AU)

Heat flow through cylinder = Heat flow through slab

$$\frac{2\pi kL\left(T_i - T_o\right)}{\ln (r_o/r_i)} = \frac{kA_m (T_i - T_o)}{(r_o - r_i)}$$

$$\therefore A_m = \frac{2\pi L (r_o - r_i)}{\ln (r_o/r_i)}$$

(or) $$A_m = \frac{A_o - A_i}{\ln (A_o/A_i)}$$

Downloaded from Ktunotes.in

A_m is called log mean area of the cylinder.

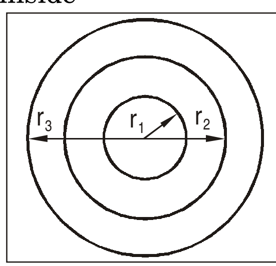
r_m = log mean radius

$$= \left[\frac{r_o - r_i}{\ln \frac{r_o}{r_i}} \right]$$

Problem 1.16 *A 3 cm OD steam pipe is to be covered with two layers of insulation each having a thickness of 2.5 cm. The average thermal conductivity of one insulation is 5 times that of other. Determine the percentage decrease in heat transfer if better insulating material is next to pipe than it in the outer layer. Assume that the outside and inside temperatures of composite insulation are fixed. (May/June 2007 - AU)*

Solution

Case I: when better insulation is inside



$$r_1 = \frac{0.03}{2} = 0.015 \text{ m}$$

$$r_2 = 0.015 + 0.025 = 0.04 \text{ m}$$

$$r_3 = 0.04 + 0.025 = 0.065 \text{ m}$$

$$k_B = 5k_A$$

Heat lost through pipe is given by

$$Q_1 = \frac{2\pi L (T_1 - T_3)}{\dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{k_A} + \dfrac{\ln\left(\dfrac{r_3}{r_2}\right)}{k_B}} = \frac{2\pi L (T_1 - T_3)}{\dfrac{\ln \dfrac{(0.04)}{0.015}}{k_A} + \dfrac{\ln\left(\dfrac{0.065}{0.04}\right)}{5k_A}}$$

$$= 0.9277\, \pi L\, k_A (T_1 - T_3)$$

Downloaded from Ktunotes.in

Case II: When better insulation is outside

$$Q = \frac{2\pi L (T_1 - T_3)}{\dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{k_A} + \dfrac{\ln\left(\dfrac{r_3}{r_2}\right)}{k_B}} = \frac{2\pi L (T_1 - T_3)}{\dfrac{\ln\dfrac{(0.04)}{0.015}}{k_A} + \dfrac{\ln\left(\dfrac{0.065}{0.04}\right)}{5k_A}}$$

$$= 1.4672\ \pi L k_A (T_1 - T_3)$$

$$Q_2 / Q_1 = 1.5807$$

% decrease in heat transfer

$$= \frac{Q_2 - Q_1}{Q} = \frac{Q_2}{Q} - 1 = 1.5807 - 1 = 0.581 \text{ (or) } 58.1\%$$

Problem 1.17 *A 30 mm OD steam pipe is to be covered with two layers of insulation each having a thickness of 20 mm. k. for one insulating material is 5 times that of the other. Which material should be next to the pipe to minimise the heat loss and also determine the percentage of heat decrease in this arrangement. Assume the inside and outside surface temperature of the composite insulation are fixed. (April - 2001, Madras University)*

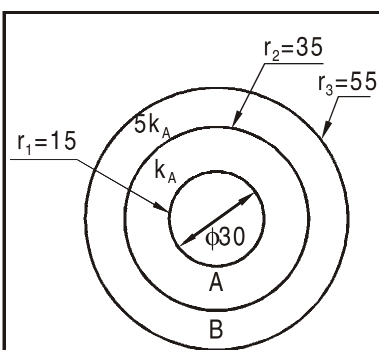
Solution:

Let k_A = Conductivity of material A

k_B = conductivity of material B.

Take $\boldsymbol{k_B = 5k_A}$

So conductivity of material B is more. Now



Case (i)

Downloaded from Ktunotes.in

assume A is nearer to pipe and find Q_1. Given

$r_1 = \frac{30}{2} = 15$ mm; $r_2 = 15 + 20 = 35$ mm

$r_3 = 35 + 20 = 55$ mm

Case (i) (A is nearer to pipe)

$$R = \frac{1}{2\pi L}\left[\frac{1}{k_A}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_B}\ln\left(\frac{r_3}{r_2}\right)\right]$$

$$Q_1 = \frac{\Delta T}{R_1} - \frac{(T_1 - T_2)}{\frac{1}{2\pi L}\left[\frac{1}{k_A}\ln\left(\frac{35}{15}\right) + \frac{1}{k_B}\ln\left(\frac{55}{35}\right)\right]}$$

$$Q_1 = \frac{2\pi L\,(\Delta T)}{\frac{1}{k_A}(0.84729) + \frac{1}{5k_A}(0.451985)}$$

$$= 2\pi k_A\, L\,(\Delta T)\,[1.06645] \quad \ldots \text{(i)}$$

Case (ii) (B is nearer to pipe)

$$R_2 = \frac{1}{2\pi L}\left[\frac{1}{k_B}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_A}\ln\left(\frac{r_3}{r_2}\right)\right]$$

$$Q_2 = \frac{\Delta T}{R_2} = \frac{(T_1 - T_2)}{\frac{1}{2\pi L}\left[\frac{1}{5\,k_A}\ln\left(\frac{35}{15}\right) + \frac{1}{k_A}\ln\left(\frac{55}{35}\right)\right]}$$

$$Q_2 = \frac{2\pi \mathrm{L}\,(\Delta T)}{\frac{1}{k_A}(0.16945 + 0.451985)}$$

$$= 2\pi \mathrm{L}\, k_A\,(\Delta T)\,(1.609178)$$

$$\ldots \text{(ii)}$$

Downloaded from Ktunotes.in

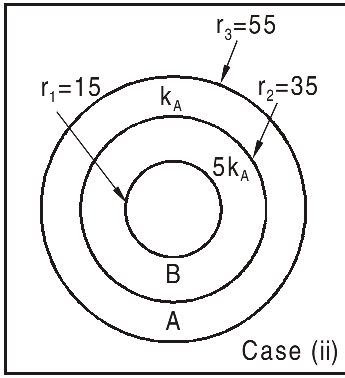
Comparing Q_2 and Q_1

$2\pi L\, k_A\, (\Delta T)\, (1.609178)$

$> 2\pi\, k_A\, L\, (\Delta T)\, (1.06645)$

$Q_2 > Q_1$, since $L, k_A, (\Delta T)$ are same for both equations

To minimise the heat loss, the material 'A' should be next to pipe since Q_1 is loss



1.26 COMPOSITE SPHERE

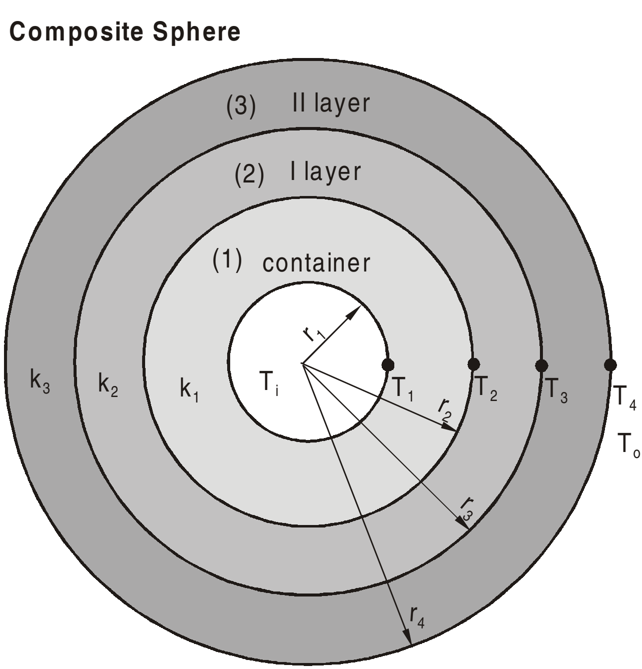


Fig. 1.13

Downloaded from Ktunotes.in

T_i and T_0 – inner and outer fluid temperature respectively.

T_1, T_2, T_3, T_4 – surface temperatures.

k_1, k_2, k_3 – Thermal conductivity of materials 1, 2, 3 respectively.

R_{ci}, R_{co} – inner and outer convective resistances.

R_1, R_2, R_3 – conductive resistance of the material 1, 2, 3 respectively.

$$R_{ci} = \frac{1}{h_i A_i} = \frac{1}{h_i (4\pi r_1^2)}$$

$$R_{co} = \frac{1}{h_0 A_0} = \frac{1}{h_0 (4\pi r_4^2)}$$

$$R_1 = \frac{1}{4\pi k_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$R_2 = \frac{1}{4\pi k_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

$$R_3 = \frac{1}{4\pi k_3}\left[\frac{1}{r_3} - \frac{1}{r_4}\right]$$

$$Q = \frac{T_i - T_0}{R}$$

where $R = R_{ci} + R_1 + R_2 + R_3 + R_{co}$

Also $Q = \dfrac{T_i - T_1}{R_{ci}} = \dfrac{T_1 - T_2}{R_1} = \dfrac{T_2 - T_3}{R_2} = \dfrac{T_3 - T_4}{R_3} = \dfrac{T_4 - T_0}{R_{co}}$

Also $Q = h_1 A_1 (T_i - T_1)$ and

$$Q = h_0 A_0 (T_4 - T_0)$$

Downloaded from Ktunotes.in

The formulae can be taken from HMT table Pg. 46 of CPK as follows:

$$R = \frac{1}{4\pi}\left[\frac{1}{h_1 r_1^2} + \frac{1}{k_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{k_2}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{k_3}\left(\frac{1}{r_3} - \frac{1}{r_4}\right) + \frac{1}{h_0 r_4^2}\right]$$

$$Q = \frac{(\Delta T)_{overall}}{R}$$

Problem 1.18 *Determine the loss of heat through the wall of a rotating sphere shaped boiling pan with an inner diameter* $d_1 = 1.5$ m *and total boiler wall thickness* $t = 20$ cm. *Inner surface temperature is* 200°C *and that of outer surface is* 50°C. *The thermal conductivity of material is* 0.13956 W/m °C. *Also find the heat flux,*

Solution

Given: $r_1 = 0.75$ m, $r_2 = 0.95$ m

From HMT Table Pg. 46 of CPK

$$R = \frac{1}{4\pi}\left[\frac{1}{h_i r_1^2} + \frac{1}{k_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h_0 r_2^2}\right]$$

(Since h_i and h_0 are not given, we can strike off these two terms).

$$R = \frac{1}{4\pi}\left[\frac{1}{0.13957}\left(\frac{1}{0.75} - \frac{1}{0.95}\right)\right]$$

$$= 0.160045 \text{ K/W.}$$

$$Q = \frac{T_1 - T_2}{R}$$

Downloaded from Ktunotes.in

$$= \frac{200 - 50}{0.160045} = 937.23 \text{ W}$$

$$\text{Heat flux } q = \frac{Q}{A}$$

$$= \frac{937.23}{4\pi(0.75)^2} = 132.59 \text{ W/m}^2$$

Problem 1.19 *A hollow sphere of 5 cm ID, 15 cm OD has inner surface temperature* 300°C, *outer surface temperature is* 30°C. *Thermal conductivity is* 18 W/m°C. *Determine*

1. Heat loss by conduction.

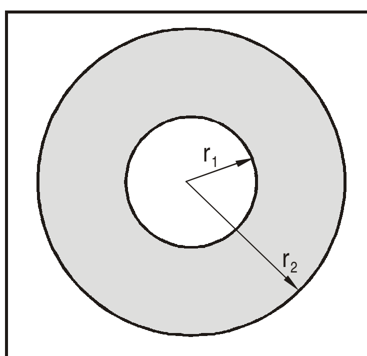
2. Heat loss by conduction, if equation for plane wall is assumed to apply to the sphere with area equal to mean of inner and outer surface area. (Apr-97, Madras University)

Solution

(1) Heat loss by Conduction

$$r_1 = \frac{0.05}{2} = 0.025 \text{ m}$$

$$r_2 = \frac{0.15}{2} = 0.075 \text{ m}$$



$$T_1 = 300°\text{C}$$

$$T_2 = 30°\text{C}$$

$$R = \frac{1}{4\pi k_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{1}{4 \times \pi \times 18}\left[\frac{1}{0.025} - \frac{1}{0.075}\right]$$

$$= 0.11789 \text{ K/W}$$

Downloaded from Ktunotes.in

$$Q = \frac{T_1 - T_2}{0.11789} = \frac{300 - 30}{0.11789} = 2290.22 \text{ W}$$

$$Q = 2290.22 \text{ W.}$$

(2)

$$A = \frac{A_i + A_0}{2}$$

$$= \frac{4\pi r_1^2 + 4\pi r_2^2}{2}$$

$$A = 2\pi r_1^2 + 2\pi r_2^2$$

'R' equation for plane wall $= \dfrac{L}{kA} = \dfrac{r_2 - r_2}{kA}$

$$= \frac{r_2 - r_1}{k\ [\ 2\pi r_1^2 + 2\pi r_2^2\ ]}$$

$$= \frac{0.075 - 0.025}{18 \times [\ 2\pi \times 0.025^2 + 2\pi \times 0.075^2\ ]}$$

$$= \frac{0.05}{0.70685}$$

$$= 0.070735 \text{ K/W}$$

$$Q = \frac{300 - 30}{0.070735} = 3817.03 \text{ W}$$

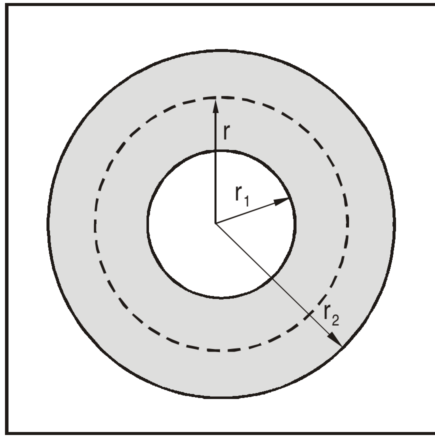
$$\boxed{\mathbf{Q = 3817.03\ W.}}$$

Problem 1.20: *A hollow sphere 1 m ID, 1.6 m OD is having a thermal conductivity of* 1 W/m°C. *The inner surface temperature is* 70 K, *outer surface temperature is* 300 K.

Downloaded from Ktunotes.in

Determine (1) heat transfer rate (2) temperature at a radius of 650 mm.

Solution



$$r_1 = 0.5 \text{ m}$$

$$r_2 = 0.8 \text{ m}$$

$$r = 0.65 \text{ m}$$

$$k = 1 \text{ W/m°C}$$

$$T_1 = 70 \text{ K}$$

$$T_2 = 300 \text{ K}$$

(1)

$$R = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{1}{4\pi \times 1}\left[\frac{1}{0.5} - \frac{1}{0.8}\right] = 0.05968 \text{ K/W}$$

$$\boldsymbol{Q = \frac{70 - 300}{0.05968} = -\ 3853.68 \text{ W}}$$

'–' sign indicates heat is flowing from outer surface (hot side) to inner surface.

(2) When $r = 0.65$ m

$$R_m = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right]$$

$$= \frac{1}{4\pi \times 1}\left[\frac{1}{0.5} - \frac{1}{0.65}\right] = 0.03673 \text{ K/W}$$

In this case also, Q is same.

$$Q = \frac{T_1 - T}{R_m} = \frac{70 - T}{0.03673} = -\ 3853.68 \text{ W}$$

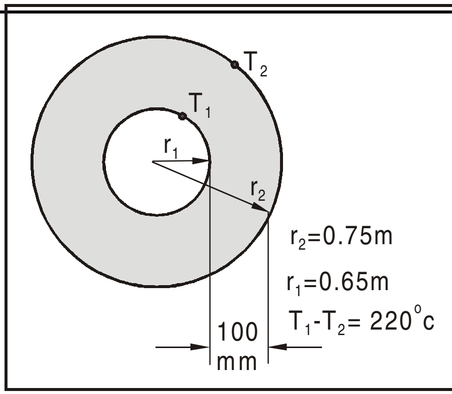
Downloaded from Ktunotes.in

$$T - 70 = 3853.68 \times 0.03673 = 141.53$$

$$\boldsymbol{T = 70 + 141.53 = 211.54 \text{ K.}}$$

Problem 1.21 *A spherical shaped vessel of 1.5 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is* 220°C. *Thermal conductivity of the material of the sphere is* 0.083 W/m°C.

Solution



$$r_2 = 0.75 \text{ m}$$

$$r_1 = 0.65 \text{ m}$$

$$T_1 - T_2 = 220°\text{C}$$

$$R = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{1}{4\pi \times 0.083}\left[\frac{1}{0.65} - \frac{1}{0.75}\right] = 0.19667 \text{ K/W}$$

$$Q = \frac{T_1 - T_2}{R} = \frac{220}{0.19667} = 1118.63 \text{ W}$$

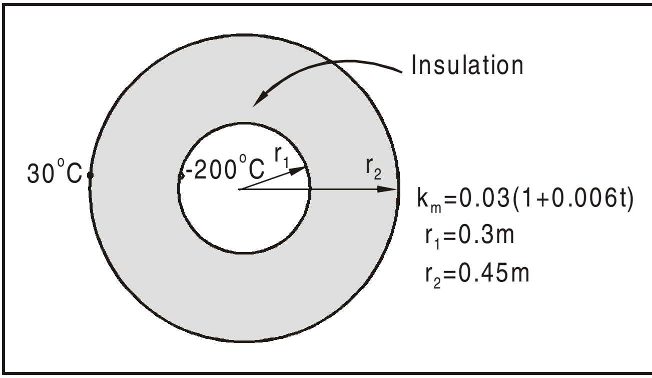
$$\boldsymbol{Q = 1118.63 \text{ W.}}$$

Problem 1.22 *A spherical container having outer diameter 600 mm is insulated by 150 mm thick layer of material with thermal conductivity* k = 0.03(1 + 0.006t) W/m°C, *where t is in* °C. *If the surface temperature of sphere is* – 200°C *and temperature of outer surface is* 30°C, *determine the heat flow*

$$k_m = 0.03(1 + 0.006t)$$

$$r_1 = 0.3 \text{ m} ; r_2 = 0.45 \text{ m}$$

Downloaded from Ktunotes.in



$$k_m = 0.03\left[1 + 0.006\left(\frac{-200 + 30}{2}\right)\right] = 0.0147 \text{ W/m°C}$$

$$Q = \frac{T_1 - T_2}{R}$$

where $R = \dfrac{1}{4\pi k_m}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right] = \dfrac{1}{4\pi \times 0.0147}\left[\dfrac{1}{0.3} - \dfrac{1}{0.45}\right]$

$$= 6.0149 \text{ K/W}$$

$$Q = \frac{-200 - 30}{6.0149} = -38.238 \text{ W}$$

'–' sign indicates heat is conducted from outer surface to inner surface.

1.27 Critical Thickness of Insulation

Insulation prevents heat flow from the system to surroundings. Insulation prevents heat flow from the surroundings to the system. But in some situation, insulation increases the heat transfer rate. Let us see how it is occurred. Heat transfer is directly proportional to the surface area of the cylinder and indirectly proportional to the thickness of the layer.

Downloaded from Ktunotes.in

If we add the insulator around a cable or cylinder, its thickness is increased. So the heat transfer is reduced. At the same time, since the surface area is (in case of cylinder and sphere only) increased, the heat transfer is increased. The Net heat transfer increases. If we go on increasing the thickness of insulator, at particular thickness, the heat transfer starts decreasing.

The thickness upto which heat flow increases and after which heat flow decreases is termed as critical thickness. In case of cylinders and spheres, it is called critical radius.

r_c - The critical radius is defined as outer radius of insulation for which the heat transfer rate is maximum.

1.27.1 Critical thickness:

It is defined as the thickness of insulation for which the heat transfer rate is maximum.

So,

- ❖ Heat transfer increases, when r_0 (outer radius) $\le r_c$.
- ❖ Heat transfer decreases, when $r_0 > r_c$

In case of steam pipe, $r_0 > r_c$ so that heat leakage can be avoided.

In case of electrical cables, insulation is used to increase heat transfer rate in order to safeguard the cable.

$$\text{So, } r_0 < r_c$$

Downloaded from Ktunotes.in

1.27.2 Critical radius of insulation for cylinder

$$R_1 = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right)\right]$$

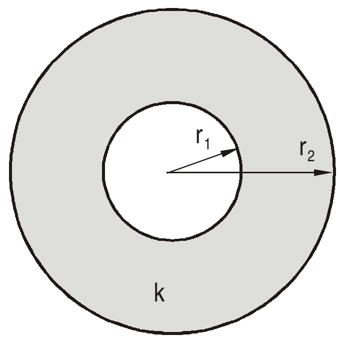


Fig. 1.14 For cylinder

$$R_{co} = \frac{1}{h_0(2\pi r_2 L)}$$

$$R = R_1 + R_{co}$$

$$Q = \frac{\Delta T}{R} = \Delta T(R)^{-1}$$

$$Q = \Delta T\left[\left(\frac{1}{2\pi L}\left(\frac{1}{k_1} ln\left[\frac{r_2}{r_1}\right]\right)\right) + \frac{1}{h_0\,(2\pi r_2 L)}\right]^{-1}$$

To get $Q_{\max}$, $\dfrac{dQ}{dr_2} = 0$

$$\frac{dQ}{dr_2} = \Delta T\,(-1)\left[\left(\frac{1}{2\pi l}\left(\frac{1}{k_1} ln\left[\frac{r_2}{r_1}\right]\right)\right) + \frac{1}{h_0\,(2\pi r_2 L)}\right]^{-2} \times$$

$$\left[\frac{1}{2\pi k_1 L}\frac{1}{\left(r_2/r_1\right)} \times \frac{1}{r_1} + \frac{1}{h_0 2\pi L}\left(\frac{1}{r_2^2}\right)\right] = 0$$

i.e. $$\frac{1}{2\pi k_1 L r_2} - \frac{1}{h_0 2\pi L r_2^2} = 0$$

$$\frac{1}{k_1 r_2} - \frac{1}{h_0 r_2^2} = 0$$

$$\frac{1}{k_1 r_2} = \frac{1}{h_0 r_2^2}$$

Downloaded from Ktunotes.in

$$\frac{1}{k_1} = \frac{1}{h_0 r_2} \Rightarrow r_2 = \frac{k_1}{h_0}$$

$$r_2 = r_0 = r_c = \text{critical radius} \Rightarrow r_c = \frac{k}{h_0}$$

1.27.3 Critical radius of insulation for sphere

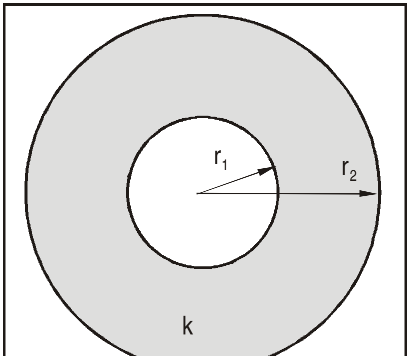


Fig. 1.15

$$R = R_1 + R_{co}$$

$$R_1 = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$R_{co} = \frac{1}{h_0(4\pi r_2^2)}$$

$$Q = \frac{\Delta T}{R} = \Delta T (R)^{-1}$$

$$= \Delta T \left[\left[\frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right] + \frac{1}{h_0(4\pi r_2^2)}\right]^{-1}$$

To get $Q_{\max}$,

$$\frac{dQ}{dr_2} = 0$$

$$= \Delta T(-1)\left[\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + \frac{1}{h_0(4\pi r_2^2)}\right]^{-2} \times$$

$$\left[\frac{1}{4\pi k}\left(0 + \frac{1}{r_2^2}\right) + \frac{1}{h_0 4\pi}\left(\frac{-2}{r_2^3}\right)\right] = 0$$

$$\frac{1}{4\pi k r_2^2} - \frac{2}{h_0 4\pi r_2^3} = 0$$

Downloaded from Ktunotes.in

$$\frac{1}{4\pi k r_2^2} = \frac{2}{h_0 4\pi r_2^3}$$

$$\frac{1}{k} = \frac{2}{h_0 r_2}$$

$$r_2 = \frac{2k}{h_0}$$

$$\boxed{\boldsymbol{r_2 = r_c = \frac{2k}{h_0}}}$$

1.27.4 Important Points to be noted

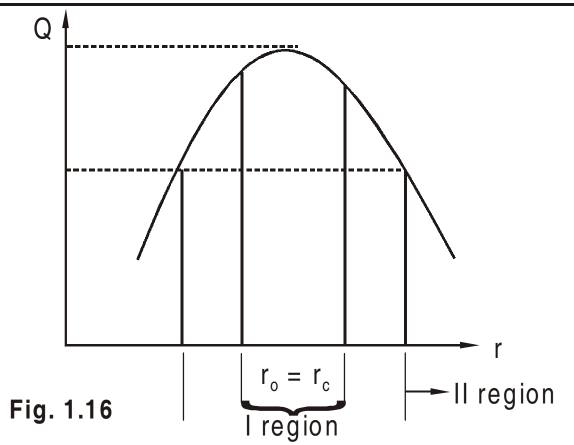


Fig. 1.16

1. When $r_o = r_c$, then Q will be Q_{max}.
2. Region I is the recommended region i.e., recommended outer radius of insulation for electrical cable so that heat transfer is increased.
3. Region II is the recommended outer radius of insulation for steam carrying pipe (for decreasing heat transfer).

Downloaded from Ktunotes.in

Problem 1.23 *A steam carrying pipe of 100 mm ID, 110 mm OD is carrying steam at* 300°C. *This pipe is covered with asbestos of* $k = 1$ W/m°C. *The surrounding air temperature is* 20°C *with heat transfer coefficient of* $8\ W/m^2$°C. k *for pipe* $= 53.605$ W/m°C.

1. Determine heat transfer rate when the thickness of insulation are 50 mm, 70 mm, 90 mm.
2. Find critical thickness and corresponding Q_{max}.

$k_1 = k$ for pipe $= 53.605$ W/m°C

Since h_i is not given, $R_{ci} = 0$; $L = 1$ m

$$r_1 = \frac{\text{Pipe inner dia}}{2} = \frac{100}{2} = 50 \text{ mm} = 0.05 \text{ m}$$

$$r_2 = \frac{\text{Pipe outer dia}}{2} = \frac{110}{2} = 55 \text{ mm} = 0.055 \text{ m}$$

$$h_0 = 8 \text{ W/m}^2\text{°C}$$

Q – when layer thickness is 50 mm ($r_0 = 0.105$)

$$r_0 = r_2 + \text{thickness of layer}$$

$$= 55 + 50 = 105 \text{ mm} = 0.105 \text{ m}$$

$$\mathbf{r_0 = 0.105 \text{ m}}$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_0}{r_2}\right) + \frac{1}{h_0 r_0}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{53.605} ln\left(\frac{0.055}{0.05}\right) + \frac{1}{1} ln\left(\frac{0.105}{0.055}\right) + \frac{1}{8 \times 0.105}\right]$$

$$= 0.293 \text{ K/W}$$

$$Q = \frac{T_i - T_0}{R} = \frac{300 - 20}{0.293} = \mathbf{955.63 \text{ W.}}$$

Downloaded from Ktunotes.in

Q when layer thickness is 70 mm ($r_0 = 0.125$ m)

$r_0 = 55 + 70 = 125 \text{ mm} = 0.125 \text{ m}$

$$R = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_0}{r_2}\right) + \frac{1}{h_0 r_0}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{53.605} ln\left(\frac{0.055}{0.05}\right) + \frac{1}{1} ln\left(\frac{0.125}{0.055}\right) + \frac{1}{8 \times 0.125}\right]$$

$$= \frac{1}{2\pi}[\,1.778 \times 10^{-3} + 0.82098 + 1\,]$$

$$= 0.2901 \text{ K/W}$$

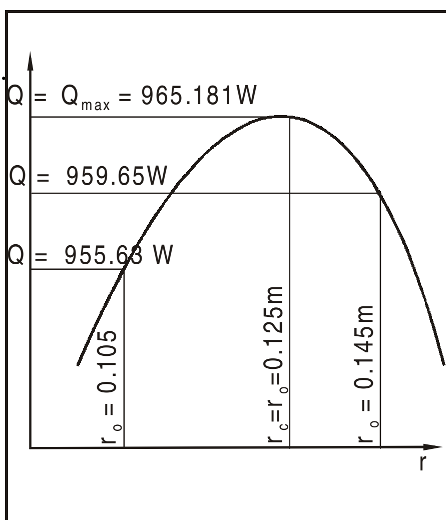
$$Q = \frac{300 - 20}{0.2901} = \mathbf{965.181.}$$

Q when layer thickness is 90 mm ($r_0 = 0.145$ m)

$$R = \frac{1}{2\pi \times 1}\left[1.778 \times 10^{-3} + \frac{1}{1} ln\left(\frac{0.145}{0.055}\right) + \frac{1}{8 \times 0.145}\right]$$

$$= 0.29177 \text{ K/W}$$

$$Q = \frac{280}{0.29177} = \mathbf{959.65\ W.}$$



To find critical radius 'r_c'

$$r_c = \frac{k_2}{h_0} = \frac{1}{8} = 0.125 \text{ m}$$

[k for asbestos]

when $r_c = r_0 = 0.125$ m

Q will be

$Q_{max} = 965.181$ W.

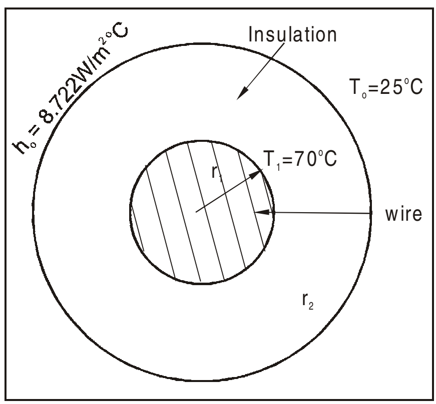
Downloaded from Ktunotes.in

To find critical thickness

$$r_c - r_2 = 0.125 - 0.55 = 0.7 \text{ m}$$

Critical thickness = 70 mm.

Problem 1.24 *A wire of 8 mm diameter at a temperature of* 70°C *is to be insulated by a material having* $k = 0.174$ W/m°C. *Convection heat transfer coefficient* $(h_0) = 8.722$ W/m^2°C. *The ambient temperature is* 25°C. *For maximum heat dissipation, what is the minimum thickness of insulation and heat loss per metre length? Also find the % increase in the heat dissipation too.*



Solution

$$r_1 = \frac{8}{2} = 4 \text{ mm} = 0.004 \text{ m}$$

$$k_2 = 0.174 \text{ W/m°C.}$$

For maximum heat dissipation, critical radius of insulation $(r_0 = r_c)$ is recommended:

$$\text{So, } r_0 = r_c = \frac{k_2}{h_0} = \frac{0.174}{8.722}$$

$$= 0.01995 \text{ m.}$$

$$\text{Minimum thickness } = r_0 - r_1 = 0.01995 - 0.004$$

$$= 0.01595 \text{ m.}$$

Downloaded from Ktunotes.in

Case (i) Heat the loss without insulation Q_1

$$Q = \frac{\Delta T}{R} = \frac{T_1 - T_0}{R} = \frac{70 - 25}{R}$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left(\frac{r_1}{r_i}\right) + \frac{1}{h_0 r_1}\right]$$

From Pg. 46 of CPK

[Since k_1 is not given, we can strike off the term in the formula]

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{8.722 \times 0.004}\right]$$

$$= 4.56188 \text{ K/W.}$$

$$Q = \frac{45}{4.56188} = 9.864 \text{ W/m.}$$

Case (ii) Q_2 – Heat loss with insulation of $r_0 = r_c$

$$R = \frac{1}{2\pi l}\left[\frac{1}{k_2} ln\left(\frac{r_0}{r_1}\right) + \frac{1}{h_0 r_0}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{0.174} ln\left(\frac{0.01995}{0.004}\right) + \frac{1}{8.722 \times 0.01995}\right]$$

$$= 2.3845 \text{ K/W}$$

$$Q_2 = \frac{70 - 25}{2.3845} = 18.872 \text{ W/m}$$

% increase in heat dissipation $= \dfrac{Q_2 - Q_1}{Q_1}$

$$= \frac{18.872 - 9.864}{9.864} = 91.32\%.$$

Downloaded from Ktunotes.in

Problem 1.25: *A pipe having OD 40 mm is required to be thermally insulated. The outside air film coefficient of heat transfer is* 12 W/m^2°C. k for insulation = 0.3 W/m°C.

1. Determine whether the insulation will be effective.

2. Find the maximum value of thermal conductivity of insulating material to reduce the heat transfer.

Solution

Given: $r_0 = \dfrac{40}{2} = 20 \text{ mm} = 0.02 \text{ m}$

$$h_0 = 12 \text{ W/m}^2{}^\circ\text{C}$$

$$k_{\text{insulation}} = 0.3 \text{ W/m}^\circ\text{C}$$

To find critical radius r_c

$$r_c = \frac{k_{\text{ins}}}{h_0} = \frac{0.3}{12} = 0.025 \text{ m}$$

i.e. 25 mm.

Since $\underset{(20\text{ mm})}{r_0} < \underset{(25\text{ mm})}{r_c}$, heat transfer will increase by adding insulation and thus it is not effective.

(2) For insulation to be effective what is *k*?

For insulation to be effective, $r_0 > r_c$

$$\text{i.e.,} \quad 0.02 > \frac{k_{insulation}}{h_0}$$

$$0.02 > \frac{k_{\text{insulation}}}{12}$$

$$k_{\text{insulation}} < 0.02 \times 12$$

Downloaded from Ktunotes.in

i.e., $k_{insulation} < 0.24$ W/m°C

k for insulation should be less than 0.24 W/m°C.

1.28 *Q* – HEAT GENERATION RATE

When electrical current passes through a conductor, heat is generated and it is given by $Q = I^2 R_e$

where $R_e = \rho \times \dfrac{L}{a}$

where ρ= resistivity or specific resistance

a = cross sectional area (πr^2) of the conductor

L = length of the conductor

I = current flowing in the conductor

R_e = electrical resistance

k_e = electrical conductivity (reciprocal of ρ).

Problem 1.26 *An electrical wire of 10 mm dia is covered with 10 mm thickness of plastic insulation. The insulation is exposed to air at* 35°C *with heat transfer coefficient of* 8 W/m²°C. *The temperature of wire surface is* 180°C. *Determine heat transfer rate, current carrying capacity, max. heat transfer rate max. heat carrying capacity. k for copperwire* = 400 W/m°C, k *for plastic* = 0.5 W/m°C, ρ = resistivity = 3×10^{-6} ohm–m.

Solution

To find the heat transfer rate

Downloaded from Ktunotes.in

$$r_1 = \frac{10}{2} = 5 \text{ mm}$$

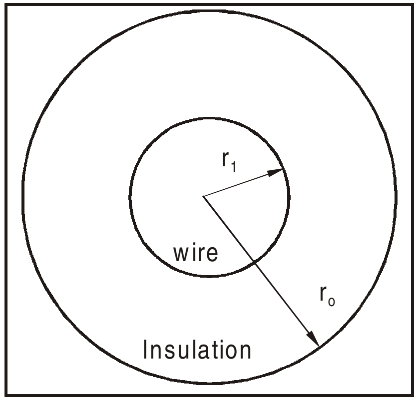
$$r_0 = 5 + 10 = 15 \text{ mm}$$

$$T_0 = 35°\text{C}$$

$$h = 8 \text{ W/m}^2°\text{C}$$

$$T_1 = 180°\text{C}$$

$$k = 0.5 \text{ W/m°C}$$



$$R = \frac{1}{2\,\pi\,L}\left(\frac{1}{k_{\text{insulator}} \cdot \ln\left(\frac{r_0}{r_1}\right) + \frac{1}{h_0\, r_0}}\right)$$

$$R = \frac{1}{2\pi \times 1}\left[\frac{1}{0.5}\, ln\left(\frac{15}{5}\right) + \frac{1}{8 \times 0.015}\right]$$

$$= \frac{1}{2\pi}\,[\,2.19722 + 8.333\,]$$

$$= 1.6759 \text{ K/W.}$$

$$Q = \frac{T_1 - T_0}{R} = \frac{180 - 35}{1.6759} = \mathbf{86.51628\ \ W}.$$

Heat transfer rate $= 86.51628$ W and this heat is generated.

To find current carrying capacity '*I*'

$$Q = 86.51628 = I^2 R_e$$

$$I^2 = \frac{86.51628}{R_e}$$

Downloaded from Ktunotes.in

To find R_e (Electrical Resistance)

$$R_e = \rho \frac{L}{A}$$

$$= 3 \times 10^{-6} \times \frac{1}{\pi(0.005)^2}$$

$$= 0.038197 \text{ ohm}$$

$$I^2 = \frac{86.51628}{R_c}$$

$$= \frac{86.51628}{0.038197} = 2264.99$$

$$\boldsymbol{I = 47.592} \textbf{ amp.}$$

To find max. heat transfer rate Q_{max}

For $Q_{\max}$, $r_0 = r_c$

$$r_c = \frac{k}{h_0} = \frac{0.5}{8} = 0.0625 \text{ m}$$

$$r_1 = 0.005 \text{ m.}$$

$$R = \frac{1}{2\pi \times 1}\left[\frac{1}{k} ln\left(\frac{r_0}{r_1}\right) + \frac{1}{h_0 r_0}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{0.5} ln\left(\frac{0.0625}{0.005}\right) + \frac{1}{8 \times 0.0625}\right]$$

$$= \frac{1}{2\pi \times 1}[5.05145 + 2]$$

$$= 1.12227 \text{ K/W.}$$

$$Q_{\max} = \frac{T_1 - T_0}{R} = \frac{180 - 35}{R} = \frac{145}{1.12227} = 129.202$$

Downloaded from Ktunotes.in

$$\boldsymbol{Q_{max} = 129.202 \text{ W.}}$$

To find max. heat carrying capacity I_{max}

$$Q_{max} = 129.202 = I_{max}^2 R_e$$

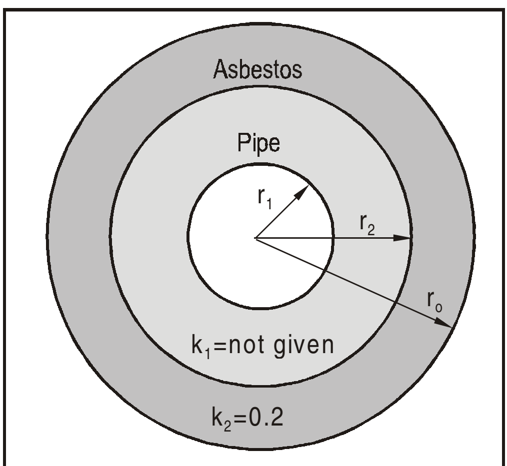
where $R_e = 0.038197$ ohm (already found out).

$$I_{max}^2 = \frac{129.202}{0.038197} = 3382.52$$

$$\boldsymbol{I_{max} = 58.1594 \text{ amp.}}$$

Problem 1.27 *Steam at* 150°C *passes through a 20 mm OD pipe which is insulated with asbestos* ($k = 0.2$ W/m°C). *It is exposed to air at* 35°C *with heat transfer coefficient* (6 W/m^2°C). *Determine (1) Critical thickness, (2) Corresponding* $\boldsymbol{Q_{max}}$.

Solution



$$r_2 = \frac{20}{2} = 10 \text{ mm}$$

$$= 0.01 \text{ m}$$

$$T_i = 150\,°\text{C}$$

$$T_0 = 35°\text{C}$$

$$h_0 = 6 \text{ W/m}^2\text{°C}$$

1. To find critical thickness $(r_0 - r_2)$ or $(r_c - r_2)$

$$\text{Critical radius } r_c = \frac{k_{insulator}}{h_0} = \frac{k_2}{h_0} = \frac{0.2}{6} = 0.03333$$

Downloaded from Ktunotes.in

$$\text{Critical thickness} = r_c - r_2$$

$$= 0.0333 - 0.01$$

$$= 0.02333 \text{ m}$$

$$= 23.33 \text{ mm}$$

2. To find corresponding Q_{max}

$$Q_{max} = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi \times L}\left[\frac{1}{k_2} ln\left(\frac{r_0}{r_2}\right) + \frac{1}{h_0 r_0}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{0.2} ln\left(\frac{33.3}{10}\right) + \frac{1}{6 \times 0.0333}\right]$$

$$= \frac{1}{2\pi}[\ 5.9696 + 5\ ]$$

$$= 1.745$$

$$Q_{max} = \frac{T_1 - T_0}{R}$$

$$= \frac{150 - 35}{1.74595} = 65.867 \text{ W}$$

$$\boldsymbol{Q_{max} = 65.867 \text{ W.}}$$

Problem 1.28 *A sphere of 10 mm dia is exposed to a convection environment with* $h = 10$ W/m^2°C, *by enclosing it in a spherical sheath of* $k = 0.04$ W/m°C. $\Delta T = 120$°C

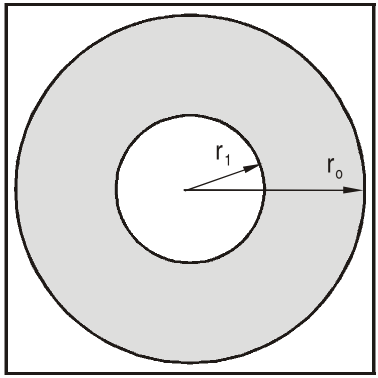
Find out (1) the thickness of insulation for which heat transfer rate is maximum

(2) Heat transfer rate from bare sphere.

(3) Heat transfer rate from sheathed sphere.

Downloaded from Ktunotes.in

1. To find the critical thickness



$$r_1 = \frac{10}{2} = 5 \text{ mm}$$

$$h_0 = 10 \text{ W/m}^2\text{°C}$$

$$k = 0.04 \text{ W/m°C}$$

For Q_{max} , r should be $= r_c$.

$$\text{For sphere, } r_c = \frac{2k}{h_0} = \frac{2 \times 0.04}{10} = 8 \times 10^{-3} \text{ m} = r_0$$

Critical radius $= 8 \times 10^{-3}$ m

$$\text{Critical thickness} = r_c - r_1 = r_0 - r_1$$

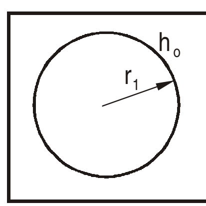
$$= 8 \times 10^{-3} - 0.005$$

$$= 3 \times 10^{-3} \text{ m}$$

$$= 3 \text{ mm}$$

Thickness of insulation = 3 mm

Q_1 Heat transfer from bare sphere



$$R_{co} = \frac{1}{h_0(4\pi r^2)}$$

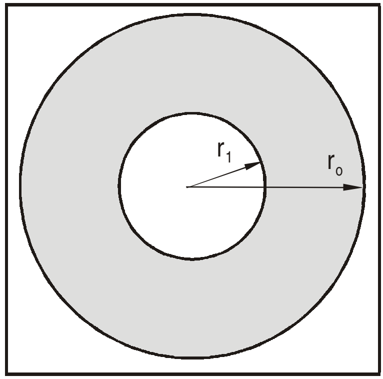
$$= \frac{1}{10(4\pi \times 0.005^2)} = 318.309 \text{ K/W}$$

$$Q_{\text{bare}} = \frac{T_1 - T_0}{R_{co}} = \frac{120}{318.309} = 0.37699 \text{ W}$$

$$\mathbf{Q_{bare} = 0.37699 \text{ W.}}$$

Downloaded from Ktunotes.in

Q_2 – Heat transfer from sheathed sphere



$r_0 = 0.008$ m

$r_1 = 0.005$ m

$k = 0.04$

$h_0 = 10$

$$R_1 = \frac{1}{4\pi k_2}\left[\frac{1}{r_1} - \frac{1}{r_0}\right]$$

$$R_{co} = \frac{1}{h_0(4\pi r_0^2)}$$

$$R_1 = \frac{1}{4\pi \times 0.04}\left[\frac{1}{0.005} - \frac{1}{0.008}\right]$$

$$= 149.207 \text{ K/W}$$

$$R_{co} = \frac{1}{10(4\pi \times 0.008^2)} = 124.339 \text{ K/W}$$

$$R = R_1 + R_{co} = 149.207 + 124.333$$

$$= 273.5467 \text{ K/W}$$

$$Q = \frac{\Delta T}{R} = \frac{120}{273.5467}$$

$$= 0.43868 \text{ W}$$

$\boldsymbol{Q_{\text{sheathed}} = 0.43868}$ **W.**

So Q will be maximum (0.4386 W), when sphere is sheathed.

Downloaded from Ktunotes.in

1.29 PLANE WALL HEAT GENERATION

Heat is generated within the system in many cases. The heat generation rate should be controlled, otherwise equipment (or) device may fail. Therefore, while designing thermal equipments, temperature distribution within the system and the rate of heat dissipation to the surroundings should be considered. Heat generating systems are 1. fuel rod (nuclear reactor), 2. electrical conductor, 3. chemically reacting systems, etc. Figure 1.17 shows the plane wall with uniform heat generation.

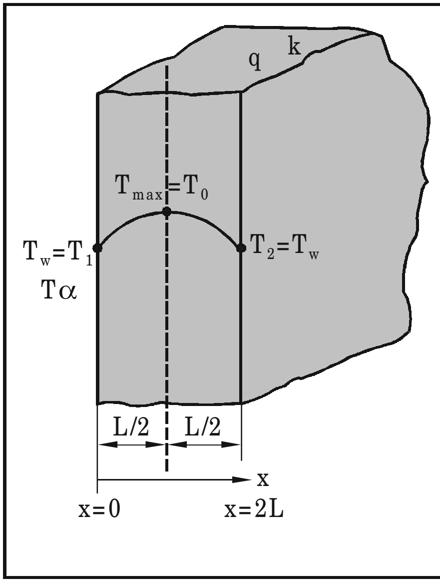


Fig. 1.17 Plane wall with uniform heat generation: both the surfaces of the wall have the same temperature

Consider a plane wall of thickness 2 L with uniform internal generation of heat and with uniform thermal conductivity. The wall surfaces are maintained at temperatures T_1 and T_2. Now, consider heat flow in one direction and steady-state conditions.

The equation for heat generation is

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0 \left[\because \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial z^2} = 0 \text{ and } \frac{\partial T}{\partial t} = 0 \right] \quad ...(1)$$

Downloaded from Ktunotes.in

Integrating this Eq. (1) twice, we get

$$\frac{dT}{dx} = \frac{-q_x}{k} + C_1 \qquad ...(2)$$

$$T = \frac{-\dot{q}\, x}{k} + C_1$$

$$T = \frac{-\dot{q}\, x^2}{2k} + C_1 x + C_2 \qquad ...(3)$$

Case A: When both the surfaces of the wall have the same temperature [Fig. 1.17]

$$(T_1 = T_2 = T_w)$$

The boundary conditions are

At $x = 0, T = T_1$

and at $x = 2L, T = T_1$

Substituting the above boundary conditions in Eq. (3)

At $x = 0, T = T_1$: $\boldsymbol{T_1 = C_2}$ and at $x = 2L, T = T_1$: $\boldsymbol{C_1 = \frac{\dot{q}\, L}{k}}$

Substituting C_1 and C_2 into Eq. (3), we get

$$T = \frac{-\dot{q}\, x^2}{2k} + \frac{\dot{q}\, Lx}{k} + T_1$$

$$.T = \frac{\dot{q}}{2k}(Lx - x^2) + T_1 \qquad ...(4)$$

Downloaded from Ktunotes.in

Equation (4) is the temperature distribution eqn. To determine the maximum temperature and its location, we can differentiate Eq. (4) w.r.t x and equate to zero, i.e.

$$\frac{dT}{dx} = \frac{d}{dx}\left(\frac{q}{2k}(Lx - x^2)\right) = 0 \quad ...(5)$$

$$2L - 2x = 0; L = x$$

Hence, the maximum temperature occurs at $x = L$. Substituting this in Eq. (4) to get the maximum temperature (T_{max}),

$$T_{max} = \frac{\dot{q}}{2k}(2L^2 - L^2) + T_1$$

$$T_{max} = \frac{\dot{q}\,L^2}{2k} + T_1 = \frac{qL^2}{2k} + T_w \quad ...(6)$$

$$(\because T_1 = T_w)$$

Heat transfer takes place from both the sides (i.e. $x = 0, x = L$) and is equal. This can be given by Fourier's equation,

$$Q = -kA\frac{dT}{dx}$$

$$\frac{dT}{dx} = \frac{\dot{q}}{k}(L - x)$$

$$Q = -kA\frac{\dot{q}}{k}(L - x)$$

When $x = 0$ and $x = 2L$, we get

i.e. $Q = AL\dot{q}$ for each surface. ...(7)

Downloaded from Ktunotes.in

Here (Q) at each surface is further convected to the surroundings at temperature T_α

$$Q = AL_{\dot{q}} = hA\,(T_1 - T_\alpha)$$

$$T_1 = T_\alpha + \frac{\dot{q}}{h} L = T_w \quad \text{...(8)}$$

Refer formula from HMT DB Pg No. 48

Case B: When the surfaces of the wall have different temperatures [Figure 1.18]

Two boundary conditions are sufficient for the determination of the solution for temperature distribution

At $x = 0$, $T = T_1$;

At $x = 2L$, $T = T_2$

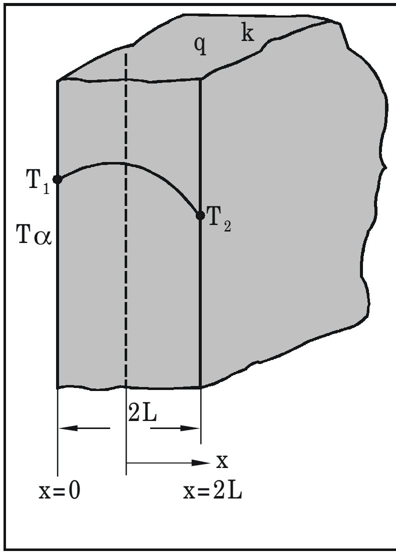


Fig. 1.18 Plane wall with uniform heat generation: - both the surfaces of the wall have different temperatures

Substituting the above boundary conditions in Eqn (3), we get

$C_2 = T_1$ and

$$C_1 = \frac{T_2 - T_1}{2L} + \frac{\dot{q}\,L}{k} \quad \text{...(9)}$$

Substituting C_1 and C_2 in Eq. (3), we get

Downloaded from Ktunotes.in

$$T = \frac{-\dot{q}\, x^2}{2k} + \left[\frac{T_2 - T_1}{2L} + \frac{\dot{q}\, L}{k} \right] x + T_1$$

$$T = \frac{\dot{q}\, Lx}{2k} - \frac{\dot{q}\, x^2}{k} + \frac{x}{2L} (T_2 - T_1) + T_1$$

$$T = x \left[\frac{\dot{q}\, (2L - x)}{2k} + \frac{(T_2 - T_1)}{2L} \right] + T_1 \qquad ...(10)$$

Equation (10) gives the temperature distribution T, if both the sides of the plane have different temperatures.

Heat transfer takes place from both the sides. This can be given by the Fourier's equation

$$Q = -kA \frac{dT}{dx} \qquad ...(11)$$

Differentiate equ (10), we get

$$\frac{dT}{dx} = \frac{\dot{q}}{2k} (2L - 2x) + \frac{T_2 - T_1}{2L}$$

Substituting the above equation in Eq. (11) (Fourier equation),

$$Q = -kA \left[\frac{\dot{q}}{2k} (2L - 2x) + \frac{T_2 - T_1}{2L} \right]$$

$$\text{At } x = 0, \Rightarrow Q = -kA \left[\frac{\dot{q}}{k} L + \frac{T_2 - T_1}{2L} \right] \qquad ...(12)$$

$$\text{At } x = 2L, \Rightarrow Q = -kA \left[\frac{\dot{q}\, (-L)}{k} + \frac{T_2 - T_1}{2L} \right] \qquad ...(13)$$

Heat (Q) at each surface is further convected to the surroundings at temperature T_∞. Hence.

Downloaded from Ktunotes.in

$$Q = -kA\left[\frac{\dot{q}L}{k} + \frac{T_2 - T_1}{2L}\right] = hA\ (T_1 - T_\alpha),\ \text{at } x = 0 \quad ...(14)$$

$$Q = -kA\left[\frac{\dot{q}L}{k} + \frac{T_1 - T_2}{2L}\right] = hA\ (T_2 - T_\alpha),\ \text{at } x = 2L \quad ...(15)$$

For formulae, refer pg 48 of HMT DB.

Problem 1.29 *Heat is generated in a slab of thickness 150 mm and* $k = 125$ W/m – K *at the rate of* 1.5×10^7 W/m^3. *The temperature on either side of the slab is* 130°C. *Calculate.*

(a) The maximum temperature

(b) The temperature at one-fourth distance from left end.

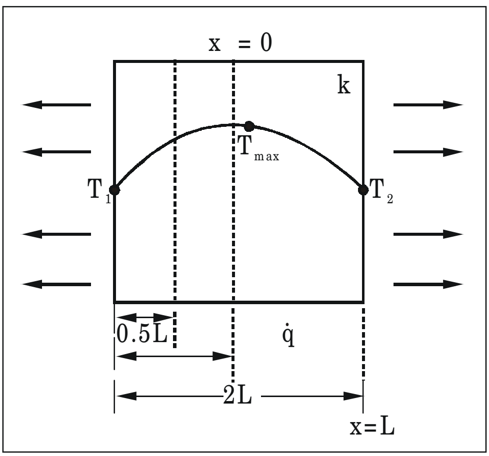
(c) The heat flow rate at quarter plane

Solution

$k = 125$ W/m – K, $\dot{q} = 1.5 \times 10^7$ W/m^3,

$$L = \text{Half of thickness} = \frac{150}{2} = 75 \text{ mm} = 0.075 \text{ m}$$

$T_1 = T_2 = 130°C$



Downloaded from Ktunotes.in

(a) The maximum temperature (T_{max})

It is clear that maximum temperature occurs at $x = 0$

$$\therefore \qquad x = \frac{150}{2} = 75 \text{ mm}$$

$$T_0 = T_{max} = \frac{\dot{q}}{2k} L^2 + T_1$$ [refer HMT DB Pg. 48]

$$\text{or } T_0 = T_{max} = \frac{1.5 \times 10^7}{2 \times 125} \times 0.075^2 + 130$$

$$= 467.5\,°\text{C}$$

(b) The temperature at one-fourth distance from left end, i.e. $x = 0.5L = 0.5 \times 0.075 = 0.0375$ m

$$T = T_0 - \frac{\dot{q}x^2}{2k} = 467.5 - \frac{1.5 \times 10^7 \times 0.0375^2}{2 \times 125}$$

$$\therefore T = 383.125\,°\text{C}$$

(c) The heat flow rate at quarter plane (q_x at 0.25L). Heat transfer at any distance is given by

At Quarter plane, $q_x = \dot{q} \times x$

$$\therefore \qquad = 1.5 \times 10^7 \times 0.0375 \text{ m} = \mathbf{5.625 \times 10^5 \ W/m^2}$$

Problem 1.30 *A Plane wall 10 cm thick generates heat at the rate of* 4×10^4 W/m^3 *when an electric current is passed through it. The convective heat transfer co-efficient between each face of the wall and the ambient air is* 50 W/m^2 K. *Determine.*

(a) the surface temperature

(b) the maximum air temperature on the wall. Assume the

Downloaded from Ktunotes.in

ambient air temperature to be 20°C and the thermal conductivity of the wall material to be 15 W/mK.

[Madras University April 98]

Solution

Given:

Half Thickness $= L = \frac{10}{2}$ cm $= 0.05$ m

Heat generation, $\dot{q} = 4 \times 10^4$ W/m^3

Convective heat transfer co-efficient, $h = 50$ W/m^{2}K'

Ambient air temperature, $T_\infty = 20°C + 273 = 293\ K$

Thermal conductivity, $k = 15$ W/mK.

From HMT DB page 48

Surface temperature, $T_w = T_\infty + \frac{\dot{q}L}{h}$

$$= 293 + \frac{4 \times 10^4 \times 0.05}{50}$$

$$\boxed{T_w = 333\ K.}$$

Maximum temperature occurs at mid of slab.

Maximum temperature, $T_{max} = T_w + \frac{\dot{q}L^2}{2k}$

$$= 333 + \frac{4 \times 10^4 \times (0.05)^2}{2 \times 15}$$

$$\boxed{T_{max} = 336.3\ K}$$

Downloaded from Ktunotes.in

Problem 1.31 *A concrete wall of 1 m thick is poured with concrete. The hydration of concrete generates* 150 W/m^3 *heat. If both the surfaces of the wall are maintained at* $35°\text{C}$*, find the maximum temperature in the wall.*

[Madras University April 99]

Solution

Given:

Thickness, $L =$ Half of thickness $= \frac{1}{2} = 0.5$ m

Heat generation, $\dot{q} = 150 \text{ W/m}^3$

Surface temperature, $T_w = 35°\text{C} + 273 = 308$ K

From HMT data book, page 48

Maximum temperature present in mid of the wall.

$$T_{max} = T_0 = T_w + \frac{\dot{q}L^2}{2k}$$

Thermal conductivity of concrete, $k = 1.279$ W/mK

[From HMT data book P.No.19]

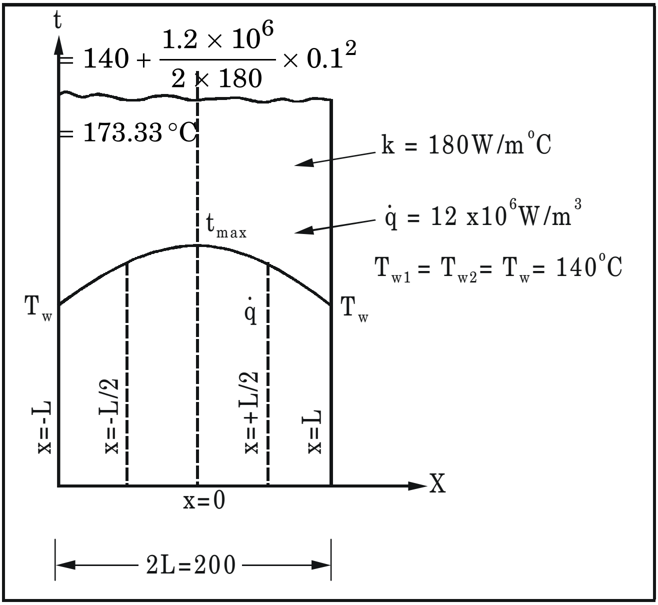
$$T_{max} = 308 + \frac{150 \times 0.5^2}{2 \times 1.279}$$

$$\boxed{\boldsymbol{T_{max} = 322.6 \text{ K}}}$$

Problem 1.32 *The rate of heat generation in a slab of thickness 200 mm (* $k = 180 \text{ W/m°C}$ *is* $1.2 \times 10^6 \text{ W/m}^3$*. If the temperature of each of the surface of solid is* $140°\text{C}$*, determine the temperature at the mid and quarter planes:*

Solution

Downloaded from Ktunotes.in



Refer to Fig.

Thickness of slab $2L = 200$ mm

$= 0.2$ m

The rate of heat generated $\dot{q} = 1.2 \times 10^6$ W/m^3

Thermal conductivity of slabs, $k = 180$ W/m°C

The temperature of each surface, $T_1 = T_2 = T_w = 140°C$

(Where $T_w =$ temperature of the wall surface)

The temperature at the mid and quarter planes:

The temperature distribution is given by:

(Refer pg 48 of HMT DB)

At mid plane: $x = 0$

$$T_0 = T_w + \frac{\dot{q}}{2k}L^2$$

Downloaded from Ktunotes.in

At Quarter plane

$$\text{At } x = -\frac{L}{2} = -\frac{80}{2} = -40 \text{ mm}$$

$$\frac{T_x - T_0}{T_2 - T_0} = \left(\frac{x}{L}\right)^2$$

$$\frac{T_x - 173.33}{140 - 173.33} = \left(-\frac{40}{80}\right)^2 = 0.25$$

$$T_x - 173.33 = -8.3325$$

$$T_x = 164.9975\,°\text{C}$$

At Quarter plane, $x = +\dfrac{L}{2}$

$$\frac{T_x - 173.33}{140 - 173.33} = \left(\frac{40}{80}\right)^2 = 0.25$$

$$T_x - 173.33 = -8.3325$$

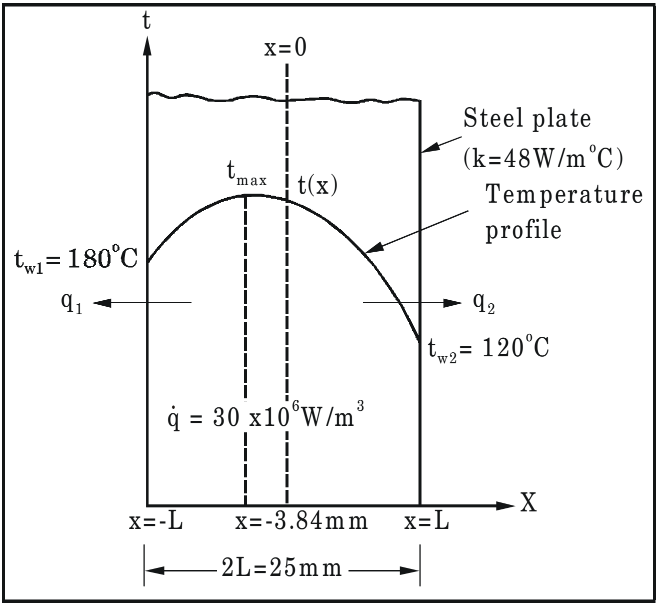
$$T_x = 164.9975\,°\text{ C}$$

Problem 1.33 *The temperatures on the two surfaces of a 25 mm thick steel plate,* ($k = 48$ W/m°C) *having a uniform volumetric heat generation of* 30×10^6 W/m^3, *are* 180°C *and* 120°C. *Neglecting the end effects, determine the following:*

(i) The temperature distribution across the plate;

(ii) The value and position of the maximum temperature,

Downloaded from Ktunotes.in



and

(iii) The flow of heat from each surface of the plate.

Solution

Refer to Fig.

$L = 25 \text{ mm} = 0.025 \text{ m}$

$t_{w1} = 180°\text{C}$; $t_{w2} = 120°\text{C}$

$\dot{q} = 30 \times 10^6 \text{ W/m}^3$

$k = 48 \text{ W/m°C}$

$$T_{\max} = \frac{\dot{q}\,L^2}{2k} + \frac{k}{8\,\dot{q}\,L^2} \times (T_{w1} - T_{w2})^2 + \frac{1}{2}\,(T_{w1} + T_{w2})$$

$$\left[L = \frac{25}{2} = 12.5 \text{ mm} \right]$$

Downloaded from Ktunotes.in

$$T_{max} = \frac{30 \times 10^6 \times 0.0125^2}{2 \times 48}$$

$$+ \frac{48}{8 \times 30 \times 10^6 \times 0.0125^2} \times (180 - 120)^2 + \frac{1}{2}(180 + 120)$$

$$= 48.828 + 4.608 + 150$$

$$= 203.436\,°C$$

Position of Maximum Temperature

$$x_{max} = \frac{k}{2\,\dot{q}\,L}(T_{w2} - T_{w1})$$

$$= \frac{48}{2 \times 30 \times 10^6 \times 0.0125}(120 - 180)$$

$$= -3.84 \times 10^{-3}\ \text{m}$$

Maximum Temperature occurs at 3.84 mm left to mid plane (or) 8.6 mm from left side of the slab.

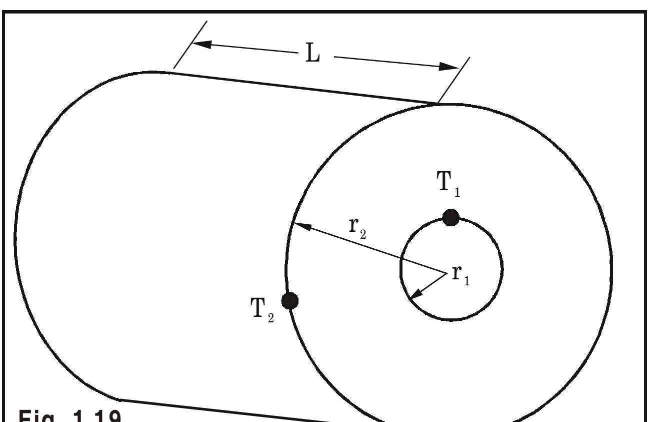


Fig. 1.19

Downloaded from Ktunotes.in

1.30 Conduction through hollow cylinder with internal heat generation

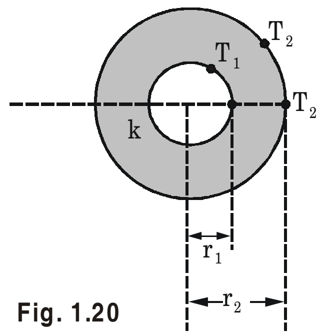


Fig. 1.20

Figure 1.19 shows a long hollow cylinder of length L and inside and outside radii r_1 and r_2 respectively. Temperatures T_1 and T_2 are maintained at r_1 and r_2 respectively. A constant rate of heat $\dot{q}$ (W/m^3) is generated within the cylinder.

The one-dimensional, steady state with heat generation is

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}\,r}{k} = 0, \quad r_1 \leq r_2 \qquad ...(16)$$

Integrating Equation. 16, once,

$$\frac{r\,dT}{dr} + \frac{\dot{q}\,r^2}{2k} = C_1$$

$$\frac{dT}{dr} + \frac{\dot{q}\,r}{2k} = \frac{C_1}{r}$$

or $$\frac{dT}{dr} = \frac{C_1}{8} - \frac{\dot{q}\,r}{2k} \qquad ...(16\ (a))$$

Integrating once again,

$$\boxed{T = \frac{-\dot{q}\,r^2}{4k} + C_1 \ln r + C_2} \qquad ...(17)$$

Downloaded from Ktunotes.in

Substituting boundary conditions into Eq.17,

$T = T_1$ at $r = r_1$

$T = T_2$ at $r = r_2$

$$T_1 + \frac{\dot{q}\, r_1^2}{4k} = C_1 \ln r_1 + C_2 \qquad \text{...(18)}$$

$$T_2 + \frac{\dot{q}\, r_2^2}{4k} = C_1 \ln r_2 + C_2 \qquad \text{...(19)}$$

$$T_{max} + \frac{\dot{q}\, r^2}{4k} = C_1 \ln r + C_2$$

[r = radial distance where the temperature is maximum]

Solving Eqs. 18 and 19, we get

$$C_1 \ln \frac{r_2}{r_1} = (T_2 - T_1) + \frac{\dot{q}\,(r_2^2 - r_1^2)}{4k}$$

$$C_1 = \frac{(T_2 - T_1) + \dfrac{\dot{q}}{4k}(r_2^2 - r_1^2)}{\ln \dfrac{r_2}{r_1}}$$

and $$C_2 = T_1 + \frac{\dot{q}\, r_1^2}{4k} - \left[\frac{(T_2 - T_1) + \dfrac{\dot{q}}{4k}(r_2^2 - r_1^2)}{\ln \dfrac{r_2}{r_1}} \right] \times \ln r_1$$

Substituting C_1 and C_2 from the above equations into Eq. 17, we get

Downloaded from Ktunotes.in

$$T = \frac{-\dot{q}\, r^2}{4k} + \left[(T_2 - T_1) + \frac{\dot{q}}{4k}(r_2^2 - r_1^2) \right] \frac{\ln r}{\ln \dfrac{r_2}{r_1}} + T_1 + \frac{\dot{q}\, r_1^2}{4k}$$

$$- \left[\frac{(T_2 - T_1) + \dfrac{\dot{q}}{4k}(r_2^2 - r_1^2)}{\ln \dfrac{r_2}{r_1}} \right] \times \ln r_1$$

$$(T - T_1) = \frac{\dot{q}}{4k}(r_1^2 - r^2) + \left[(T_2 - T_1) + \frac{\dot{q}}{4k}(r_2^2 - r_1^2) \right] \frac{\ln r/r_1}{\ln r_2/r_1}$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{\dot{q}}{4k} \left[\frac{(r_1^2 - r^2)}{(T_2 - T_1)} + \frac{(r_2^2 - r_1^2)\ln r/r_1}{(T_2 - T_1)\ln r_2/r_1} \right] + \frac{\ln r/r_1}{\ln r_2/r_1}$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln r/r_1}{\ln r_2/r_1} + \frac{\dot{q}}{4k}\frac{r_2^2 - r_1^2}{T_2 - T_1} \left[\frac{\ln r/r_1}{\ln r_2/r_1} - \frac{(r/r_1)^2 - 1}{(r_2/r_1)^2 - 1} \right]$$

The radial heat flow can be found out by Fourier's equation.

$$= -kA\frac{dT}{dr} = k2\pi rL\frac{dT}{dr}$$

Substituting dT/dr from Eq. 16 (a) into the above equation.

$$Q = -k2\pi rL \left[\frac{C_1}{r} - \frac{\dot{q}\, r}{2k} \right] = 2\pi L \left[\frac{\dot{q}\, r^2}{2} - C_1 k \right]$$

$$Q = 2\pi L \left[\frac{\dot{q}\, r^2}{2} - \left\{ \frac{(T_2 - T_1) + \dfrac{\dot{q}}{4k}(r_2^2 - r_1^2)}{\ln \dfrac{r_2}{r_1}} \right\} \times k \right] \quad \ldots(20)$$

Refer formulae from Pg 49 of HMT Database.

Downloaded from Ktunotes.in

Also for solid sphere, take formula from pg 49 of HMT DB

Problem 1.34 *Heat is generated within a wire of 3 mm in diameter by passing a current of 350 A. The thermal conductivity and resistivity of the wire are 25 W/m-K and* $80 \times 10^{-8}\ \Omega - \text{m}$ *and the length of the wire is 2.2 m. This wire is immersed in a water bath maintained at* 30°C.*The heat transfer coefficient at the outer surface of the wire is* 4500 W/m *Calculate the temperature at the centre of the wire and at the surface of the wire.*

Solution

$$R = 1.5\ \text{mm},\ L = 2.2\ \text{mm},\ h = 4500\ \text{W/m}^2 - \text{K},$$

$$k = 25\ \text{W/m} - \text{K},\ T_\alpha = 30°\text{C},\ I = 30\ \text{A},$$

$$\rho = \text{resistivity of the wire} = 80 \times 10^{-8}\ \Omega - \text{m}$$

$$\dot{q} = \frac{\dot{Q}}{AL} = \frac{I^2 \rho L}{A} \times \frac{1}{AL} = \rho \left(\frac{I}{A}\right)^2$$

$$\text{Area} = \frac{\pi}{4} \times 0.003^2 = 7.068 \times 10^{-}6\ \text{m}^2$$

$$= 80 \times 10^{-8} \left(\frac{350}{7.068 \times 10^{-6}}\right)^2$$

$$= 1.9614 \times 10^9\ \text{W/m}^3$$

(a) Temperature at the surface of the wire

From pg 48 of HMT Databook

$$T_2 = T_\infty + \frac{\dot{q}\, R}{2h}$$

Downloaded from Ktunotes.in

(b) Temperature at the centre of the wire

$$T_{r=0} = T_w + \frac{\dot{q}}{4k}(R^2 - r^2)$$

[Here $r = 0$]

$$T_0 = 356.9 + \frac{1.9614 \times 10^9}{4 \times 25} \times 0.0015^2$$

$$= 401.03\,°C$$

Problem 1.35 *A current of 200 A is passed through a stainless steel wire* ($k = 19$ W/mK) *3 mm in diameter. The resistivity of the steel may be taken as* $70\ \mu\ \Omega$ *cm and the length of the wire is submerged in a liquid at* 110°C *with heat transfer co-efficient* $h = 4$ kW/m^{2}C. *Calculate the centre temperature of the wire.* *[Madras University, April 2000]*

Solution

Given:

Current $A = 200\ A$

Thermal conductivity $k = 19\ W/mK$

Diameter $d = 3$ mm $= 0.003$ m

Resistivity $= 70\ \mu\ \Omega - \text{cm} = 0.7\ \mu\ \Omega\ \text{m} = 0.7 \times 10^{-6}\ \Omega\ \text{m}$

Liquid Temperature $T_w = 110\,°C + 273 = 383$ K

Downloaded from Ktunotes.in

$T_\infty = 30°C$

Heat transfer co-efficient $h = 4 \text{ kW/m}^2 \text{ C}$

$= 4 \times 10^3 \text{ W/m}^2\text{C}$

$$T_w = 30 + \frac{1.961 \times 10^9 \times 0.0015}{2 \times 4500}$$

Length of wire $= 1$ m

$T_0 =$ Centre temp = max. temp

$= 356.896\,°C$

The maximum temperature in the wire occurs at the centre.

$= 356.9\,°C$

$$T_{max} = T_0 = T_w + \frac{\dot{q}\, R^2}{4k} \qquad ...(1)$$

[From Pg 48 of HMT]

$$\text{Resistance of wire, } R = \frac{\text{Resistivity} \times \text{Length}}{\text{Area}}$$

$$= \frac{0.7 \times 10^{-6} \times 1}{\frac{\pi}{4} \times 0.003^2}$$

$$\boxed{R = 0.099\ \Omega}$$

We know that,

$$Q = I^2 R$$

$$= (200)^2 \times (0.099)$$

$$\boxed{Q = 3961.189\ \text{W}}$$

$$\text{Heat generated, } \dot{q} = \frac{Q}{V} = \frac{3961.189}{\frac{\pi}{4} d^2 \times L}$$

$$\dot{q} = \frac{3961.189}{\frac{\pi}{4}(0.003)^2 \times 1}$$

Downloaded from Ktunotes.in

$$\boxed{\dot{q} = 560.394 \times 10^6}$$

Substituting $\dot{q}$ value in Equation

$$T_{max} = T_w + \frac{\dot{q}\,R^2}{4k}$$

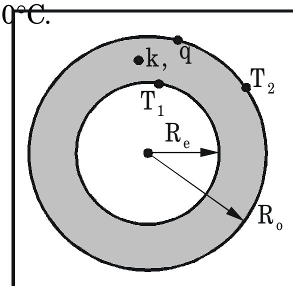
$$T_{max} = T_0 = 383 + \frac{560.394 \times 10^6 \times (0.0015)^2}{4 \times 19}$$

$$= 383 + 16.59$$

$$\boxed{\mathbf{T_0 = 399.59\ K}}$$

Centre temperature of wire, $T_0 = 399.59$ K

Problem 1.36 *A hollow cylinder has a conductivity of 0.2 W/m-K. Its inner and outer radii are 5 mm and 7 mm, respectively. It has an electric resistance of 0.03 Ω per metre. It is insulated at the outer radius. The inner radius is maintained at 30°C. Calculate the maximum current if the temperature is not to exceed 60°C*



Solution

Data:

$k = 20$ W/m – K, $T_2 = T_{max} = 60°C$, $r_1 = 5$ mm,

$r_2 = 7$ mm, $R_e = 0.03\ \Omega/\text{m}$, $T_1 = 30°C$

The outer surface is insulated, hence it has maximum temperature, i.e., $T_2 = T_{max}$

Downloaded from Ktunotes.in

To find $\dot{q}$

From Pg 49 of HMT Databook

For Hollow cylinder, outside adiabatic

$$T_0 - T_i = \frac{\dot{q}}{2k} \cdot R_0^2 \ln \frac{R_0}{R_i} - \frac{\dot{q}}{4k} (R_0^2 - R_i^2)$$

$$60 - 30 = \frac{\dot{q}}{2 \times 20} \times 0.007^2 \ln\left(\frac{7}{5}\right) - \frac{\dot{q}}{4 \times 20} (0.007^2 - 0.005^2)$$

$$30 = \dot{q} \times 4.1218 \times 10^{-7} - 3 \times 10^{-7}\, \dot{q}$$

$$\dot{q} = \frac{30}{1.1218 \times 10^{-7}} = 2.674 \times 10^8 \frac{W}{m^3}$$

Heat generated per meter length:

$$\frac{\dot{q}}{1 \text{ m length}} = \dot{q} \times A = 2.674 \times \frac{\pi}{4} \times (0.007^2 - 0.005^2)$$

$$= 5040.9 \frac{W}{m}$$

The maximum Current (I)

$$\frac{\dot{q}}{1 \text{ m length}} = I^2 R_e$$

$$5040.9 = I^2 \times 0.03$$

$$I = \sqrt{\frac{5040.9}{0.03}} = 409.914 \text{ A}$$

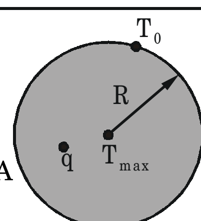


Fig. 1.21

Downloaded from Ktunotes.in

1.31 Solid Sphere with Heat Generation

Consider a solid sphere of **Figure (1.21)** with internal heat generation. The outer radius is R and the outer temperature is T_o. The solid sphere has uniform thermal conductivity k. The one-dimensional, steady state with heat generation equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{q}\,r^2}{k}=0$$

Integrating the above equation twice, we get

$$r^2\frac{dT}{dr}=-\frac{\dot{q}\,r^3}{3k}+C_1$$

or $$\frac{dT}{dr}=-\frac{\dot{q}\,r}{3k}+\frac{C_1}{r^2} \quad ...(38)$$

or $$T=-\frac{\dot{q}\,r^2}{6k}-\frac{C_1}{r}+C_2 \quad ...(39)$$

Boundary conditions:

AT centre, $r=0$, $dT/dr=0$; At $r=R$, $T=T_0$

At centre, $r=0$, $\frac{dT}{dr}=0$, hence $C_1=0$

At $r=R$, outer surface, $T=T_0$

$$T_0=\frac{-\dot{q}\,R^2}{6K}+C_2 \quad \therefore \quad C_2=T_0+\frac{\dot{q}\,R^2}{6k}$$

Downloaded from Ktunotes.in

Substituting the values of constant C_1 and C_2 in Equation. 39, we get

$$T = T_0 + \frac{\dot{q}}{6k}(R^2 - r^2) \qquad ...(40)$$

This equation gives the temperature distribution in solid sphere with heat generation.

$$\frac{dT}{dr}(r) = 0$$

The temperature is maximum at $r = 0$

$$T_i = T_{max}$$

$$\therefore \qquad T_i = T_{max} = T_0 + \frac{\dot{q}R^2}{6k} \qquad ...(41)$$

The heat flow rate can be calculated from Fourier's equation,

$$Q = -kA\frac{dT}{dr} = -k4\pi R^2 \left.\frac{dT}{dr}\right|_{r=R}$$

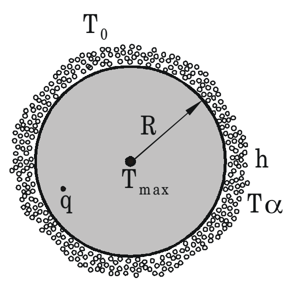


Fig. 1.22 Solid sphere with convection.

Substituting for dT/dr from equation.38 into the above equation, we get

$$Q = -k4\pi R^2\left(\frac{-\dot{q}R}{3k}\right) = \frac{4\pi R^3 \dot{q}}{3} \qquad ...(42)$$

or $\qquad Q = \text{Volume} \times \dot{q}$

Downloaded from Ktunotes.in

The heat generated within the sphere is conducted at the outer surface, which is convected to the surroundings.

We have

Heat generated within the solid sphere

= Heat conducted at the outer surface of the solid sphere.

= Heat convected from outer surface into the surroundings.

$$\dot{q}\ \frac{4}{3}\pi R^2 h\ (T_0 - T_\infty) \qquad ...(43)$$

$$T_0 = T_\infty + \frac{\dot{q}\ R}{3h}$$

Substituting the above equation in Eq. 40, we get

$$T = \left[T_\infty + \frac{\dot{q}\ R}{3h} \right] + \frac{\dot{q}\ (R^2 - r^2)}{6k} \qquad ...(44)$$

Equation (44) gives the temperature distribution within the solid sphere with convective heat transfer coefficient h and surrounding temperature T_∞

Problem 1.37 *An approximately spherical shaped orange* ($k = 0.23$ W/m°C), *100 mm in diameter undergoes riping process and generates* 5100 W/m^3 *of energy. If external surface of the orange is at* 10°C *determine:*

(i) Temperature at the centre of the orange, and

(ii) Heat flow from the outer surface of the orange.

(May/June 2005 - AU)

Solution

Downloaded from Ktunotes.in

Outside radius of the orange,

$$R = \frac{100}{2} = 50 \text{ mm} = 0.05 \text{ m}$$

Rate of heat generation, $\dot{q} = 5100 \text{ W/m}^3$

The temperature at the outer surface of the orange, $T_w = 10°\text{C}$

(i) Temperature at the centre of the orange T_{max}

$$T_{max} = T_w + \frac{\dot{q}}{6k} R^2$$

From pg. 49 of HMT DB

$$\text{or } T_{max} = 10 + \frac{5100}{(6 \times 0.23)} \times (0.05)^2 = 19.24\,°\text{C}$$

(ii) Heat flow from the outer surface of the orange, Q

Heat conducted = Heat generated

$$\therefore \qquad Q = \dot{q} \times \frac{4}{3} \pi R^3$$

$$Q = 5100 \times \frac{4}{3} \pi \times (0.05)^3 = 2.67 \text{ W}$$

Downloaded from Ktunotes.in

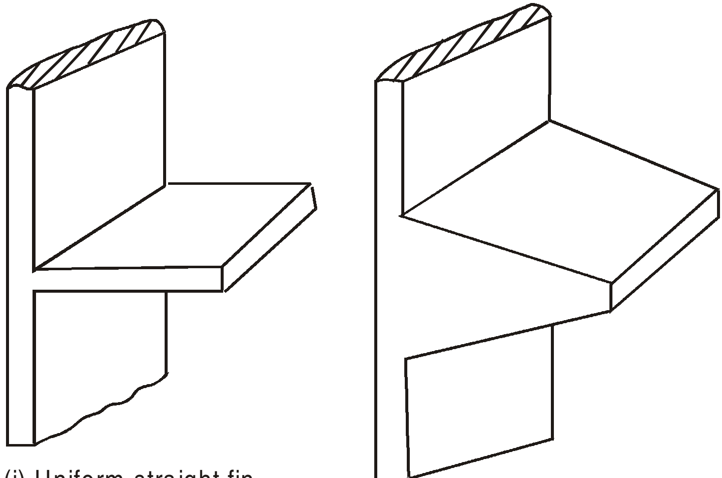
1.32 HEAT TRANSFER THRO' FINS

Extended surfaces or fins are used to transfer the required quantity of heat with the available temperature drop and convective heat transfer coefficient.

The fins are commonly used to increase the heat transfer by increasing the surface area. By available surface area, the heat transfer may be less. If we add fins to the surfaces of heat transfer equipment, then the surface area will be increased, hence heat transfer will be increased.

The fins are used for scooters, motor-cycles and compressors. They are used in evaporators and condensors of refrigerating system by increasing surface area of heat transfer to increase the heat transfer rate.

1.33 TYPES OF FINS



(i) Uniform straight fin

Fig. 1.23 (ii) Tapered straight fin

Downloaded from Ktunotes.in

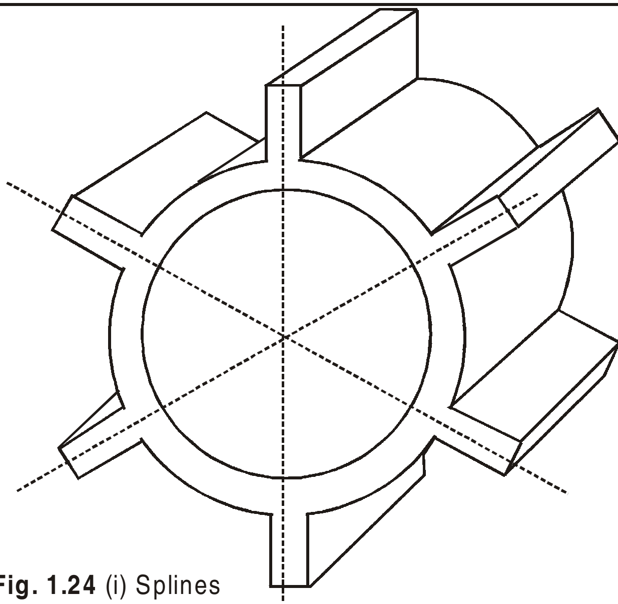
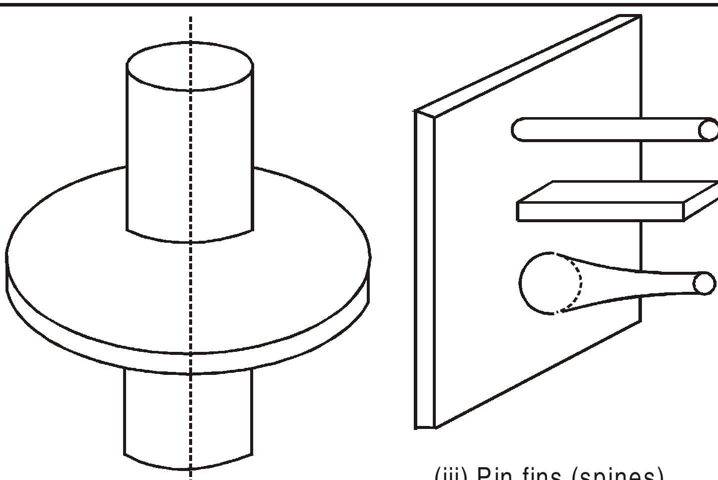


Fig. 1.24 (i) Splines



(ii) Annualar fin **Fig. 1.24** (iii) Pin fins (spines)

Downloaded from Ktunotes.in

There are two types of fins used in industries.

1. Straight fin-It is attached to a wall.
2. Annular or Spline fin - It is attached to the circumference of the cylindrical surface.

1.34 HEAT FLOW THROUGH RECTANGULAR FINS

Figure 1.25 shows a rectangular fin of uniform cross section having temperature T_b at the base and the surrounding atmosphere temperature T_∞. The assumptions made for the analysis of heat flow through the fins.

- ❖ One-dimensional conduction in x-direction.
- ❖ Homogeneous and fin material has uniform thermal conductivity (isotropic).
- ❖ No internal heat generated in the fin.
- ❖ Steady state conduction prevails.

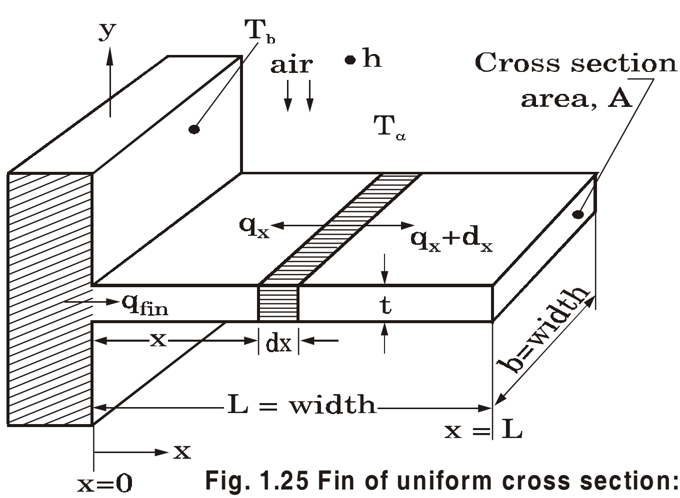


Fig. 1.25 Fin of uniform cross section:
(a) rectangular fin,

Downloaded from Ktunotes.in

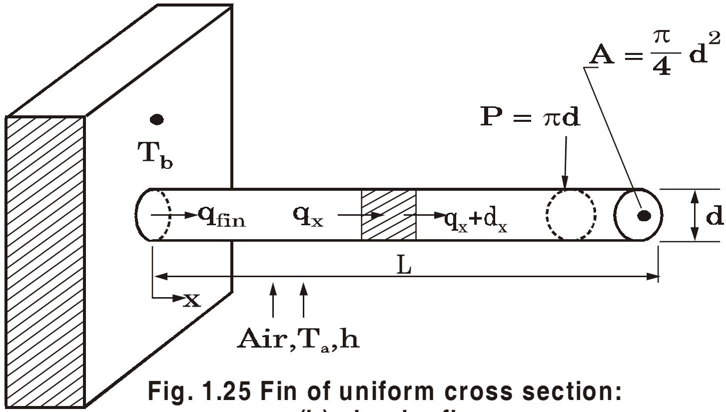


Fig. 1.25 Fin of uniform cross section: (b) circular fin.

- ❖ Uniform convective heat transfer coefficient over the entire surface of the fin.
- ❖ Radiation heat transfer is neglected.

Let L = Length of the fin

b = Width of the fin

t = Thickness of the fin (m)

P = Perimeter of the rectangular fin $= 2\,(b + t)\,(m)$

A = Cross-sectional area of the fin (area perpendicular to the direction of heat flow) (m^2)

T_b = Temperature of the base of the fin (°C or K)

T_∞ = Surrounding ambient temperature (°C)

h = Convective heat transfer coefficient ($W/m^2 - K$)

k = Thermal conductivity of fin ($W/m - K$)

Downloaded from Ktunotes.in

Energy balance to an element of the fin of length dx at a distance x from the surface is applied.

Heat conducted into the element = Heat conducted out of the element + Heat convected away from the element.

i.e $\quad Q_x = Q_{x+dx} + Q_{\text{convection}}$

or $$-kA \left.\frac{dT}{dx}\right|_x = -kA \left.\frac{dT}{dx}\right|_{x+dx} + h\,(P\,dx)\,[T - T_\infty] \quad \text{...(21)}$$

We know $$\left.\frac{dT}{dx}\right|_{x+dx} = \left.\frac{dT}{dx}\right|_x + \frac{d}{dx}\left(\frac{dT}{dx}\right)_x dx$$

Substituting the above expression in Eq.(21), we have

$$-kA\frac{dT}{dx} = -kA\frac{dT}{dx} - kA\frac{d}{dx}\left(\frac{dT}{dx}\right)dx + hP\,dx\,[T - T_\infty]$$

$$kA\frac{d}{dx}\left(\frac{dT}{dx}\right)dx = hP\,dx\,[T - T_\infty]$$

$$\frac{d^2T}{dx^2} - \frac{hp}{kA}[T - T_\infty] = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}\theta = 0 \quad \text{...(22)}$$

Where $\theta = (T - T_\infty)$; T_∞ = constant ; Let $m = \sqrt{\dfrac{hP}{kA}}$

$$\frac{d\,\theta}{dx} = \frac{dT}{dx} ; \frac{d^2\,\theta}{dx^2} = \frac{d^2T}{dx^2}$$

Downloaded from Ktunotes.in

Substituting $m = \frac{hP}{kA}$ and $\frac{d^2\,\theta}{dx} = \frac{d^2 T}{dx^2}$

We get $\frac{d^2\,\theta}{dx} - m^2\,\theta = 0$ ...(23)

The parameter m is constant, because h and k are assumed constant. Then, the general solution for the above equation (second order differential equation) is

$$\theta = C_1 e^{mx} + C_2 e^{-\,mx}$$ ...(24)

This equation can be written in the form of hyperbolic sine and hyperbolic cosine functions. Thus, we have

$$\theta = C_1 \cosh mx + C_2 \sinh mx$$ ...(25)

$$\theta = C_1 \cosh m\,(L - x) + C_2 \sinh m\,(L - x)$$ ...(26)

Three different conditions are possible for this fin. They are:

1. The fin is infinitely long [temperature at the tip of the fin = surrounding temperature, i.e. $T = T_\infty$ at $x = \infty$
2. The tip of the fin is insulated, and short fin.
3. The length of the fin is short and heat transfer is by convection and end not insulated.

Case (i)

Fin is Infinitely Long ($L \rightarrow \infty$)

$$\theta = T - T_\infty$$

Downloaded from Ktunotes.in

The boundary conditions are:

At $x = 0, T = T_b,\ \theta = T_b - T_\infty = \theta_b$

At $x = \infty, T = T_\infty, \theta = T_\infty - T_\infty = 0$

Substituting the above boundary conditions into Eq. (24)

$\theta = C_1 e^{mx} + C_2 e^{-mx}$

At $x = 0,\ C_1 + C_2 = \theta_b = T_b - T_\infty$

At $x = \infty, C_1 e^{m\infty} + C_2 e^{-m\infty} = 0,\ C_1 = 0$

$\therefore \qquad C_2 = \theta_b = (T_b - T_\infty)$

Substituting the values of C_1 and C_2 in Eq. (24), we get

$$\theta = \theta_b e^{-mx} = (T_b - T_\infty) e^{-mx}$$

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \qquad \text{...(27)}$$

Refer page 50 of HMT DB.

Equation 27 gives the temperature distribution along the length of the fin.

(a) The rate of the heat flow through the fin can be analyzed as follows:

Heat is convected away from the tip of the fin $(L \to \infty)$. Hence,

Downloaded from Ktunotes.in

$$Q = h\ (Pdx)\ [T - T_{\infty}] = \int_0^{\infty} hP\ dx\ [T - T_{\infty}]$$

$(\because Pdx = \text{Surface Area})$

From Eq. 27,

$$T - T_{\infty} = (T_b - T_{\infty})\ e^{-mx}$$

$$Q = \int_0^{\infty} hP\ (T_b - T_{\infty})\ e^{-mx}\ dx = hP\ (T_b - T_{\infty}) \times \frac{1}{m}$$

$$Q = hp\ (T_b - T_{\infty}) \sqrt{\frac{kA}{hP}} = \sqrt{hPkA}\ (T_b - T_{\infty}) \qquad ...(28)$$

Refer formula from pg.50 of HMT Databook.

Case (ii) Short fin-end insulated.

$$\frac{d^2\theta}{dx} - m^2\theta = 0, \quad 0 \le x \le L \qquad [\theta = T - T_{\infty}]$$

Boundary Conditions

At $x = 0$, $T = T_b$; Then $\theta = (T_b - T_{\infty}) = \theta_b$

Substituting the above boundary conditions in the following equation (26), we get

$$\theta = C_1 \cosh m\ (L - x) + C_2 \sinh m\ (L - x)$$

At $x = 0$, $\theta = \theta_b = C_1 \cosh (mL) + C_2 \sinh\ (mL)$ ...(29)

At $x = L$, $\dfrac{dT}{dx} = \dfrac{d\theta}{dx} = 0$ [end is insulated]

Downloaded from Ktunotes.in

Differentiating θ Eq. (26), putting $x = L$ and $\frac{d\theta}{dx} = 0$, we have

$$\frac{d\theta}{dx} = C_1 \sinh [m (L - L)] + C_2 \cosh [m (L - L)]$$

$$= C_1 \times 0 + C_2 m = 0$$

$$C_2 = 0$$

Substituting $C_2 = 0$ in Eq. (29)

$$C_1 = \frac{\theta_b}{\cosh (mL)}$$

Substituting C_1 and C_2 values in θ Eq. (26), we get

$$\theta = [T - T_\infty] = \frac{\theta_b}{\cosh (mL)} \cosh [m (x - L)] + 0$$

$$= \frac{\theta_b \cosh [m (L - x)]}{\cosh (mL)}$$

$$\text{or} \quad \frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh [m (L - x)]}{\cosh (mL)} \quad \text{...(29)}$$

The above equation gives the temperature distribution in a fin, with end insulated.

To find Q

Equation (29) can be written as

$$\theta = \theta_b \frac{\cosh [m (L - x)]}{\cosh(mL)}$$

$$\frac{d\theta}{dx} = \frac{- m \, \theta_b \sinh (L - x) \, m}{\cosh (mL)}$$

Downloaded from Ktunotes.in

$$\left.\frac{d\,\theta}{dx}\right|_{x=0} = -\,m\,\theta_b \frac{\sinh\,(mL)}{\cosh(mL)} = -\,m\,\theta_b \tanh\,(mL)$$

The rate of heat flow through the short fin (end insulated) is given by Fourier's equation,

$$Q = -\,kA\left.\frac{dT}{dx}\right|_{x=0} = -\,kA\left.\frac{d\,\theta}{dx}\right|_{x=0} = -\,kA\,[-\,m\,\theta_b \tanh\,(mL)]$$

$$Q = k\,Am\,\theta_b \tanh\,(mL) = kA\sqrt{\frac{hp}{kA}}\,(T_b - T_\alpha)\tanh(mL)$$

$$Q = \sqrt{hPkA}\,(T_b - T_\alpha)\tanh\,(mL) \qquad \text{...(30)}$$

Refer page 50 of HMT DB for formula $[\theta = T - T_\alpha]$

Case (iii) Short fin end not insulated

$\theta = T - T_\infty$

At $x = 0$, $T = T_b$, $\theta = \theta_b = T_b - T_\alpha$

At $x = L$

Heat conducted to the tip

= Heat convected into the surroundings

$$\therefore \quad -\,kA\,\frac{dT}{dx} = h\,A\,[T - T_\alpha] \qquad [\because\ \theta = T - T_\alpha]$$

$$\frac{dT}{dx} = \frac{-\,h}{k}\,\theta \qquad \text{...(31)}$$

Putting $x = L$ in Eq. (26)

Downloaded from Ktunotes.in

$$\theta = C_1 \cosh m\,(L - x) + C_2 \sinh m\,(L - x)$$

$$\theta_{(L)} = C_1 \cosh\,[m\,(0)] + C_2 \sinh\,[m\,(0)]$$

or $\theta_{(L)} = C_1 = T_L - T_\alpha$ ...(32)

Differentiating Eq. (26) and putting $x = L$,

$$\frac{d\,\theta}{dx} = C_1 \sinh\,[m\,(L - x)] + C_2 \cosh\,[m\,(L - x)]$$

Put $x = L$,

$$\frac{d\,\theta_{(L)}}{dx} = C_1 \sinh\,[m\,(0)] + C_2 \cosh\,[m\,(0)] = C_2\,m \quad \text{...(33)}$$

From Eqs. (31), (33) and $x = L$, we get

$$\frac{dT_{(L)}}{dx} = \frac{d\,\theta_{(L)}}{dx} = C_2\,m = \frac{h}{k}\,\theta_{(L)}$$

$$\therefore \quad C_2 = \frac{h\,\theta_{(L)}}{mk} \quad \text{...(34)}$$

Substituting (C_1) and (C_2) back in Eq. (26), we get

$$\theta = \theta_{(L)} \cos h\,[m\,(L - x)] + \frac{h\,\theta_{(L)}}{mk} \sin h\,[m\,(L - x)] \quad \text{...(35)}$$

Dividing Eq. (35) by θ_b

$$\frac{\theta}{\theta_b} = \frac{T - T_\alpha}{T_b - T_\alpha} = \frac{\cosh\,[m\,(L - x)] + \dfrac{h}{mk} \sinh\,[m\,(L - x)]}{\cosh\,(mL) + \dfrac{h}{mk} \sinh\,(mL)} \quad \text{...(36)}$$

Downloaded from Ktunotes.in

Equation (36) gives the temperature distribution in a short fin (end not insulated) when heat is convected from the tip.

$$\therefore \; Q = \frac{(T_b - T_\alpha)\left[\tanh(mL) + \dfrac{h}{mk}\right]}{1 + \dfrac{h}{mk}\tanh(mL)} \sqrt{hPkA} \qquad \text{...(37)}$$

In simple words, there are three types of fins as follows

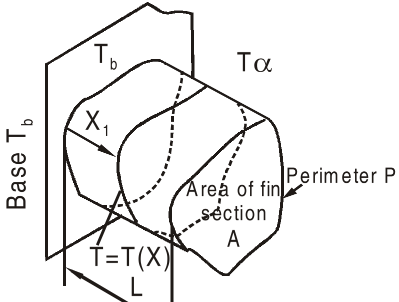
1. Sufficiently long fin (or) Infinitely long fin
2. Short fin - end (tip of the fin) insulated
3. Short fin - end (tip of the fin) not insulated

If the type of fin is not clearly given in the problem, then use the following formula.

$I_f \dfrac{L}{d} > 30$, then it is case (1) ie it is infinitely long fin.

Downloaded from Ktunotes.in

1.35 FINS OR EXTENDED SURFACES [For Formulae, refer page 50 of HMT Data Book]

Type of FIN boundary	Temperature distribution $\dfrac{T - T_\infty}{T_b - T_\infty}$	Heat transferred by Fin Q	
Long Fin ($T_b = T_\infty$)	e^{-mx}	$(T_b - T_\infty)\sqrt{(hPkA)}$	 Fig.1.26
SHORT FIN (end insulated)	$\dfrac{\cosh m(L-X)}{\cosh(mL)}$	$(hPkA)^{1/2}(T_b - T_\infty)\tanh(mL)$	
SHORT FIN (end not insulated)	$\dfrac{\cosh[m(L-X)] + (h_L/mk)\sinh[m(L-X)]}{\cosh(mL) + (h_L/mk)\sinh(mL)}$	$(T_b - T_\infty)\dfrac{\sinh(mL) + (h_L/mk)\cosh(mL)}{\cosh(mL) + (h_L/mk)\sinh(mL)}(hPkA)^{1/2}$	$\eta_{fin} = \dfrac{\tanh(mL)}{mL}$

* Ratio of the actual heat transferred by fin to the heat transferable by fin, if the entire fin area were at base temperature.

$$m = \sqrt{(hP/kA)}$$

Downloaded from Ktunotes.in

Problem 1.38: *Find the amount of heat transfer through a fin of thickness 5mm, height 50 mm and width 100 mm. Also determine the temperature at the tip of the fin. The atmospheric temperature is* 28°C. *The temperature at the base of the fin is* 80°C $k = 58.139$ W/m°C; $h = 11.63$ W/m^2°C; $T_b = 80$°C. *The end of the fin is insulated.*

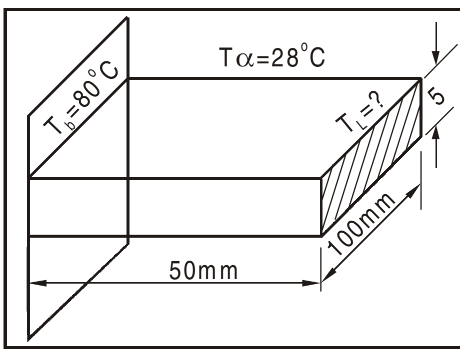
Solution

Short fin - End insulated.
Refer Pg 50 of HMT DB

Cross Sectional Area

$A_c = 0.1 \times 0.005$

$= 5 \times 10^{-4}$ m^2



Perimeter

$P = 2\,(100 + 5)$

$= 0.21$ m

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$= \sqrt{\frac{11.63 \times 0.21}{58.139 \times 5 \times 10^{-4}}}$$

$= 9.16602$

$$Q = \sqrt{hPkA_c} \times (T_b - T_\alpha)\tanh(mL)$$

$mL = 9.166018 \times 0.05 = 0.4583$

$$Q = \sqrt{11.63 \times 0.21 \times 58.139 \times 5 \times 10^{-4}} \times (80 - 28)\tanh(0.4583)$$

$= 0.26645 \times 52 \times \tanh\,(0.4583)$

$= 5.9397$ Watts

Downloaded from Ktunotes.in

Temperature Distribution

$$\frac{T - T_\alpha}{T_b - T_\alpha} = \frac{\cosh m(L - x)}{\cosh (mL)} \quad \text{(Pg 50 of CPK)}$$

Here $x = L$ (Since we need temperature at the tip of the fin)

$$\frac{T - 28}{80 - 28} = \frac{\cosh m(L - L)}{\cosh (0.4583)}$$

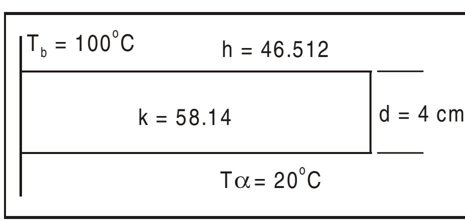
$$T - 28 = 0.903448 \times (52) \quad = 46.979$$

$$T = 46.979 + 28 = 74.979 \ °C$$

Temperature at the tip of the fin $= 75°C$

Problem 1.39: *Find the heat loss from a rod of 4 cm in diametre and infinitely long, when its base is maintained at* $100°C$. *The conductivity of the material is* $58.14 \ W/m°C$ *and the heat transfer coefficient on the surface of the rod is* $46.512 \ W/m^2 \ °C$. *The temperature of the air surrounding the rod is* $20°C$

Solution



Infinitely long fin

$$\frac{T - T_\alpha}{T_b - T_\alpha} = e^{-mx}$$

$$Q = (T_b - T_\alpha)\sqrt{hPkA_c}$$

from Pg 50 of HMT DB.

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.04^2 = 1.2567 \times 10^{-3} \ m^2$$

$$P = \pi d = \pi \times 0.04 = 0.12567 \ m$$

Downloaded from Ktunotes.in

$$Q = (100 - 20)\sqrt{46.512 \times 0.12567 \times 58.14 \times 1.2567 \times 10^{-3}}$$

$$= 52.28 \text{ W}$$

$$\boxed{\text{Heat loss } Q = 52.28 \text{ W}}$$

Problem 1.40: *The stainless steel blades of an aerofoil cross section are to be designed to carry* 81.3953 W *of heat by each blade in a gas turbine system. Find the height of the blade-if* A_c = cross sectonal area of blade = 2×10^{-4} m^2; P = perimeter of aerofoil blade = 0.06 m; T_α = *Temperature of the gas flowing over the blade* = 800°C; T_b = *temperature of the root of the blade* = 300°C; k = 23.256 W/m°C; h = 116.279 W/m^2 °C; *End face of the blade is insulated.*

Solution

Short fin end insulated; L = height of the blade = ?.

$$Q = \sqrt{hPkA_c}\,(T_b - T_\alpha)\tanh(mL)$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{116.279 \times 0.06}{23.256 \times 2 \times 10^{-4}}} = 38.73$$

$$Q = 81.3953 = \sqrt{116.279 \times 0.06 \times 23.256 \times 2 \times 10^{-4}} \times (800 - 300) \times \tanh(38.73\,L)$$

$$81.3953 = 0.18014 \times 500 \times \tanh(38.73\,L) \times$$

$$\tanh(38.73L) = 0.9037$$

$$38.73L = \tanh^{-1} 0.9037 = 1.49198$$

$$L = 0.0385 \text{ m}$$

$$\boldsymbol{L = 3.85 \text{ cm}}$$

Downloaded from Ktunotes.in

Problem 1.41: *Two long rods of the same diameter, one made of brass ($k = 85$ W/m° C) and other made of copper ($k = 375$ W/m° C) have one of their ends inserted into the furnace. Both of the rods are exposed to the same environment. At a distance 105 mm away from the furnace end, the temperature of the brass rod is 120°C. At what distance from the furnace end, the same temperature would be reached in copper rod?*

Solution

For infinitely long rod: (when $x = l$)

$$\frac{T - T_\alpha}{T_b - T_\alpha} = e^{-mx}$$

For Brass ($x = 0.105$ m)

$$\frac{120 - T_\alpha}{T_b - T_\alpha} = e^{-m_1 0.105} \qquad \ldots (1)$$

For Copper ($x = l$)

$$\frac{120 - T_\alpha}{T_b - T_\alpha} = e^{-m_2 l} \qquad \ldots (2)$$

Divide (1) by (2)

$$\frac{120\ T_\alpha}{T_b - T_\alpha} \times \frac{T_b - T_\alpha}{120 - T_\alpha} = \frac{e^{-m_1 0.105}}{e^{-m_2 l}}$$

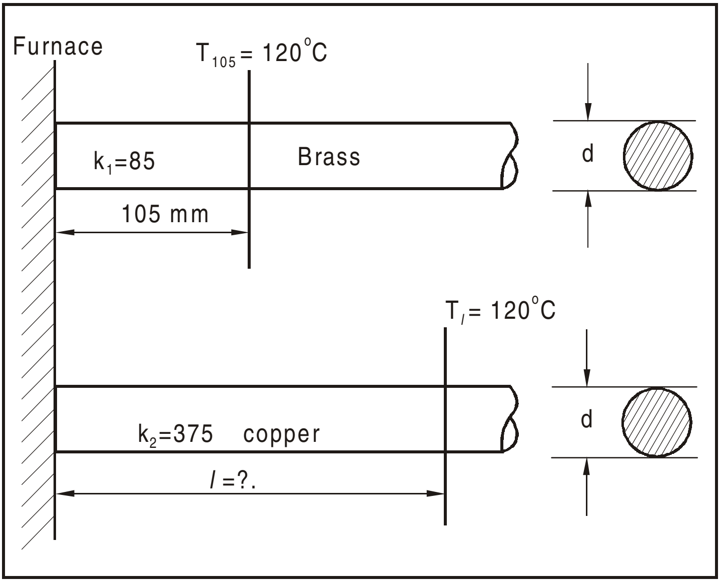
$$e^{-m_2 l} = e^{-m_1 0.105}$$

Taking ln on both sides

$$-m_2 l = -m_1 0.105$$

$$l = \frac{m_1}{m_2} \times 0.105 \qquad \ldots (3)$$

Downloaded from Ktunotes.in



$$\text{Now } \frac{m_1}{m_2} = \frac{\sqrt{\dfrac{hP}{k_1 A_c}} \text{ for Brass}}{\sqrt{\dfrac{hP}{k_2 A_c}} \text{ for Copper}}$$

$$= \sqrt{\frac{1}{k_1}} \times \sqrt{k_2}$$

$$= \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{375}{85}} = 2.1004$$

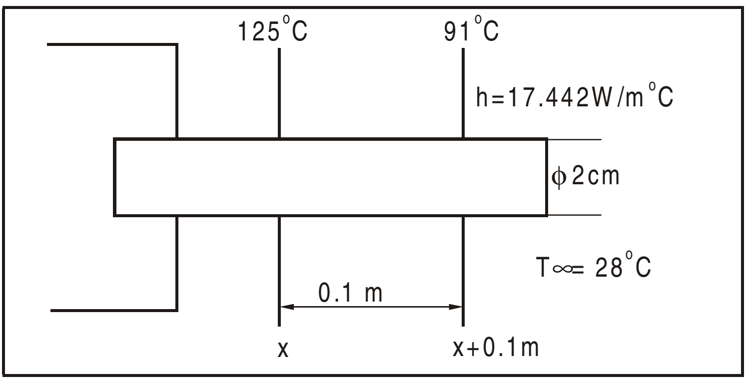
Substitute $\dfrac{m_1}{m_2}$ value in equation (3).

$l = 2.1004 \times 0.105 = 0.2205$ m

ie., $\boldsymbol{l = 22.05}$ **cm**

Downloaded from Ktunotes.in

Problem 1.42: *One end of the long rod is inserted into a furnace with the other end projecting into the outside air. After steady state is reached, the temperature of the rod is measured at two points 10 cm apart and found to be 125°C and 91°C, when the ambient temperature is 28°C. If the rod is 2cm in diameter and* $h = 17.442\ \mathrm{W/m^2\ °C}$, *what is the thermal conductivity of the rod?*



Solution

Temperature distribution along the rod is given by

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \text{ for long fin.}$$

$$P = \pi d = \pi \times 0.02 = 0.06283$$

$$A_c = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.02^2 = 3.14159 \times 10^{-4}\ \mathrm{m^2}$$

When $T = 125°\mathrm{C}$

$$\frac{125 - 28}{T_b - T_\infty} = e^{-mx} \qquad (1)$$

Downloaded from Ktunotes.in

When $T = 91°C$, $x = x + 0.1$ m

$$\frac{91-28}{T_b - T_\alpha} = e^{-m\,(x+0.1)} \quad (2)$$

Divide (1) by (2)

$$\frac{125-28}{T_b - T_\alpha} \times \frac{T_b - T_\alpha}{91-28} = \frac{e^{-m\,x}}{e^{-m\,(x+0.1)}}$$

$$1.53968 = \frac{e^{-mx}}{e^{-mx} \cdot e^{-0.1\,m}}$$

$$e^{-0.1\,m} = \frac{1}{1.53968} = 0.64948$$

Taking ln on both sides

$$-0.1\text{ m} = \ln 0.64948 = -0.431576$$

$$\text{m} = \frac{0.431576}{0.1} = 4.31576$$

$$\sqrt{\frac{hP}{kA_C}} = 4.31576$$

$$\sqrt{\frac{17.442 \times 0.06283}{k \times 3.14159 \times 10^{-4}}} = 4.31576$$

$$k = \frac{17.442 \times 0.06283}{4.31576^2 \times 3.14159 \times 10^{-4}}$$

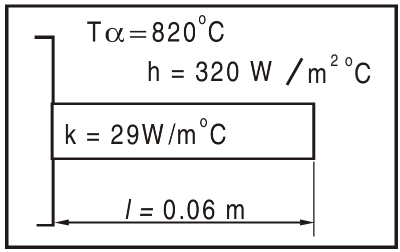
$$= 187.2832 \text{ W/m°C}$$

Problem 1.43: *A turbine blade made of steel* ($k = 29$ W/m °C) *is 60 mm long,* 500 mm^2 *cross-sectional area and* 120 mm *perimeter. The temperature of the root of blade*

Downloaded from Ktunotes.in

is 480°C and it is exposed to 820°C. If the film heat transfer coefficient between the blade and the gases is 320 W/m^2 °C, determine 1. The temperature at the middle of the blade. 2. The rate of heat flow from the blade.

Solution



$$A_c = 500 \times 10^{-6}\ \text{m}^2$$

$$P = 120 \times 10^{-3}\ \text{m}$$

$$T_b = 480°\text{C}$$

$$T_\alpha = 820°\text{C}$$

Temperature distribution for short fin end not insulated.

$$\frac{T - T_\alpha}{T_b - T_\alpha} = \frac{\cosh\,[m\,(L-x)] + (h/mk)\sinh m\,(L-x)]}{\cosh\,(mL) + (h/mk)\sinh\,(mL)}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{320 \times 120 \times 10^{-3}}{29 \times 500 \times 10^{-6}}} = 51.46$$

$$mL = 51.46 \times 0.06 = 3.08768$$

$$\frac{h}{km} = \frac{320}{29 \times 51.46} = 0.2144$$

To find temperature at the middle

$$x = \frac{L}{2} = 0.03\ \text{m}$$

$$\frac{T - 820}{480 - 820} = \frac{\cosh\,(\,51.46\,(0.03)\,) + (\,0.2144\,)\sinh\,(\,51.46 \times 0.03\,)}{\cosh\,(3.08768) + (0.2144)\sinh\,(3.087)}$$

$$= \frac{2.4479 + 0.47905}{10.9858 + 2.34399} = 0.21957$$

Downloaded from Ktunotes.in

$$T = [\ 0.21957 \times (480 - 820)\ ] + 820$$

$$= 745.34\ 295°C$$

$$\boldsymbol{T = 745.343\ °C}$$

at $\boldsymbol{x = 0.03}$ **m**

The heat transfer rate Q

$$Q = (T_b - T_\alpha) \left\{ \frac{\tanh\ (mL) + (h/mk)}{1 + (h/mk) \tanh\ (mL)} \right\} \sqrt{hPkA_c}$$

$$\sqrt{hPkA_c} = \sqrt{320 \times 120 \times 10^{-3} \times 29 \times 500 \times 10^{-6}}$$

$$\sqrt{hPkA_c} = 0.7462$$

$$Q = (480 - 820) \left\{ \frac{\tanh\ (3.08768) + (0.2144)}{1 + (0.2144) \tanh\ (3.08768)} \right\} \times 0.7462$$

$$= -340 \left[\frac{1.2102}{1.2135} \right] \times 0.7462$$

$= -253.0249$ W   ['-' sign indicates that heat flows from gas to turbine blades]

1.36 Effectiveness of fin (ε_{fin})

It is the ratio of heat transfer rate with fin to the heat transfer rate without fin.

$$\varepsilon = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

1.37 Efficiency of fin η

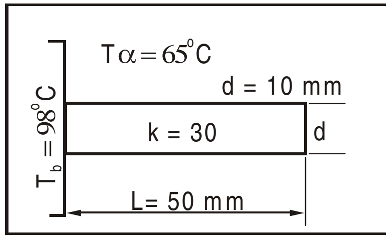
$$\eta_{\text{fin}} = \frac{\text{Heat lost from fin}}{\text{Heat lost from the fin if Whole surface of the fin is maintained at root (base) temperature}} = \frac{\tanh\ (mL)}{mL}$$

Downloaded from Ktunotes.in

Problem 1.44: *A circular pocker rod of 10 mm dia, 50 mm long is having thermal conductivity of 30* W/m K, h = 50 W/m^2°C. *Base temperature is* 98°C. *Ambient air temperature is* 65°C.

Determine

1. *tip temperature*
2. *Heat transfer rate*
3. *fin efficiency*
4. *length at which* $T = 90$°C



Assume, the rod end is insulated.

Solution

Short fin-end insulated

$h = 50$ W/m^2 °C ; $T_b = 98$°C

$k = 30$ W/m °C ; $T_\infty = 65$°C

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.01^2 = 7.8539 \times 10^{-5} \text{ m}^2$$

$$P = \pi d = \pi \times 0.01 = 0.03142 \text{ m}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{50 \times 0.03142}{30 \times 7.8539 \times 10^{-5}}} = 25.82$$

Q - Heat transfer rate

$$Q = \sqrt{hPkA_c}\,(T_b - T_\alpha) \tanh(mL).$$

$$mL = 25.82 \times 0.05 = 1.291$$

$$Q = \sqrt{50 \times 0.03142 \times 30 \times 7.8539 \times 10^{-5}}$$

$$\times (98 - 65) \times \tanh(1.291)$$

Downloaded from Ktunotes.in

$$= 0.0608403(28.3598) = 1.72542 \text{ W}$$

$$Q = 1.72542 \text{ W}$$

Fin Efficiency η_{fin}

$$\eta_{fin} = \frac{\tanh (mL)}{mL}$$

$$= \frac{\tanh (1.291)}{1.291} = 0.665676 = 66.57\%$$

Effectiveness of the fin ε

$$\varepsilon_1 = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}} = \frac{Q_{\text{with fin}}}{h\,A\,(\Delta T)} = \frac{1.72542}{50 \times 7.8539 \times 10^{-5} \times (98 - 65)}$$

$$= 13.3145$$

x-Length at which temperature is 90 °C

Temperature distribution

$$\frac{T - T_\alpha}{T_b - T_\alpha} = \frac{\cosh m(L - x)}{\cosh (mL)}$$

$$\frac{90 - 65}{98 - 65} = \frac{\cosh 25.82\,(0.05 - x)}{\cosh (1.291)}$$

$$\left(\frac{25}{331}\right) \times \cosh (1.291) = \cosh [\,25.82\,(0.05 - x)\,]$$

$$1.4815 = \cosh\,[\,25.82(0.05 - x)]$$

$$25.82(0.05 - x) = \cosh^{-1} 1.4815 = 0.94577$$

$$0.05 - x = \frac{0.94577}{25.82}$$

$$x = 0.05 - \frac{0.94577}{25.82} = 0.01337 \text{ m.}$$

Downloaded from Ktunotes.in

At 13.37 mm from the base, the temperature will be 90°C.

Problem 1.45: *A 6 cm long copper rod (k = 300 W/mK) 6 mm in diameter is exposed to an environment at* 20°C. *The base temperature of the rod is maintained at* 160°C. *The heat transfer co-efficient is* 20 W/m^2 K. *Calculate the heat given by the rod and efficiency and effectiveness of rod. (May & June 2007 AU)*

Solution:

Length of the rod, $L = 6$ cm $= 0.06$ m

Diameter of the rod, $d = 6$ mm $= 0.06$ m

Base Temperature of rod, $T_b = 160°$ C

Environment temperature, $T_\infty = 20°$ C

Thermal conductivity of rod material, $k = 300$ W/mk

Convective heat transfer coefficient $h = 20$ W/m^2 K

Heat given by the rod, from HMT DB page 50,

$$P = \pi d = \pi \times 0.006 = 0.01885$$

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.006^2 = 2.827 \times 10^{-5}$$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{20 \times 0.01885}{300 \times 2.827 \times 10^{-5}}}$$

$$m = 6.67$$

$$Q = \sqrt{hPkA_c}\,(T_b - T_\infty) \tanh(mL)$$

$$Q \text{ (with fin)} = \sqrt{20 \times 0.01885 \times 300 \times 2.827 \times 10^{-5}} \times (160 - 20) \tanh(6.67 \times 0.06)$$

Downloaded from Ktunotes.in

$$= 0.0565 \times 140 \times 0.37995$$

$$= 3.007 \text{ W}$$

$$Q \text{ (without fin)} = hA\,(T_b - T_\infty)$$

$$= 20 \times 2.827 \times 10^{-5} \times (160 - 20)$$

$$= 0.079156 \text{ W}$$

$$\text{Efficiency of fin} = \frac{\tanh\,(mL)}{mL}$$

$$= \frac{\tanh\,(6.67 \times 0.06)}{6.67 \times 0.06}$$

$$= 0.9498 = 94.98\%$$

$$\text{Effectiveness of fin} = \frac{Q_{with\ fin}}{Q_{without\ fin}}$$

$$= \frac{3.007}{0.079156}$$

$$= 37.98$$

Downloaded from Ktunotes.in

1.38 TRANSIENT CONDUCTION (or) UNSTEADY STATE CONDUCTION

1.38.1 Introduction

In the previous section, we have discussed conductive heat transfer problems in which the temperatures are time independent. However, if there is an rapid change in exterior temperature (or) the surroundings, it takes little later time to attain the steady state conditions, after the heat transfer begins. During this period, the temperature varies with time. The process of conduction where the temperature varies with time as well as position is called Transient conduction. In rectangular coordinates, this variation is expressed as $T(x, y, z, \tau)$ where x, y, z indicates variation in the x, y and z directions and τ indicates variation with time. In this section, we are discussing the variation of temperature with time as well as position in one and multi dimensional systems.

The Applications of Transient heat conduction are

(i) Cooling of IC engines.

(ii) Automobile Engine.

(iii) Heating and cooling of metal billets.

(iv) Cooling and freezing of food.

(v) Heat treatment of metals by quenching.

(vi) Starting and stopping of various heat exchange units in power installation.

(vii) Vulcanization of rubber.

(viii) Time responses of thermocouples and thermometers, etc.,

(ix) Brick Burning

Downloaded from Ktunotes.in

During the transient heat transfer, the change in temperature may follow a periodic or nonperiodic variation.

1.38.2 Periodic variation

In a periodic variation, temperature changes periodically within the system which are either regular (or) irregular but definitely cyclic. In a regular periodic variation, the variation is harmonic sinusoidal or nonsinusoidal function. In irregular periodic variation, the variation follows by any function which is cyclic but not necessarily harmonic. The examples of periodic variations are the temperature variations in cylinder of an IC engine, building during a period of 24 hours, surface of earth during a period of 24 hours and heat processing of regenerators.

1.38.3 Non periodic variation

In non periodic variation, the temperature varies non-linearly with time. The examples are heating of ingot in furnace and cooling of bars, blanks and metal billets in steel works, etc.,

1.39 LUMPED SYSTEM ANALYSIS

In this process, the internal conduction resistance of the system is so small that the temperature with in the system is substantially uniform at any time. Such analysis is called lumped system analysis.

Consider a small body whose surface are 'A'$_s$, m^2 whose initial temperature is To throughout and which is suddenly placed in a new environment at constant

Downloaded from Ktunotes.in

temperature T_∞. The response of the body can be determined by relating its rate of change of internal energy convective exchange at the surface.

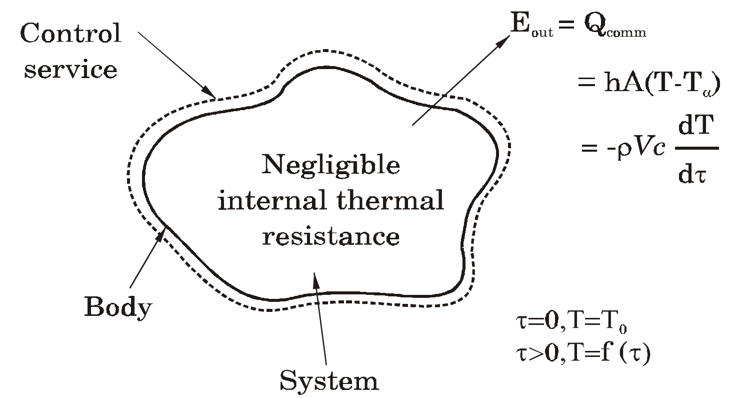


Fig. 1.27 General system for unsteady heat conduction -Lumped heat capacity system

The lumped heat capacity of the solid is $\rho\, CV$, where ρ is the density of solid in kg/m^3, c = specific heat of solid in J/kg k and v = volume of the solid in m^3. The convective heat transfer coefficient between the solid surroundings is h in W/m^2 K.

$$\therefore\ Q = -h\, A_s\, (T - T_\infty) = \rho\, CV \frac{dT}{d\,\tau} \qquad \ldots(1)$$

$$\therefore\ \frac{dT}{T - T_\infty} = \frac{-h\, A_s}{\rho\, CV}\, d\,\tau \qquad \ldots(2)$$

Integrating above equation

Downloaded from Ktunotes.in

$$ln\ (T - T_{\infty}) = \frac{-h\,A_s}{\rho\,CV}\,\tau + C_1 \qquad ...(3)$$

The constant of integration, C_1 can be found by applying the initial condition

The initial boundary condition is

At $\tau = 0, T = T_o$

$\therefore C_1 = \ln\,(T_o - T_{\infty})$

Substituting in equation

$$\ln\,(T - T_{\infty}) = \frac{-h\,A_s}{\rho\,CV}\,\tau + \ln\,(T_o - T_{\infty})$$

$$\therefore \ln\left(\frac{T - T_{\infty}}{T_o - T_f}\right) = \frac{-h\,A_s}{\rho\,CV}\,\tau$$

$$\therefore \frac{T - T_{\infty}}{T_o - T_{\infty}} = \exp\left[\frac{-h\,A_s}{\rho\,CV}\,\tau\right] \qquad ...(4)$$

The above equation helps us to determine the temperature "T" of the system for Newtonian heating (or) cooling, at time "τ" or the time "τ" required for the temperature to reach the temperature "T".

The temperature of the system falls or rises to the surrounding temperature "T_{∞}" exponentially. The temperature of the system changes drastically at the begining but slowly later on as shown in **Figure 1.28**.

The term $\dfrac{\rho\,VC}{h\,A_s}$ is known as thermal time constant "τ_{th}". This indicates the rate of response of a system to a sudden change in the surrounding temperature.

Downloaded from Ktunotes.in

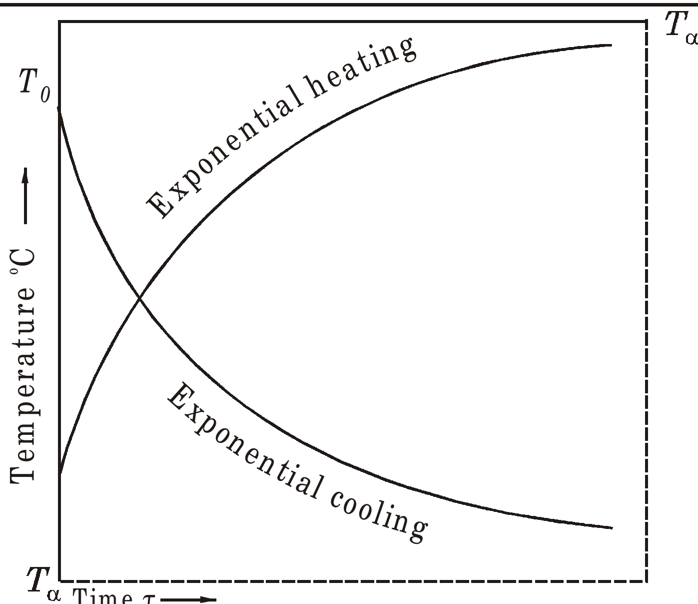


Fig 1.28 Newtonian heating or cooling

$$\text{(ie)}\quad \tau_{th} = \left(\frac{1}{h\,A_s}\right)\cdot(\rho\,V_c)$$

$$= R_{th}\cdot C_{th}$$

where

R_{th} = resistance to convection heat transfer

C_{th} = lumped thermal capacitance of solid

Application of eqn (4) depends on several factors, but the condition can be checked by using relative temperature drop within the solid compared to temperature drop from surface to the fluid.

Heat conducted within the solid can be obtained considering the solid as slab.

Downloaded from Ktunotes.in

$$Q = \frac{k\, A_s\, (\Delta\, T)_s}{L_c} \quad \text{...(5)}$$

where $(\Delta\, T)_s$ is the temperature drop in solid,

L_c is the characteristic length

k is the thermal conductivity.

The heat convected at the surface is given by

$$Q = h\, A_s\, (\Delta\, T)_c \quad \text{...(6)}$$

where $(\Delta\, T)_c$ is the convection drop, h is convective heat transfer coefficient

Equating (5) & (6)

$$\frac{k\, A_s\, (\Delta\, T)_s}{L_c} = h\, A_s\, (\Delta\, T)_c$$

$$\therefore \frac{(\Delta\, T)_s}{(\Delta\, T)_c} = \frac{hL_c}{k}$$

The term $\frac{hL_c}{k}$ is the dimensionless number.

This dimensionless number is called as **Biot Number (Bi)**. It gives the ratio of internal resistance to surface resistance. If Biot number is small then $(\Delta\, T)_s$ is small (ie) internal resistance is small, so this condition is taken as the check for applicability of lumped analysis. If Biot number is less than 0.1, it is proved that this analysis can be used without appreciable error (ie) around 5%.

Downloaded from Ktunotes.in

Figure 1.29 shows an analogus electric network, thermal capacity of system is charged initially at potential "T_o" by closing the switch "S". Then, when the switch is opened, the energy stored in the thermal capacitance "C_{th}" is dissipated through the resistance $\frac{1}{h\,A_s}$.

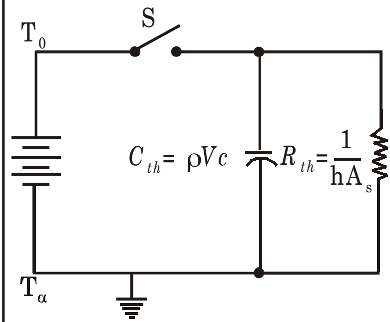


Fig. 1.29 Equivalent thermal circuit for lumped capacitance solid

The term $\frac{h\,A_s}{\rho\,VC}\tau$ can be represented in non dimensional form as

$$\frac{h\,A_s}{\rho\,VC}\tau = \left(\frac{h\,v}{k\,A_s}\right)\left(\frac{A_s^2\,k}{\rho\,v^2\,c}\tau\right)$$

$$= \left(\frac{h\,L_c}{k}\right)\left(\frac{\propto\,\tau}{L_c^2}\right) = (B_i)\,(F_o)$$

where $\propto = \frac{k}{\rho C} =$ thermal diffusivity of solid

The characteristic length "L_c" is defined as the ratio of volume of the solid "V" to surface area of solid "A_s".

For simple geometrical shapes, the values of characteristic length are given as follows.

Downloaded from Ktunotes.in

For a flat plate with length "L", breadth "B", height "H" the heat transfer occurs from both sides, hence the area exposed for heat transfer 2BH and the volume of flat plate is LBH.

$$\therefore L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = \frac{L}{2}$$

For a cylinder with

Height "L", Radius "R"

$$L_c = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2}$$

For sphere, with Radius of sphere "R",

$$L_c = \frac{4/3\,\pi R^3}{4\pi R^2} = \frac{R}{3}$$

For cube, with side of cube "L",

$$L_c = \frac{L^3}{6L^2} = \frac{L}{6}$$

The term $\frac{\alpha\,\tau}{L_c^2}$ is a non dimensional factor which is known as Fourier number. "F_o". This number signifies the rate of passing of heat through the body with respect to the body dimensions. Fourier number should be higher for sudden response in heating (or) cooling.

1.40 SUMMARY

Steady state $\Rightarrow T =$ constant

Unsteady state $\Rightarrow T =$ variable with time (t)

Downloaded from Ktunotes.in

1.41 TYPES OF UNSTEADY STATE CONDUCTION

1. Lumped heat conduction ($B_i \simeq 0, B_i < 0.1$)
2. Infinite solids ($0.1 < B_i < 100$)
3. Semi-infinite solids ($B_i = \infty$)

where B_i = Biot Number = $\dfrac{R}{R_C}$

Biot number is defined as the ratio of internal conductive resistance to its surface convective resistance.

$$B_i = \frac{R}{R_C} = \frac{\Delta x / kA}{1/hA} \ (slab)$$

$$= \frac{h\,\Delta x}{k}$$

$$= \frac{h\,L_c}{k} \quad \text{[Refer page 58, 112 for formula]}$$

where L_c = characteristic length or significant length

$$L_c = \frac{\text{Volume}}{\text{Surface Area}}$$

L_c for slab

$$L_c = L = \text{half thickness}$$

$$L_c = \frac{A \times L}{2A} = \frac{L}{2}$$

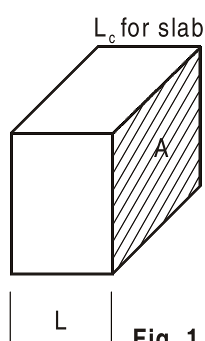


Fig. 1.30 (i)

Downloaded from Ktunotes.in

L_c for cylinder of radius R

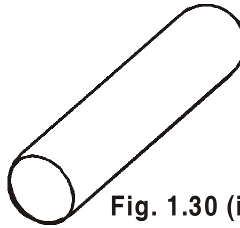


Fig. 1.30 (ii)

Surface Area = $2\pi RL$

Volume = $\pi R^2 \times L$

$$L_c = \frac{\text{Vol}}{\text{S.A.}} = \frac{\pi R^2 \times L}{2\pi RL} = \frac{R}{2}$$

L_c for sphere

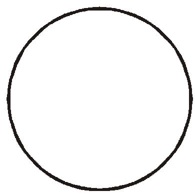


Fig. 1.30 (iii)

Surface Area = $4\pi R^2$

Volume = $\frac{4}{3}\pi R^3$

$$L_c = \frac{\text{Vol}}{\text{S.A}} = \frac{4}{3}\frac{\pi R^3}{4\pi R^2} = \frac{R}{3}$$

L_c for cube of side 'L'.

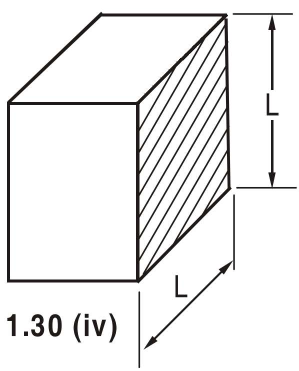


Fig. 1.30 (iv)

Surface Area = $6L^2$

Volume = L^3

$$L_c = \frac{\text{Vol}}{\text{S.A.}} = \frac{L^3}{6L^2} = \frac{L}{6}$$

L_c-Characteristic length

for slab = $\frac{L}{2}$

for Cylinder = $\frac{R}{2}$

Downloaded from Ktunotes.in

$$\text{for Sphere} = \frac{R}{3}$$

$$\text{for Cube} = \frac{L}{6}$$

Problem 1.46: *A Copper slab of* $400\text{mm} \times 400\text{mm} \times 5\text{mm}$ *thick is initially at* 250°C, *suddenly its surface temp. is lowered to* 30°C *with heat transfer coefficient* $90\ \text{W/m}^2\,°\text{C}$. *Determine (a) Time required for the slab to reach* 90°C. *(b) Instantaneous heat transfer rate at* 90°C *(c) Total heat flow upto* 90°C *(d) Temp. after 1 minute.*

Solution

$$t = L = 5\text{mm};\ T_0 = 250\,°\text{C}$$

$$T_\alpha = 30°\text{C};\ L_c = \frac{L}{2} = 2.5\text{mm}$$

$$h = 90\ \text{W/m}^2\,°\text{C}$$

For Copper (Take from Pg 2 of HMT Table - CPK)

$$\rho = 8954\ \text{kg/m}^3$$

$$C_p = 381\ \text{J/KgK}$$

$$k = 386\ \text{W/m-K}$$

$$L = \text{Thickness} = 5\ \text{mm}$$

$$L_c = 2.5\text{mm}$$

$$B_i = \frac{h\,L_c}{k} = \frac{90 \times 2.5 \times 10^{-3}}{386} = 0.5829 \times 10^{-3} < 0.1$$

So it is Lumped conduction.

Downloaded from Ktunotes.in

From Pg 58

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{hA}{CV\rho}\tau\right]}$$

where $\frac{A}{V} = \frac{1}{L_c}$

$$\frac{90 - 30}{250 - 30} = e^{\left[\frac{-90 \times \tau}{381 \times 8954 \times 2.5 \times 10^{-3}}\right]}$$

Take ln on both sides

$$ln\left(\frac{60}{220}\right) = [0.01055 \ \tau]$$

$$\boldsymbol{\tau = 123.12146 \text{ sec}}$$

The time required for the slab to reach $90°C = \tau = 123.124$ sec

(b) Instantaneous heat transfer rate at 90°C

$$q = h\ A\ (T - T_\alpha)$$

$A = 2A$ (for slab) from both sides of slab

$$= 2 \times (0.4 \times 0.4) = 0.32 \text{ m}^2$$

$$q = 90 \times 0.32\ (90 - 30) = 1728 \text{W}.$$

$$\boxed{q = 1728 \text{ W}}$$

(c) Total heat flow upto 90°C

$$q_t = m\ C_P(T - T_0) = \rho\ V\ C_P\ (T - T_0)$$

$$V = A \times L$$

$$= 0.16 \times (5 \times 10^{-3}) = 8 \times 10^{-4} \text{ m}^3$$

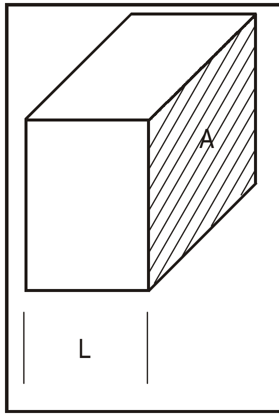
Downloaded from Ktunotes.in

$q_t = 8954 \times 381 \times 8 \times 10^{-4}\ (90 - 250)$

$= -\ 436668.6723$

$\boldsymbol{q_t = -\ 436668.672\ \ J}$

'-' sign indicates heat is coming out of the slab.



(d) Temp after 1 minute

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{hA}{\rho VC}\tau\right]}$$

where $\frac{A}{V} = \frac{1}{L_c}$

$\tau = 60$ sec

$$\frac{T - 30}{250 - 30} = e^{\left[-\frac{90 \times 60}{8954 \times 381 \times 2.5 \times 10^{-3}}\right]}$$

$$\boxed{T = 146.8008\ °C}$$

Problem 1.47: *An aluminium alloy plate of 200 mm × 200 mm × 2 mm size at* 180°C *is suddenly quenched in liquid oxygen at* – 192°C *determine the time required for the plate to reach a temperature of* – 90°C. *Assume* $h = 5.56$ kw/m^2 °C, $c_p = 0.8$ kJ/kg°C, *and* $\rho = 3000$ kg/m^3.

Given

$L = 2$ mm $= 0.002$ m; $B = 400$ mm $= 0.4$ m

$H = 400$ mm $= 0.4$ m

$T_o = 180°$ C; $T_\infty = -\ 192°$ C; $T = -\ 90°$ C;

Downloaded from Ktunotes.in

$h = 5.56$ kw/m^2 °C; $C_p = 0.8$ kJ/kg°C;

$\rho = 3000$ kg/m^3

To find

τ

Solution

From Heat and Mass transfer table,

Pg.No.1,

for Aluminium Alloy

$k =$ 177 W/mk = 0.177 kW/mk

$\therefore$ Bi (Biot number) for slab

$$= \frac{hL_c}{k}$$

where L_c = characteristic length

$$= \frac{V}{A_s} = \frac{L}{2} = \frac{0.002}{2}$$

$$L_c = 0.001 \text{ m}$$

$$\therefore Bi = \frac{hL_c}{k} = \frac{5.56 \times 0.001}{0.177} = 0.03141$$

Since Bi is less than 0.1, hence lumped capacitance method may be applied for the solution.

From HMT table pg.no. 58,

the temperature distribution is given

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\frac{h\,A_s}{CV\,\rho}\,\tau\right]$$

Downloaded from Ktunotes.in

$$\frac{-90-(-192)}{180-(-192)} = \exp\left[\frac{-5.56}{0.8\times 3000}\times\frac{1}{0.001}\times\tau\right]$$

$$0.2741 = e^{-2.3166\,\tau}$$

$$0.2741 = \frac{1}{e^{2.3166\,\tau}}$$

$$e^{2.3166\,\tau} = \frac{1}{0.2741} = 3.647$$

$$2.3166\,\tau = \ln 3.647$$

$$2.3166\,\tau = 1.2939$$

$$\therefore\ \tau = \frac{1.2939}{2.3166}$$

$$\tau = 0.5585 \text{ secs}$$

Problem 1.48: *Aluminium Cube of* 60 mm *side is at* 500°C *initially and it is suddenly immersed in water at* 100°C *with* $h = 120\ \text{W/m}^2\text{°C}$.

Determine

(a) time required for the cube to reach 250°C

(b) Instantaneous heat transfer rate.

(c) Total heat flow upto 250°C.

for Aluminium From Pg 1 of HMT Table

$k = 204$ W/m K ; $\rho = 2707$ Kg/m^3

Thermal diffusivity

$$\alpha = \frac{k}{\rho\,C} = 0.34\text{m}^2/\text{hr} = 8.39\times 10^{-5}\ \text{m}^2/\text{sec}$$

$$L = 60\text{mm and } L_c = \frac{L}{6}$$

Downloaded from Ktunotes.in

$$L_c = \frac{60 \times 10^{-3}}{6} = 0.01\,\text{m}$$

$$T_0 = 500°\text{C} \; ; \; T_\alpha = 100°\text{C}$$

$$h = 120 \text{ W/m}^2 \text{ °C} \; ;$$

$$\alpha = \frac{k}{\rho\, C} = 8.39 \times 10^{-5}$$

$$\rho C = \frac{k}{8.39 \times 10^{-5}} = \frac{204}{8.39 \times 10^{-5}}$$

$$= 2.431466 \times 10^{6}$$

$$\text{Biot Number} = \frac{hL_c}{k}$$

$$= \frac{120 \times 0.01}{204}$$

$$= 5.8823 \times 10^{-3} < 0.1$$

So it is lumped heat conductions.

From Pg 58

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{hA}{\rho CV} \tau \right]}$$

$$A = 6L^2 = 6 \times 0.06^2$$

$$= 0.0216 \text{ m}^2$$

$$V = L^3 = 0.06^3$$

$$\frac{A}{V} = \frac{1}{L_c} = \frac{1}{0.01}$$

Downloaded from Ktunotes.in

$$T = 250°\text{C}$$

$$\frac{250 - 100}{500 - 100} = e^{\left[\frac{-120 \times \tau}{2.4314 \times 10^6 \times 0.01}\right]}$$

Taking ln on both sides

$$\ln(0.375) = \left[-4.93529 \times 10^{-3}\tau\right]$$

$$\tau = 198.73775 \text{ sec}$$

(b) Instantaneous heat transfer at 250°C

From Pg 58

$$q = h\,A\,(T - T_\alpha)$$

$$A = 6L^2 = 6 \times 0.06^2 = 0.0216 \text{ m}^2$$

$$q = 120 \times 0.0216\,(250 - 100) = 388.8 \text{ W}$$

$$\boxed{q = 388.8 \text{ W}}$$

(c) Total heat flow upto 250°C

From Pg 58

$$q_t = \rho C\,V\,(T - T_0)$$

$$= 2.431466 \times 10^6 \times (0.06)^3\,(250 - 500) = -131295.6 \text{ J}$$

$$\boldsymbol{q_t = -131295.6\ J}$$

'-' sign indicates heat is flowing from the cube.

Time Constant (τ_0)

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{h\,A\,\tau}{\rho C V}\right]}$$

Downloaded from Ktunotes.in

Substitute $\tau = \dfrac{\rho c v}{h A}$

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{h A}{\rho C V} \times \frac{\rho C V}{h A}\right]}$$

$$= e^{[-1]}$$

$$= 0.3678$$

$$-\left[\frac{T - T_\alpha}{T_0 - T_\alpha}\right] = -0.3678$$

$$+1 - \left[\frac{T - T_\alpha}{T_0 - T_\alpha}\right] = -0.3678 + 1$$

$$\frac{T_0 - T_\alpha - T + T_\alpha}{T_0 - T_\alpha} = 0.6322$$

$$\frac{T_0 - T}{T_0 - T_\alpha} = 0.6322$$

$$T_0 - T = 0.6322\,(T_0 - T_\alpha)$$

$T_0 - T =$ Temp. drop and $T_0 - T_\alpha =$ Initial temp. difference.

So, time Constant (τ_0) is defined as time required for temperature drop to reach 63.22% of initial temperature difference.

It is used in determining the sensitivity of thermometer.

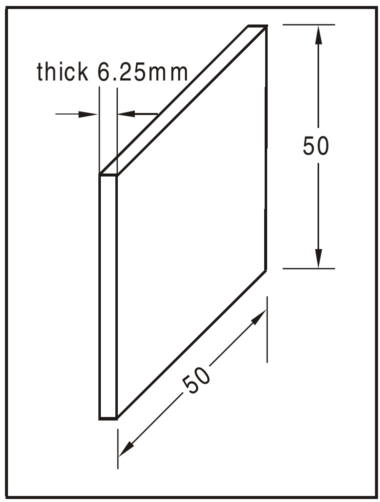
If $\tau_0 < 0.625$ sec, then it is sensitive.

Downloaded from Ktunotes.in

Problem 1.49: *A* 50 cm × 50 cm *copper slab 6.25 mm thick has a uniform temp. of* 300°C. *Its temperature is suddenly lowered to* 36°C. *Calculate the time required for the plate to reach the temp. of* 108°C. *Take* $\rho = 9000\ \text{Kg/m}^3$; $C = 0.38\text{KJ/Kg°C}$, $k = 370\ \text{W/m°C}$, $h = 90\ \text{W/m}^2\text{°C}$; *thickness* $= 6.25 \times 10^{-3}$ m

Solution

Surface Area (Both Sides)



$$A = 2 \times 0.5 \times 0.5$$

$$= 0.5\ \text{m}^2$$

Characteristic length L_c

$$L_c = \frac{\text{Volume}}{\text{Surface Area}}$$

$$= \frac{0.5 \times 0.5 \times 6.25 \times 10^{-3}}{0.5}$$

$$= 3.125 \times 10^{-3}\ \text{m}$$

$$\text{Biot Number } B_i = \frac{hL_C}{k}$$

$$= \frac{90 \times 3.125 \times 10^{-3}}{370}$$

$$= 7.60135 \times 10^{-4} < 0.1$$

Since $B_i < 0.1$, lumped capacitance method should be applied for the solution

Downloaded from Ktunotes.in

Temperature distribution

from Pg. 58

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{hA}{\rho CV}\tau\right]}$$

$$\frac{108 - 36}{300 - 36} = e^{\left[-\frac{90 \times 0.5}{9000 \times (0.5 \times 0.5 \times 6.25 \times 10^{-3}) \times 380}\tau\right]}$$

$$0.272727 = e^{\left[-8.421 \times 10^{-3}\tau\right]}$$

$$\ln 0.272727 = -8.421 \times 10^{-3}\tau$$

$$\tau = \frac{-1.299}{-8.421 \times 10^{-3}} = 154.29 \text{ sec}$$

$$\boldsymbol{\tau = 154.29 \text{ sec}}$$

Problem 1.50: *Alloy steel of 12 mm diameter heated to* 800°C *are quenched in a bath at* 100°C. *The material properties of the ball are* $k = 205$ KJ/mhr°C, *density* $\rho = 7860$ Kg/m^3, *diffusivity* $\alpha = 0.06\, m^2/hr$; *Sp. heat* $C_p = 0.45$ KJ/KgK. *The Connective heat transfer coefficient h is* 180 KJ/hr m^2 K. *Determine temperature for ball after 10 sec and time for the ball to cool to* 400°C.

(April-1999, Madras University)

Solution

$$k = \frac{205 \times 10^3}{3600} \text{ W/m°C} = 59.44 \text{ W/m°C}$$

$$h = \frac{150 \times 10^3}{3600} \text{ W/m}^2\text{°C} = 41.67 \text{ W/m°C}; \quad B_i = \frac{hL_c}{k}$$

Downloaded from Ktunotes.in

$$L_c = \frac{R}{3} \text{ for sphere}$$

$$L_c = \frac{6}{3} = 2\text{mm} = 2 \times 10^{-3}\text{m}$$

$$B_i = \frac{41.67 \times 2 \times 10^{-3}}{59.44} = 1.402 \times 10^{-3} < 0.1$$

Since $B_i < 0.1$, the Lumped Capacity analysis is applied.

T_0 = initial temp = 800°C

T_α = Surface temp = 100°C

To find T when $\tau = 10$ sec

From Pg 58

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{hA\,\tau}{\rho\, CV}\right]}$$

$$\left[\frac{A}{V} = \frac{1}{L_c}\right]$$

$$\frac{T - 100}{800 - 100} = e^{\left[-\frac{41.67 \times 10}{7860 \times 450 \times (2 \times 10^{-3})}\right]}$$

$$= 0.9428$$

$$T - 100 = (0.9428 \times 700)$$

$$T = (0.9428 \times 700) + 100 = 759.96\ ^\circ\text{C}$$

$$\boldsymbol{T = 759.96\,^\circ\text{C}}$$

Downloaded from Ktunotes.in

To find τ for $T = 400°C$

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{h A \tau}{\rho C V}\right]}$$

$$\frac{400 - 100}{800 - 100} = e^{\left[-\frac{41.67 \times \tau}{7860 \times 450 \times 2 \times 10^{-3}}\right]}$$

$$\ln 0.42857 = -5.8906 \times 10^{-3}\,\tau$$

$$\tau = \mathbf{143.84\ sec}$$

Problem 1.51: *A solid copper cylinder of 9 cm diameter is initially temperature of* 28°C *and it is suddenly dropped into ice water. After 5 minutes the temperature of cylinder again measured as* 1°C. *Determine unit surface conductance by using lumped heat analysis method.*

Given

$D = 9$ cm $= 0.09$ m; $T_o = 28°$ C $+ 273 = 301$ K;

$T_\infty = 0°$ C $+ 273 = 273$ K; $T = 1°$ C $+ 273 = 274$ K;

$\tau = 5$ min $= 300\ S$

To find

h

Solution

from HMT data book, the properties of copper, from Pg No.2

$\rho = 8954$ kg/m^3

$c_p = 383$ J/kg k

Downloaded from Ktunotes.in

$k = 386$ w/mk

for cylinder,

the characteristic length, $L_c = \dfrac{R}{2}$

$$\therefore L_c = \frac{0.045}{2} = 0.0225 \text{ m}$$

$$\therefore L_c = 0.0225 \text{ m}$$

from HMT data book, Pg No. 58

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[\frac{-h A_s}{CV\rho}\tau\right]$$

$$\frac{274 - 273}{301 - 273} = \exp\left[\frac{-h \times \tau}{CL_c \rho}\right]$$

$$0.03571 = \exp\left[\frac{-h \times 300}{383 \times 0.0225 \times 8954}\right]$$

$$0.03571 = \exp[-h \times 3.8879 \times 10^{-3}]$$

$$\ln[0.03571] = -h \times 3.8879 \times 10^{-3}$$

$$\therefore h = 857.1 \text{ w/m}^2 \text{ k}$$

Problem 1.52: *An egg with a mean diameter of 35 mm and initially at 22°C is placed in boiling water pan for 5 minutes and found to be boiled to the consumer's taste. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5°C. Take the following properties for egg.*

$k = 10$ w/m°C, $\rho = 1200$ kg/m^3,

$c = 2$ kJ/kg°C, $h = 100$ W/m^2 °C,

Downloaded from Ktunotes.in

use lumped analysis.

Given:

$$D = 35 \text{ mm} = 0.035 \text{ m} \Rightarrow R = \frac{0.035}{2} = 0.0175 \text{ m};$$

$$T_o = 22° \text{ C}, \tau = 5 \text{ min} = 300 \text{ s}; k = 10 \text{ w/m° C};$$

$$\rho = 1200 \text{ kg/m}^3, h = 100 \text{ W/m}^2 \text{ °C}$$

$$T_\infty = 100° \text{ C} = 373 \text{ K}, C = 2 \text{ kJ/kg°C} = 2000 \text{ J/kg°C}$$

To find

τ

Solution:

for sphere,

the characteristic length $L_c = \dfrac{R}{3}$

$$\therefore L_c = \frac{0.0175}{3}$$

$$L_c = 5.833 \times 10^{-3} \text{ m}$$

$\therefore Bi$ (biot number)

$$= \frac{hL_c}{K} = \frac{100 \times 5.833 \times 10^{-3}}{10}$$

$$= 0.05833 < 0.1$$

Since Bi is less than 0.1, hence lumped capacitance method may be applied for the solution

from HMT data book, Page No. 58,

Downloaded from Ktunotes.in

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[\frac{-h\, A_s\, \tau}{PVC}\right]$$

$$\frac{T - 373}{295 - 373} = \exp\left[\frac{-100 \times 300}{1200 \times 5.833 \times 10^{-3} \times 2000}\right]$$

$$\therefore\ T = 363.85\ \text{K}$$

Keeping the given data same

(ie) $T_o = 5°\text{ C} = 278\text{ K}, T_\infty = 100°\text{ C} = 373\text{ K};$

$$T = 363.85\ \text{K}$$

Substituting in

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[\frac{-100 \times \tau}{1200 \times 5.833 \times 10^{-3} \times 2000}\right]$$

$$[0.09631] = \exp\,[-7.1432 \times 10^{-3}\,\tau]$$

$$\ln 0.09631 = -7.1432 \times 10^{-3}\,\tau$$

$$\tau = 327.6\ \text{s}$$

Problem 1.53: *When a thermocouple is moved from one medium to another medium at a different temperature, sufficient time must be given to a thermocouple to come to thermal equilibrium with the new conditions before a reading is taken. Find the temperature response (i.e. an approximate temperature Vs time for intervals, 0, 40 and 120 seconds) when this is suddenly immersed in*

(i) Water at 40°C (h = 80 W/m^2 K)

(ii) Air at 40°C (h = 40 W/m^2 K)

Assume unit length of wire

Downloaded from Ktunotes.in

Solution:

Given initial temperature $T_o = 150°$ C

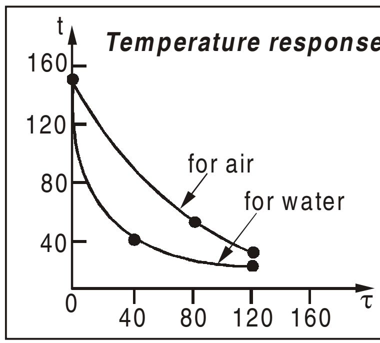
$\tau = 0, 40$ and 120 seconds,

for copper (Pg.No.2 HMT DB)

$\rho = 8954$ kg/m^3, $C = 383$ J/kgK

(i) water

$T_\infty = 40°$ C, $h = 80$ W/m^2 K



for $\tau = 0$ sec

From HMT DB page, 58

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\frac{h}{\rho} \times \frac{1}{L}\frac{\tau}{c}\right]$$

$$= \exp\left[-\frac{h}{\rho} \times \frac{2}{R}\tau\right]$$

$$\frac{T - 40}{150 - 40} = \exp\left[-\frac{80}{8954} \times \frac{2}{0.0005} \times \frac{0}{383}\right]$$

$\therefore\ t = 150°$ C

For $\tau = 40$ sec

$$\frac{T - 40}{150 - 40} = \exp\left(-\frac{80}{8954} \times \frac{2}{0.0005} \times \frac{40}{383}\right)$$

$= 0.024$

$\therefore\ T = 42.63\ °$ C

For $\tau = 120$ sec

$$\frac{T - 40}{150 - 40} = \exp\left[-\frac{80}{8954} \times \frac{2}{0.0005 \times 383} 120\right]$$

Downloaded from Ktunotes.in

$\therefore \ \tau = 40$ sec

(ii) for air $T_\infty = 40°$ C, $h = 40$ W/m^2 K

When $\tau = 0$ sec

$$\frac{T - T_\infty}{T_o - t_\infty} = \exp^0 \Rightarrow \frac{t - 40}{150 - 40} = 1 \Rightarrow t = 150°\text{ C}$$

When $t = 40$ sec

$$\frac{T - 40}{150 - 40} = \exp\left(-\frac{40}{8954} \times \frac{2}{0.0005} \times \frac{40}{383}\right)$$

$$\Rightarrow t = 57°\text{ C}$$

When $\tau = 120$ sec

$$\frac{T - T_\infty}{T_o - T_\infty} = \exp -\left(\frac{h}{\rho} \times \frac{1}{L} \times \frac{\tau}{C}\right)$$

$$\frac{T - 40}{150 - 40} = \exp -\left(\frac{40}{8954} \times \frac{2}{0.0005} \times \frac{120}{383}\right)$$

$$\therefore T = 40.4\ °\text{ C}$$

Problem 1.54: *A large steel plate of 5 cm thick initially at* 400°C *is suddenly exposed to a surrounding at* 60°C *with* h = 285 W/m^2°C

Calculate

(a) Centre line temperature

(b) temp inside the plate at a distance of 1.25 cm form midplane after 3 minutes. $k = 42.5$ W/m°C; $\alpha = 0.043$ m^2/hr

(Nov-96, Madras University)

Solution

Initial temp. $T_i = 400°$C

Downloaded from Ktunotes.in

$\tau = 3$ min

$= 180$ sec

Surface temp $T_\alpha = 60°C$

$k = 42.5$ W/m°C ; $\alpha = 0.043$ m^2/hr

$= 1.194 \times 10^{-5}$ m^2/sec

$L_c = \frac{L}{2}$ (half thickness) $= \frac{5}{2} = 2.5$ cm

$L_c = 0.025$ m

$B_i = \frac{hL_c}{k} = 0.1676 > 0.1$

Since $B_i > 0.1$, it is infinite Solid.

From Pg 66 of HMT Table-CPK

'X' axis $\rightarrow \frac{\alpha\,\tau}{L_c^2} = \frac{1.194 \times 10^{-5} \times 180}{(0.025)^2} = 3.44$

Refer Heisler's chart for plane wall centre temperature,

For X-axis 3.44 and $\frac{hL_c}{k} = 0.17$, Y *axis* $= 0.62$

Page 66 of HMT Table

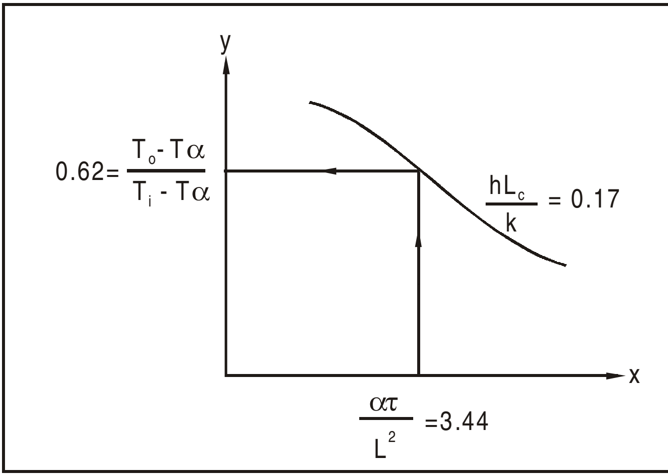
From Chart,

we found $\frac{T_0 - T_\alpha}{T_i - T_\alpha} = 0.62$

$T_0 = 0.62\ (T_i - T_\alpha) + T_\alpha$

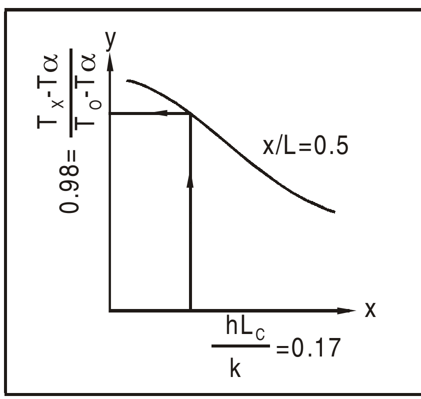
$= 0.62\ (400 - 60) + 60 = 270.8°C$

Downloaded from Ktunotes.in



So the centre line temp= 270.8 °C

To find temperature at 1.25 cm from midplane (Pg 67)



$$\frac{T_x - T_\alpha}{T_0 - T_\alpha} = 0.98$$

for $B_i = \dfrac{hL}{k} = 0.17$

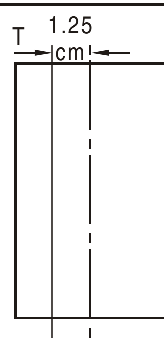
and $\dfrac{x}{L} = \dfrac{0.0125}{0.025} = 0.5$

$$\frac{T_x - T_\alpha}{T_0 - T_\alpha} = 0.98$$

$$\frac{T_x - 60}{270.8 - 60} = 0.98$$

$$T_x = 0.98\ (270.8 - 60) + 60$$

$$= 266.584\ °\mathrm{C}$$



Downloaded from Ktunotes.in

Temperature inside the plate at a distance of 1.25 cm from midplane after 3 min. is 266.584 °C

Problem 1.55: *A slab of Aluminium 5 cm thick initially at* 200°C *is suddenly immersed in a liquid at* 70°C *for which the convection heat transfer coefficient is* $525\ \text{W/m}^2\text{K}$. *Determine the temperature at a depth of 12.5 mm from one of the faces 1 minute after the immersion. Also calculate the energy removed per unit area from the plate during 1 minute of immersion. (May/June 2007 - AU)*

Take $P = 2700$ bar $C_p = 0.9$ kJ/kgK

$$k = 215 \text{ W/mK} \quad \alpha = 8.4 \times 10^{-5} \text{ m}^2/\text{s}$$

Solution:

The Heisler charts are used for solving this problem

Here 2L = 5 cm, L = 2.5 cm, $t = 1$ min = 60 s

from Pg.No. 64

$$F_o = \frac{\alpha\,\tau}{L^2} = \frac{8.4 \times 10^{-5} \times 60}{0.025^2} = 8.064$$

$$\frac{1}{B_i} = \frac{k}{hL} = \frac{215}{525 \times 0.025} = 16.38$$

From HMT DB page 64

The centre line temperature is given by

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{\theta_c}{\theta_o} = 0.6$$

$$\therefore\ \theta_c = T_o - T_\infty = 0.6\ (200 - 70)$$

$$= 88.4°\ \text{C}$$

$$\therefore\ T_o = 88.4 + 70 = 158.4°\ \text{C}$$

Downloaded from Ktunotes.in

Temperature at a depth of 12.5 mm from one of the faces 1 minutes after immersion

$$\frac{x}{L} = \frac{12.5}{25} = 0.5$$

From heisler chart, at

$$\frac{x}{L} = 0.5, \frac{k}{hL} = 16.38$$

From HMT DB page 67

$$\frac{T_{(x/L)} - T_\infty}{T_o - T_\infty} = 0.75$$

$$T_{(x/L)} = 0.75\ (158.4 - 70) + T_\infty$$

$$= 66.3° \text{ C} + T_\infty$$

$$= 66.3 + 70 = 136.3° \text{ C}$$

Energy removed per unit area, $\left(\frac{U}{A}\right)$

$$\frac{h^2 \alpha t}{k^2} = \frac{525^2 \times 8.4 \times 10^{-5} \times 60}{215^2} = 0.03$$

$$B_1 = \frac{hL}{k} = \frac{525 \times 0.025}{215} = 0.06$$

$$\frac{U}{U_o} = 0.5$$

$$\therefore \frac{U_o}{A} = \rho\, C\ (2L)\ (T_o - T_\infty)$$

$$= 2700 \times 900 \times 0.05\ (200 - 70)$$

$$= 15.8 \times 10^6 \text{ J/m}^2$$

Downloaded from Ktunotes.in

$$\Rightarrow \frac{U}{A} = 0.5 \times 15.8 \times 10^6$$

$$= 7.89 \times 10^6 \text{ J/m}^2$$

Problem 1.56: *A long steel cylinder of 12 cm dia initially at* 20°C *is placed in a furnace at* 820°C *with* $h = 140$ W/m^2°C. *Calculate*

(a) time required for the axis to reach 800 °C

(b) The corresponding temp. at a radius of 5.4 cm at same time

k = 21 W/'m °C*;* $\alpha = 6.11 \times 10^{-6}$ m^2/sec

T_i = initial temperature = 20°C *(Oct-99, Madras University)*

Solution

T_α = surface temperature = 820°C; h = 140 W/m^2°C

$$L_c = \frac{R_0}{2} = \frac{0.06}{2} = 0.03 \text{ m}$$

$$B_i = \frac{hL_c}{k} = \frac{140 \times 0.03}{21} = 0.2 > 0.1$$

Since $B_i > 0.1$, and $B_i < 100$ it is infinite solid.

From Pg 69, Refer chart

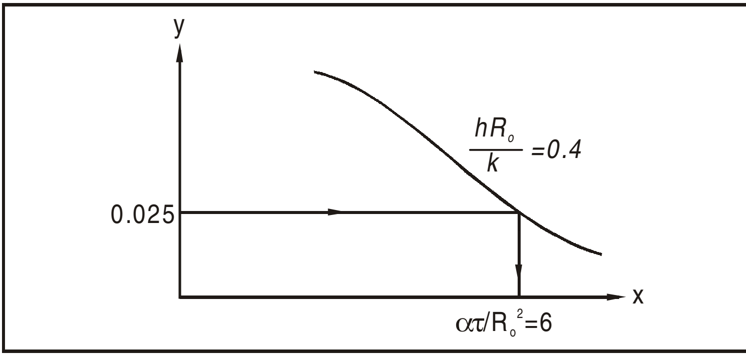
$$\text{Curve} \rightarrow \frac{hR_0}{k} = \frac{140 \times 0.06}{21} = 0.4$$

$$Y\text{–}axis \rightarrow \frac{T_0 - T_\alpha}{T_i - T_\alpha} = \frac{800 - 820}{20 - 820} = 0.025$$

So from 'x' axis, $\dfrac{\alpha\,\tau}{R_0^2} = 6$

$$\frac{6.11 \times 10^{-6} \times \tau}{0.06^2} = 6$$

Downloaded from Ktunotes.in



τ = **3535.188 sec**

Temp. at radius of 5.4 cm (Refer Chart in Pg 70 of HMT Table)

$$\frac{r}{R_0} = \frac{0.054}{0.06} = 0.9$$

$$\frac{T_r = T_\alpha}{T_0 - T_\alpha} = 0.85$$

[By seeingcurve= 0.9 and x $axis$ = 0.4, y axis will be 0.85]

$$\frac{T_r - 820}{800 - 820} = 0.85 \quad \text{[Y–axis]}$$

$$T_r = 803\,°\text{C}$$

Temperature at the radiusof 5.4 cm is = **803°C**

Extra

(c) Find the total heat flow upto 20 min. per m length.

To find heat flow, we can use the Grober's chart (page 71)

$$\rho = 7833,\ C_p = 465$$

Downloaded from Ktunotes.in

From page No. 71

$$\frac{h\,R_0}{k} = \frac{140 \times 0.06}{21} = 0.4$$

$$\frac{h^2\,\alpha\,\tau}{k^2} = \frac{140^2 \times 6.11 \times 10^{-6} \times (20 \times 60)}{21^2}$$

$$= 0.3259$$

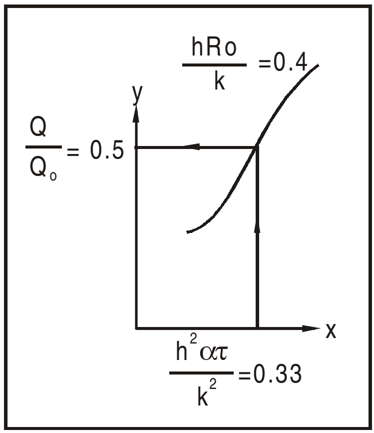
x–axis $\rightarrow$ 0.33

Curve $\rightarrow$ 0.4

Y axis $\rightarrow$ 0.5

from pg 71

So $\dfrac{Q}{Q_0} = 0.5$



where $Q_0 = \rho\, C\, \pi\, R_o^2 \left[\, T_i - T_\alpha \,\right]$

$$= 7833 \times 465 \times (0.06)^2\ [20 - 820] = -\ 10.5\ \text{MJ}$$

'-' Sign indicates heat is flowing from surrounding to cylinder.

Problem 1.57: *Aluminium slab of 100 mm thick initially at* 400°C *is exposed to a convection environment at* 90°C *with* $h = 1400$ W/m^2°C. *Determine (a) time required for the centre line temperature to reach* 180°C *(b) Temperature at a depth of 20 mm from the surface at same time.*

Solution

Initial temp $T_i = 400°C$

Surface temp. $T_\alpha = 90°C$

Downloaded from Ktunotes.in

$h = 1400\ \text{W/m}^2{}^\circ\text{C}$; $t = 0.1\text{m}$

From Pg 1 Properties of Aluminium

$\rho = 2707\,\text{kg/m}^3$

$k = 204.2\ \text{W/m}^\circ\text{C}$

$C = 896\ \text{J/KgK}$

$L_c = \text{half thickness} = \dfrac{0.1}{2} = 0.05\ \text{m}$

$$B_i = \frac{hL_c}{k} = \frac{1400 \times 0.05}{204.2} = 0.34$$

Since $B_i > 0.1$ and $B_i < 100$, it is infinite solid.

From Pg 66

$$\text{Y axis, } \frac{T_0 - T_\alpha}{T_i - T_\alpha} = \frac{180 - 90}{400 - 90} = 0.2903$$

Curve $\dfrac{h\,L_c}{k} = 0.34$, for these parameters,

corresponding Y axis, $\dfrac{\alpha\,\tau}{L^2} = 4.3$

where

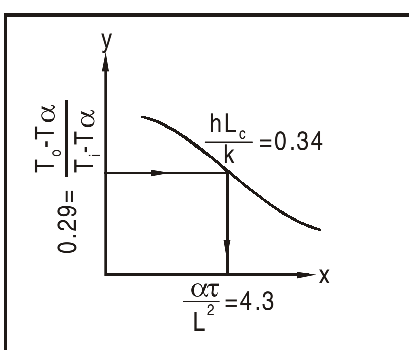
$$\alpha = \left(\frac{k}{\rho\,C}\right) = \frac{204.2}{2707 \times 896}$$

$= 8.4189 \times 10^{-5}\ \text{m}^2/\text{sec}$

$$\frac{\alpha\,\tau}{L^2} = \frac{8.4189 \times 10^{-5} \times \tau}{0.05^2}$$

$= 4.3$

$\boldsymbol{\tau = 127.687}$ **sec.**



Downloaded from Ktunotes.in

So the time required for the centre line temperature to reach 180°C is 127.687 sec.

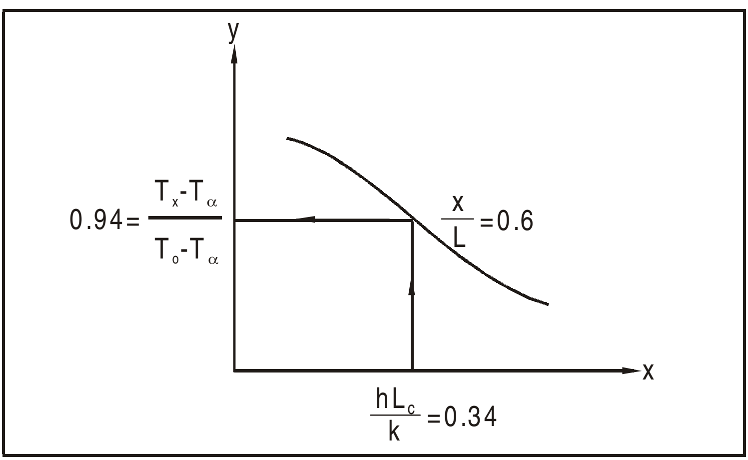
Temperature at a depth of 20 mm from surface at $\tau = 127.687$ sec

From Pg 67

$x = 20$ mm from surface means 30 mm from centre line. So $x = 0.03$ m

Curve $\dfrac{x}{L} = \dfrac{0.03}{0.05} = 0.6$

X-axis, $\dfrac{hL}{k} = 0.34$. For these two parameters, corresponding Y-axis $\dfrac{T_x - T_\alpha}{T_0 - T_\alpha} = 0.94$



$$\frac{T_x - T_\alpha}{T_0 - T_\alpha} = 0.94 \Rightarrow \frac{T_x - 90}{180 - 90} = 0.94$$

$$T_x = 90 \times 0.94 + 90 = 174.6\,°\text{C}$$

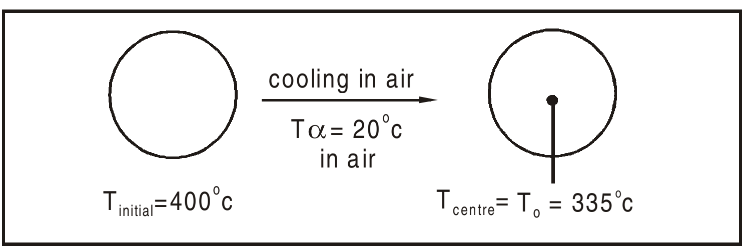
Downloaded from Ktunotes.in

Problem 1.58: *A metallic sphere of 10 mm radius is at* 400°C *initially. It is to be heat treated first by cooling it in air at* 20°C ($h = 10$ W/m^2°C) *until the centre temperature becomes* 335°C. *It is then quenched in water at* 20°C ($h = 6000$ W/m^2°C) *until the centre temperature falls from* 335°C *to* 50°C. *Determine 1. time required for cooling in air 2. time required for cooling in water 3. Surface temp after cooling in air 4. Surface temp after cooling in water*
Take the properties of sphere as follows.
$\rho = 3000$ Kg/m^3; $C = 1000$ J/KgK, $k = 20$ W/m°C

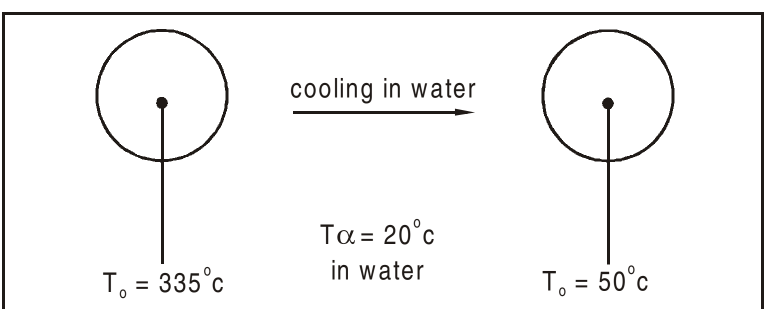
Solution

I Process: Air Cooling

$h_{air} = 10$ W/m^2°C



II process: Water Cooling



Downloaded from Ktunotes.in

$T_\alpha = 20°C$

1. I process : Air Cooling

$$L_c = \frac{R_o}{3} = \frac{10 \times 10^{-3}}{3} = 3.33 \times 10^{-3} \text{m}$$

$$B_i = \frac{h\,L_c}{k} = \frac{10 \times 3.33 \times 10^{-3}}{20} = 0.001655$$

Since $B_i < 0.1$, it is lumped

Initial temperature $T_0 = 400°C$

Air temperature $T_\alpha = 20°C$

$T = 335°C$

From Pg 58

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = e^{\left[-\frac{h\,\tau}{\rho\,C\,L_c}\right]} \qquad \left[\because \frac{A}{V} = \frac{1}{L_c}\right]$$

$$\frac{335 - 20}{400 - 20} = e^{\left[-\frac{10 \times \tau}{3000 \times 1000 \times 3.33 \times 10^{-3}}\right]}$$

$$\ln(0.8289) = -1 \times 10^{-3}\,\tau$$

$\boldsymbol{\tau = 187.468}$ **sec**

2. II Process - Water Cooling

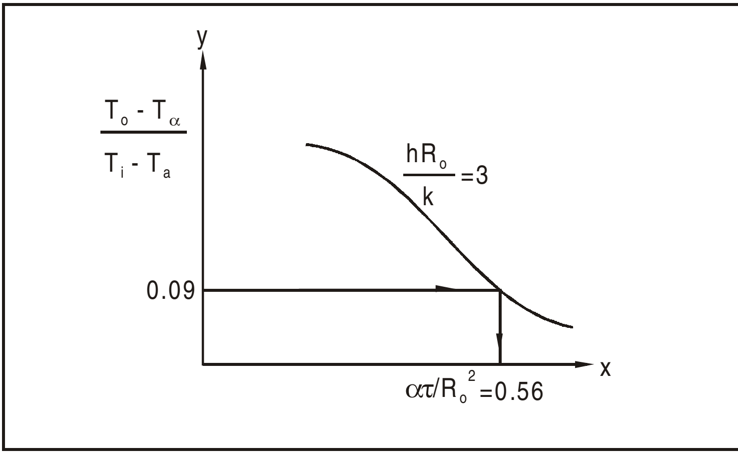
$$B_i = \frac{h\,L_c}{k} = \frac{6000 \times 3.33 \times 10^{-3}}{20}$$

$= 0.999 > 0.1$

Since $B_i > 0.1$, it is infinite solid.

Downloaded from Ktunotes.in

Use Heisler's Chart



From Pg 72

T_i = initial temperature = 335°C

T_0 = Centre temperature = 50°C

T_α = 20°C

$$\frac{h R_0}{k} = \frac{6000 \times 10 \times 10^{-3}}{20} = 3$$

$$\frac{T_0 - T_\alpha}{T_i - T_\alpha} = \frac{50 - 20}{335 - 20} = 0.09524\,.$$

For Y axis = 0.09524 and Curve = 3

The X axis → 0.56

So $\frac{\alpha\,\tau}{R_0^2} = 0.56$

Downloaded from Ktunotes.in

$$\tau = \frac{0.56 \times R_0^2}{\alpha}$$

$$= \frac{0.56 \times \left(10 \times 10^{-3}\right)^2}{\alpha}$$

$$\alpha = \frac{k}{\rho C} = \frac{20}{3000 \times 1000} = 6.67 \times 10^{-6}$$

$$\tau = \frac{0.56 \times (0.01)^2}{6.67 \times 10^{-6}} = \mathbf{8.4sec}$$

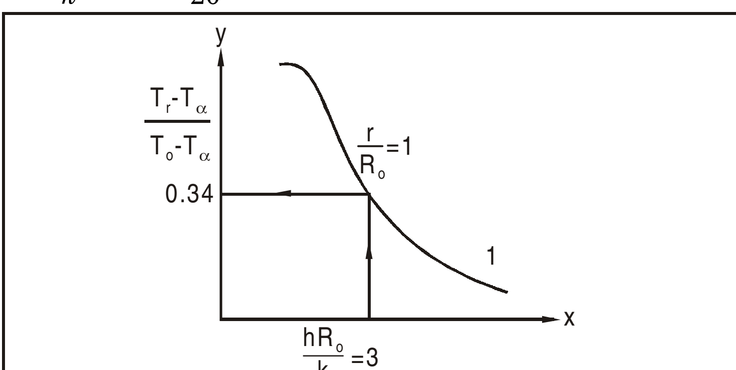
3. Surface temperature is also 335°C, since in Lumped system, the temperature at all surfaces are equal.

4. Surface temperature after cooling in water.

From Pg 73

$$\frac{r}{R_0} = \frac{0.01}{0.01} = 1 \qquad \left[\text{Since } r = R_0 \text{ at the surface}\right]$$

$$\frac{h\,R_0}{k} = \frac{6000 \times 0.01}{20} = 3$$



Downloaded from Ktunotes.in

For Curve$\rightarrow$ 1 and X–*axis*$\rightarrow$ 3

Y *axis*$\rightarrow$ 0.34

$$\frac{T_r - T_\alpha}{T_0 - T_\alpha} = 0.34$$

$$\frac{T_r - 20}{50 - 20} = 0.34$$

$T_r = (0.34 \times 30) + 20 = 30.2°C$

Surface temperature after cooling in water = 30.2°C

Downloaded from Ktunotes.in

1.42 SEMI-INFINITE SOLIDS (Refer HMT DB Page 59 to 63)

Lumped, T_o = initial temperature (Page 58)

Infinite, T_o = centre temperature (Page 64)

Semi-infinite, T_o = surface temperature (Page 59)

$$B_i = \frac{hL_c}{k} = \infty \text{ [(or) } h = \infty \text{]}$$

$$\frac{T_x - T_o}{T_i - T_o} = erf\,(z) \quad (\, erf - \text{error function (Page 59)})$$

$$z = \text{constant} = \frac{x}{2\sqrt{\alpha\tau}}$$

x = depth

α = Thermal diffusivity

τ = time

q_x = heat flux at location x at time τ

from Pg No. 59

$$q_x = \left[\frac{k\,(T_0 - T_i)}{\sqrt{\pi\,\alpha\,\tau}} \exp\,[-z^2] \right]$$

$$q_x = \frac{W}{m°C} \times \frac{°C}{\sqrt{\frac{m^2}{\sec} \times \sec}} = \frac{W/m}{m} = \frac{W}{m^2}$$

from Pg No. 59

$$q_o = \left[\frac{K\,(T_0 - T_i)}{\sqrt{\pi\,\alpha\,\tau}} \right]$$

Downloaded from Ktunotes.in

q_o = heat flux at surface

from Pg No. 59

q_t = Total heat flow into solid upto time τ per unit area

$$= 2k(T_o - T_i)\sqrt{\frac{\tau}{\pi\alpha}}$$

$$= \frac{W}{m - °C} \times \sec \sqrt{\frac{\sec}{m^2/\sec}}$$

$$= \frac{W}{m} \times \frac{\sec}{m} = W \sec/\text{m}^2 = \text{J/m}^2.$$

Problem 1.59: *A large plane wall* ($k = 0.8\ W/m°C$) ($\alpha = 0.003\ m^2/\text{hr}$) *initially at* 25°C. *Its surface temperature is* **changed to** 800°C *and maintained constant thereafter. Determine temperature at a depth of 200 mm after 10 hrs, instantaneous heat flux at that depth and total heat flow upto* 10 hrs/m^2 *area.*

Solution

Since 'h' is not given, $h = \infty$

$$\therefore B_i = \frac{hL_c}{k} = \infty$$

So, it is semi-infinite solids.

$$\alpha = 0.003\ \frac{\text{m}^2}{\text{hr}} = \frac{0.003}{3600}\ \frac{\text{m}^2}{\text{sec}} = 8.33 \times 10^{-7}\ \text{m}^2/\text{sec}$$

$$k = 0.8\ \frac{W}{m°C}$$

T_i = initial temp. = 25°C

Downloaded from Ktunotes.in

$T_o = 800°\text{C} = \text{surface temp.}$

$\tau = 36000 \text{ sec}$

$x = \text{depth} = 0.2 \text{ m}$

$$z = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.2}{2\sqrt{8.33 \times 10^{-7} \times 36000}}$$

$$= 0.5774.$$

from Pg No. 60 corresponding to z

$\text{erf}(z) = 0.58792$

from Pg No. 59

$$\frac{T_x - T_o}{T_i - T_o} = \text{erf}(z)$$

$$\frac{T_x - 800}{25 - 800} = 0.58792$$

$$T_x = 344.362°\text{C}$$

(b) q_x = instantaneous heat flux flow at 200 mm

from Pg No. 59

$$q_x = \frac{k(T_o - T_i)}{\sqrt{\pi\,\alpha\,\tau}}\, exp\,[-z^2]$$

$$= \frac{0.8(800 - 25)}{\sqrt{\pi \times 8.333 \times 10^{-7} \times 36000}} \exp[-0.5774^2]$$

$$= 2019.961 \times 0.71649$$

$$= 1447.282 \text{ W/m}^2$$

q_t = Total heat flow for 10 hrs/m^2 unit area

Downloaded from Ktunotes.in

from Pg No. 59

$$q_t = 2k(T_o - T_i)\sqrt{\frac{\tau}{\pi\,\alpha}}$$

$$= 2 \times 0.8(800 - 25)\sqrt{\frac{36000}{\pi \times 8.333 \times 10^{-7}}}$$

$$q_t = 145.411 \times 10^6 \text{ J/m}^2.$$

Problem 1.60: *A large concrete highway initially at* 55°C, *suddenly cooled by rain water such that the surface temperature is lowered to* 35°C *at night* ***time and constant thereafter****. Determine (a) time required to reach* 45°C *at a depth of 50 mm. (b) instantaneous heat transfer* rate/m^2 *at* 45°C. *(c) total heat* flow/m^2 *upto this time.*

Solution

Since 'h' is not given, so, $h = \infty$

$\therefore B_i = \infty$ So, semi-infinite

$k = 1.299$ W/mK

$\alpha = 1.77 \times 10^{-3}$ m^2/hr

T_i = initial temp. 55°C

T_o = 35°C surface temp.

$$z = \frac{x}{2\sqrt{\alpha\,\tau}} = \frac{50 \times 10^{-3}}{2\sqrt{\frac{1.77 \times 10^{-3}}{3600} \times t}}$$

when $T_x = 45°C$

$$\frac{T_x - T_o}{T_i - T_o} = \text{erf}\,(z)$$

Downloaded from Ktunotes.in

$$\frac{45-35}{55-35} = \text{erf}\,(z)$$

$$erf(z) = 0.5$$

z	$erf(z)$
0.48	0.50275

from page 60 of CPK

$$0.48 = \frac{50 \times 10^{-3}}{2\sqrt{\dfrac{1.77 \times 10^{-3}}{3600} \times \tau}}$$

$$0.2304 = \frac{2.5 \times 10^{-3}}{\dfrac{4 \times 1.77 \times 10^{-3}}{3600} \times \tau}$$

$$\tau = 5517.3022 \text{ sec.}$$

q_x = instantaneous heat flux at location

from Pg No. 59

$$q_x = \frac{k(T_o - T_i)}{\sqrt{\pi\alpha\,\tau}}\,exp(-z^2)$$

$$= \frac{1.299\,(35-55)}{\sqrt{\pi \times \dfrac{1.77 \times 10^{-3}}{3600} \times 5517.3}}\ \text{exp}\,(-0.48^2)$$

$$q_x = -223.515 \text{ W/m}^2$$

from Pg. No. 59

q_t = Total heat flow upto τ/unit area

$$= 2k(T_o - T_i)\sqrt{\frac{\tau}{\pi\alpha}}$$

Downloaded from Ktunotes.in

$$= 2 \times 1.299(-20)\sqrt{\frac{5517.30}{\pi \times \frac{1.77 \times 10^{-3}}{3600}}}$$

$$q_t = -3.077 \times 10^6 \text{ J/m}^2$$

(–) indicates heat is coming out.

Problem 1.61: *A large wall 2 cm thick has uniform temperature* 30°C. *If the walls are suddenly raised to and maintained at* 400°C. *Find (i) the temperature at a depth of 0.8 cm from the surface of wall after 10 sec (ii) instantaneous heat flow rate through that surface* $/m^2$ *hour. (Apr.97, Madras University)*

Solution:

$$\alpha = 0.008 \text{ m}^2/\text{hr}$$

$$k = 6 \text{ W/m°C}$$

$$x = 0.008 \text{ m}$$

Since 'h' is not given, $h = \infty$

$$B_i = \infty$$

$\therefore$ Semi-infinite solids.

$$T_i = \text{initial temp.} = 30°\text{C}$$

$$T_o = \text{surface temp.} = 400°\text{C}$$

$$z = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.008}{2\sqrt{\frac{0.008}{3600} \times 10}} = 0.848528$$

erf $(z) = 0.77067$ (Take from page 60)

Downloaded from Ktunotes.in

from Pg. No. 59

$$\frac{T_x - T_o}{T_i - T_o} = \text{erf}\,(z)$$

$$\frac{T_x - 400}{30 - 400} = 0.77067$$

from Pg No. 59

$$\boldsymbol{T_x = 114.852\,°C.}$$

$$q_x = \frac{k(T_o - T_1)}{\sqrt{\pi\alpha\tau}\; exp\;[-z^2]}$$

$$= \frac{6(400 - 30)}{\sqrt{\pi \times \frac{0.008}{3600} \times 10}} exp\;[-0.8485^2]$$

$$\boldsymbol{q_x = 129334.08\ \text{W/m}^2.}$$

1.43 DERIVATION FOR MAXIMUM HEAT TRANSFER CONDITION

$$q_x = \frac{k(T_o - T_i)}{\sqrt{\pi\alpha\tau}}\, exp\left[-\frac{x^2}{4\alpha\tau}\right] \text{[Here } \tau = \text{time} = \text{t]}$$

q_x and τ are variables   To get $q_{max}\ \frac{dq}{d\tau} = 0$ or $\frac{dq}{d_t} = 0$

$$q_x = \frac{k(T_o - T_i)}{\sqrt{\pi\alpha}}\, \tau^{-1/2}\, exp\left[-\frac{x^2}{4\alpha\tau}\right]$$

Downloaded from Ktunotes.in

$$\frac{d_{q_x}}{dt} = \frac{k(T_o - T_i)}{\sqrt{\pi\alpha}}\left[-\frac{1}{2}t^{-3/2}exp\left[-\frac{x^2}{4\alpha t}\right]\right.$$

$$\left.+\, t^{-1/2}exp\left[-\frac{x^2}{4\alpha t}\right]\left(-\frac{x^2}{4\alpha}\times\frac{-1}{t^2}\right)\right] = 0$$

$$= \frac{k(T_o - T_i)}{\sqrt{\pi\alpha}}\left[-\frac{1}{2}t^{-3/2}exp\left(-\frac{x^2}{4\alpha t}\right) + t^{-1/2}exp\left(-\frac{x^2}{4\alpha t}\right)\frac{x^2}{4\alpha t^2}\right] = 0$$

$$= -\frac{1}{2}t^{-3/2}exp\left(-\frac{x^2}{4\alpha t}\right) + t^{-1/2}exp\left(-\frac{x^2}{4\alpha t}\right)\frac{x^2}{4\alpha t^2} = 0$$

$$\frac{1}{2}t^{-3/2}exp\left(\frac{-x^2}{4\alpha t}\right) = t^{-1/2}exp\left(\frac{-x^2}{4\alpha t}\right)\frac{x^2}{4\alpha t^2}$$

$$t^{(-3/2 + 1/2 + 2)} = \frac{2x^2}{4\alpha}$$

$$\therefore\ t = \frac{x^2}{2\alpha}$$

$$\text{ie } \tau = \frac{x^2}{2\alpha}$$

This is the condition for maximum heat transfer.

Problem 1.62: *A large metal mass* ($\alpha = 0.405$ m^2/hr) *initially at* 100°C *is suddenly brought to* 0°C, *at its surface and maintained constant. Determine (i) the depth at which cooling rate is maximum after 1 minute (b) time required for temperature gradient at surface to reach* 4°C/cm.

Solution:

'h' is not given, $h = \infty$

$\therefore\ B_i = \infty$

Downloaded from Ktunotes.in

So, it is semi-infinite solid.

$T_i = 100°C; T_0 = 0°C$

$\tau = 1$ minute $= 60$ second

condition for q_{max} $t = \dfrac{x^2}{2\alpha}$

$$x^2 = 2\alpha t$$

$$x^2 = 2\left(\frac{0.405}{3600}\right) \times 60$$

$$x = 0.1161895 \text{ m}$$

$$= 116.189 \text{ mm}$$

At this depth, cooling is maximum.

(b) Temperature gradient $= \left(\dfrac{\Delta T}{\Delta x}\right)_{surface} = \dfrac{4°C}{cm} = \dfrac{4°C}{10^{-2} \text{ m}}$

$$\left(\frac{\Delta T}{\Delta x}\right)_o = 400°C/m$$

$$Q = kA\frac{\Delta T}{\Delta x}$$

$$\left(\frac{\Delta T}{\Delta x}\right)_o = \frac{q_o}{k} = 400°C/m$$

$$400 = \frac{(T_o - T_i)}{\sqrt{\pi\alpha\tau}}$$

$$400 = \frac{-100}{\sqrt{\pi \times \frac{0.405}{3600} \times \tau}}$$

Downloaded from Ktunotes.in

$$-0.25 = \sqrt{3.53 \times 10^{-4} \times \tau}$$

$$\tau = 176.8388 \text{ sec}$$

Time required for surface gradient to reach 4°C/cm is 176.83 sec.

Problem 1.63: *A semi infinite plate of cu initially at* 30°C *is exposed to a constant heat flux of* 300 KW/m^2 *at its surface. Determine surface temperature after 10 minutes and temperature at a depth of 200 mm after this time.*

Solution:

T_i = initial temp. = 30°C

$q_o = 300 \times 10^3$ W/m^2

$\tau = 600$ sec

T_o = surface temperature = ?

from Pg.No. 59

$$q_o = \frac{k(T_o - T_1)}{\sqrt{\pi\alpha\tau}}$$

From Page 2 For *Cu*

$\alpha = 0.404$ m^2/hr $= 1.122 \times 10^{-4}$ m^2/sec

$k = 386$ W/mk

$$300 \times 10^3 = \frac{386(T_o - 30)}{\sqrt{\pi \times 1.122 \times 10^{-4} \times 600}}$$

$$\boldsymbol{T_o = 387.45611\ °C.}$$

Downloaded from Ktunotes.in

(b) $x = 0.2$ m

from Pg.No. 59

$$q_x = \frac{k(T_o - T_i)}{\sqrt{\pi\,\alpha\,\tau}}\, exp\left[-\frac{x^2}{4\alpha t}\right]$$

$$z = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.2}{2\sqrt{1.122 \times 10^{-4} \times 600}} = 0.3854$$

$$T_x - T_i = \frac{2q_0}{k}\left(\frac{\alpha\tau}{\pi}\right)^{1/2} exp\,(-z^2) - \frac{q_o x}{k}\,[\,1 - erf(z)\,]$$

$$= \frac{2 \times 300 \times 10^3}{386}\left(\frac{1.122 \times 10^{-4} \times 600}{\pi}\right)^{0.5} \exp\,(-0.3854^2)$$

$$-\frac{300 \times 10^3 \times 0.2}{386}\,[\,1 - 0.41874\,]$$

$$= 196.1344 - 90.351$$

$$= 105.783$$

$$T_x = 105.783 + 30 = 135.783\,°\text{C}.$$

Problem 1.64: *A large slab of aluminium initially at* 250°C *suddenly exposed to a convection environment at* 50°C *with* $h = 500$ W/m^2–°C. *Determine temperature at a depth of 50 mm after 1 hour,* $k = 215$ W/m°C $\alpha = 8.4 \times 10^{-5}$ m^2/sec.

Solution

L = half thickness not given, $L = \infty$

$\therefore B_i = \dfrac{hL}{k} = \infty$. So it is semi-infinite solid.

Downloaded from Ktunotes.in

$T_i = 250°C$ $\qquad$ $x = 50$ mm

T_∞ = surface/fluid temperature = 50°C ; $\quad t = 3600$ sec

$h = 500$ W/m^2–°C

$k = 215$ W/m–°C

$\alpha = 8.4 \times 10^{-5}$ m^2/sec

$$z = \frac{x}{2\sqrt{\alpha\tau}}$$

$$= \frac{0.05}{2\sqrt{8.4 \times 10^{-5} \times 3600}}$$

$$= 0.04546206$$

$z = 0.0454 \quad ; \qquad erf(z) = 0.05637$ (page 60)

Refer Page 59, third row for formula:

$$\frac{T_x - T_i}{T_\infty - T_i} = [\,1 - \text{erf}\,(z)\,] - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha\tau}{k^2}\right)\right] \left[1 - \text{erf}\left(z + \frac{h\sqrt{\alpha\tau}}{k}\right)\right]$$

$$= [\,1 - 0.05637\,] - \left[\exp\left(\frac{500 \times 0.05}{215} + \frac{500^2 \times 8.4 \times 10^{-5} \times 3600}{215^2}\right)\right] \left[1 - \text{erf}\left(0.04546 + \frac{500\sqrt{8.4 \times 10^5 \times 3600}}{215}\right)\right]$$

$$= 0.94363 - [\,(5.7647)\ [\,1 - \text{erf}\,(1.3243)\,]\,]$$

$$= 0.94363 - (5.7647)\,(1 - 0.93806)$$

$$\frac{T_x - T_i}{T_\infty - T_i} = 0.58656$$

$$\frac{T_x - 250}{50 - 250} = 0.58656$$

$$\boldsymbol{T_x = 132.688\ °C}$$

Downloaded from Ktunotes.in

Problem 1.65: *A thick concrete wall* ($\alpha = 7 \times 10^{-7}$ m^2/sec, $k = 1.37$ W/m–°C) *is initially at* 340°C *suddenly exposed to a convection environment at* 40°C *with* $h = 100$ W/m^2–°C. *Determine temperature at a depth of 100 mm after one hour.*

Note IV: erf (3.6) = 1

$Z > 3.6$, erf $(Z) = 1$

Case III

$T_i = 340°C$	$k = 1.37$ W/m–°C
$T_\infty = 40°C$	$\alpha = 7 \times 10^{-7}$ m^2/sec
$h = 100$ W/m^2–°C	$x = 0.1$ m
	$\tau = 3600$ sec

$$z = \frac{x}{2\sqrt{\alpha\tau}} = \frac{0.1}{2\sqrt{7 \times 10^{-7} \times 3600}}$$

$$= 0.99602$$

$$\text{erf}(z) = 0.8427.$$

Refer page 59, 3rd row for formula

$$\text{Constant } 1 = 1 - \text{erf}(z) = 0.1573$$

$$\text{Constant } 3 = 1 - \text{erf}\left(z + \frac{h\sqrt{\alpha\tau}}{k}\right)$$

$$= 1 - \text{erf}\left(0.99602 + \frac{100\sqrt{7 \times 10^{-7} \times 3600}}{1.37}\right)$$

$$= 1 - \text{erf}(4.66) = 1 - 1$$

$$= 0.$$

Downloaded from Ktunotes.in

$$\text{Constant } 2 = \exp\left[\frac{hx}{k} + \frac{h^2\alpha\tau}{k^2}\right]$$

$$= \exp\left[\frac{100 \times 0.1}{1.37} + \frac{100^2 \times 7 \times 10^{-7} \times 3600}{1.37^2}\right]$$

$$= 1.$$

$$\frac{T_x - T_i}{T_\infty - T_i} = \text{Constant } 1 - \text{constant } 2 \times \text{constant3}$$

$$= 0.1573 - (1 \times 0) = 0.1573$$

$$\frac{T_x - 340}{40 - 340} = 0.1573$$

$$\boldsymbol{T_x = 292.81\,°C.}$$

1.44 TWO DIMENSIONAL CONDUCTION CONDUCTION SHAPE FACTOR

M = no. of temperature intervals

N = no. of heat transfer tubes

ΔQ = heat transfer through one tube

ΔT = temperature difference between intervals

t = thickness = 1

$$T_1 - T_2 = M(\Delta T) \qquad \ldots \text{ (a)}$$

$$Q = \Delta Q(N.) \text{ (b)}$$

$$\Delta Q = k(A)\,\frac{\Delta T}{\text{SEPARATION}}$$

Downloaded from Ktunotes.in

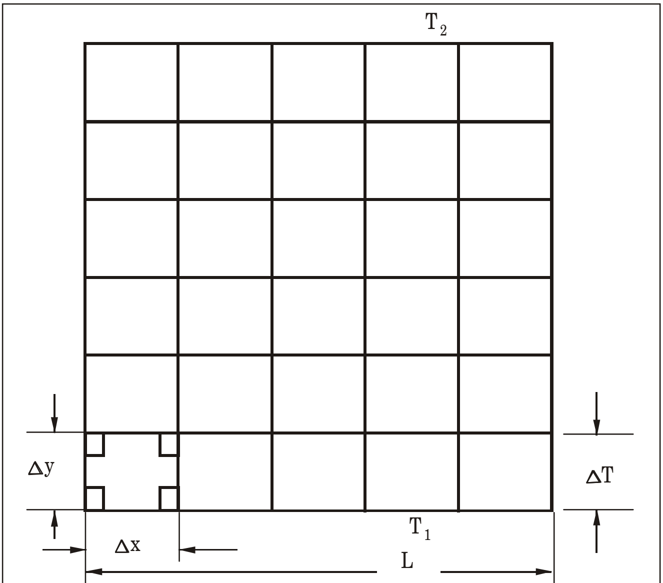


Fig. 1.32

where ΔQ = Heat transfer through one tube

$$= \frac{k(t\Delta x)\,\Delta T}{\Delta y}$$

Here = Δx is approximately equal to Δy (curvilinear)

So $\Delta Q = kt\Delta T$

Substitute ΔT from equation (a)

$$\Delta Q = \frac{kt(T_1 - T_2)}{M}$$

$$\Delta Q \cdot N = \frac{Nkt(T_1 - T_2)}{M} = Q \qquad \text{[Refer equation (b)]}$$

Downloaded from Ktunotes.in

so $Q = kt(T_1 - T_2)\,\dfrac{N}{M}$

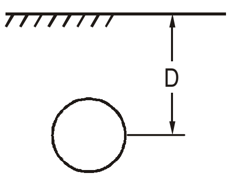
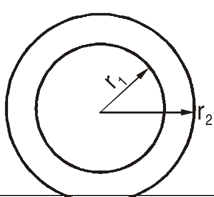
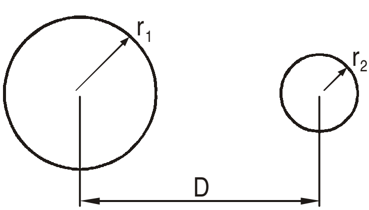
$$\left[\text{where } t \times \frac{N}{M} = \text{shape factor} = \text{S in meters}\right]$$

$Q = kS(T_1 - T_2)$

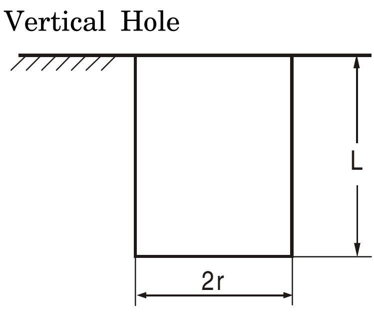
S = shape factor

Conduction shape factor is defined as (thickness × no. of tubes)/No. of temperature interceals.

Refer Page 53 of CPK.

Type	S
Buried Cylinder $\dfrac{D}{3r} > 4$  	$S = \dfrac{2\pi L}{ln\left(\dfrac{L}{r}\right)\left\{\dfrac{ln\left(\dfrac{L}{2D}\right)}{1 - ln\left(\dfrac{L}{r}\right)}\right\}}$
Concentric Cylinder  	$S = \dfrac{2\pi L}{ln\left(\dfrac{r_2}{r_1}\right)}$
Parallel Cylinder  	$S = \dfrac{2\pi L}{\cosh^{-1}\left[\dfrac{D^2 - r_1^2 - r_2^2}{2r_1r_2}\right]}$

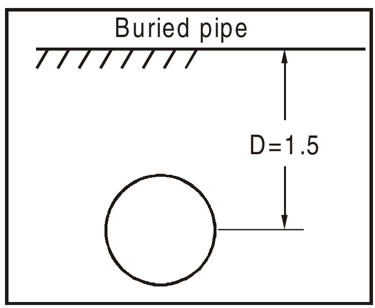
Downloaded from Ktunotes.in



$$S = \frac{2\pi L}{ln\left(\frac{2L}{\pi}\right)}$$

Problem 1.66: *A pipe of 500 mm OD is buried in earth at a depth of 1.5 m. The surface temperature of cylinder is* 85°C. *Soil temperature is* 20°C. $k_{soil} = 0.52$ W/m–°C. *Determine heat loss.*

Solution:



$D = 1.5$ m; $r = 250$ mm

$3r = 0.75$ m; So, $D > 3r$

(Refer Page 45) $L = 1$ m

$$S = \frac{2\pi L}{ln\left(\frac{2D}{r}\right)}$$

$$= \frac{2\pi \times 1}{ln\left(\frac{2 \times 1.5}{0.25}\right)} = 2.52854 \text{ m}$$

$k = 0.52$ W/m–°C

$Q = kS\,(T_1 - T_2)$

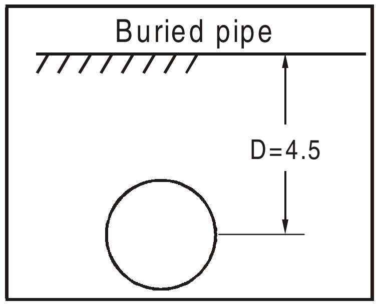
$= 0.52 \times 2.5285\,(85 - 20)$

Q = 85.4633 W.

Downloaded from Ktunotes.in

Problem 1.67: *A pipe of 500 mm OD is buried in earth at a depth of 4.5 m. The surface temperature of cylinder is* 85°C. *Soil temperature is* 20°C. $k_{soil} = 0.52$ W/m–°C. *Determine heat loss.*

Solution:



$$D = 4.5 \text{ m}$$

$$r = 0.25 \text{ m}$$

$$3r = 0.75 \text{ m}$$

$$\frac{D}{3r} = \frac{4.5}{0.75} = 6 > 4$$

$$S = \frac{2\pi L}{ln\left(\frac{L}{r}\right)\left\{\frac{ln\left(\frac{L}{2D}\right)}{1 - ln\left(\frac{L}{r}\right)}\right\}}$$

$$= \frac{2\pi \times 1}{ln\left(\frac{1}{0.25}\right)\left\{\frac{ln\left(\frac{1}{2 \times 4.5}\right)}{1 - ln\left(\frac{1}{0.25}\right)}\right\}}$$

$$= \frac{6.28}{1.3863 \times \frac{-2.19722}{-0.38629}} = 0.7964$$

$$S = 0.7964 \text{ m.}$$

$$Q = kS\,(T_1 - T_2)$$

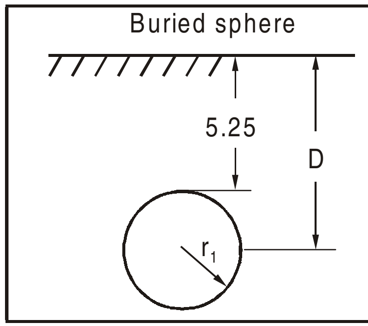
$$= 0.52 \times 0.7964 (85 - 20)$$

$$Q = 26.91832 \text{ W.}$$

Downloaded from Ktunotes.in

Problem 1.68: *A sphere of 1.5 m diameter is buried in soil such that its upper surface is 5.25 m below the earth surface. The sphere generates 600 W of heat.* $k_{soil} = 0.52$ W/m–°C. *Determine surface temperature of sphere, if soil temperature is* 5°C.

Solution:



$$S = \frac{4\pi r}{1 - \left(\frac{r}{2D}\right)}$$

$$= \frac{4\pi \times 0.75}{1 - \left(\frac{0.75}{2 \times 6}\right)} = 10.053 \text{ m}$$

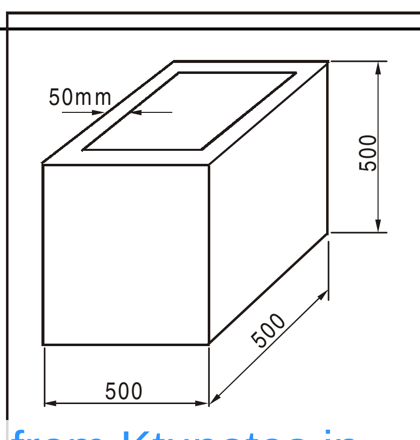
$$Q = kS(T_1 - T_2)$$

$$600 = 0.52 \times 10.053(T_1 - 5)$$

$$T_1 = 119.775 \text{ °C}$$

Problem 1.69: *A hollow cube, 500 mm is wide made up of 50 mm thick asbestos* ($k = 0.192$ W/m–°C). *The inner surface is at* 150°C *and outer surface is at* 50°C. *Determine the heat loss from the cube.*

Solution:



6 surfaces

12 edges

8 corners

Refer Page 45 of CPK.

$$S_{surface} = \frac{\text{Area}}{\text{width}}$$

Downloaded from Ktunotes.in

$$= \frac{0.5 \times 0.5}{0.05} = 5 \text{ m}$$

$$S_{\text{edge}} = 0.54 \times D = 0.54 \times 0.5$$

$$= 0.27 \text{ m}$$

$$S_{\text{corners}} = 0.15 \times \text{width}$$

$$= 0.15 \times 0.05 = 7.5 \times 10^{-3} \text{ m}$$

$$S = 6 \times 5 + 12 \times 0.27 + 8 \times 7.5 \times 10^{-3}$$

$$= 33.3 \text{ m}$$

$$Q = kS(T_1 - T_2)$$

$$= 0.192 \times 33.3 (150 - 50)$$

$$= 639.36 \text{ W.}$$

Problem 1.70: *A furnace of* $4 \text{ m} \times 3 \text{ m} \times 2 \text{ m}$ *is having a thickness of 200 mm. It is made up of material with* $k = 1.6$ W/m–°C. *Determine (a) heat loss from it if its inner surface is at* 220°C *and its outer surface temperature is* 100°C.

Surfaces

$2\ (4 \times 3)$ = Top and Bottom area

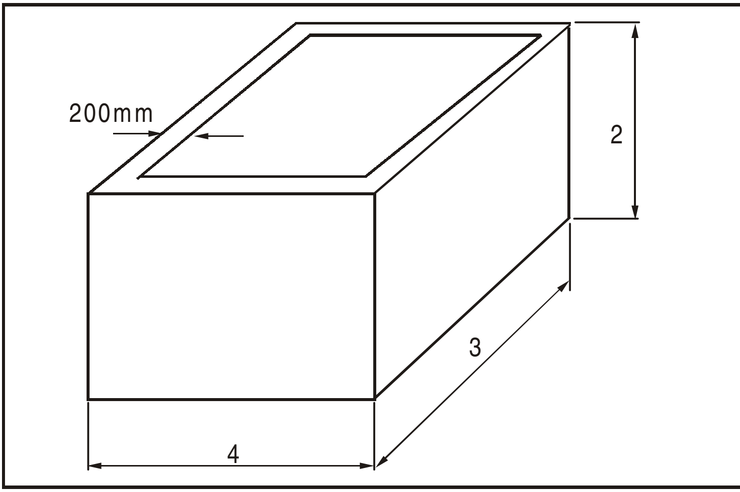
$2\ (3 \times 2)$ = Left and Right area

$2\ (2 \times 4)$ = Front and Rear area

$$S_{\text{surface}} = \frac{\text{Area}}{\text{width (or) thickness}}$$

$$= \frac{2}{0.2} [4 \times 3 + 3 \times 2 + 2 \times 4] = 260 \text{ m}$$

Downloaded from Ktunotes.in



Edges

$$\text{Front and Rear} = 4 \times 4$$

$$\text{Left and Right} = 4 \times 3$$

$$\text{Vertical} = 4 \times 2$$

$$S_{\text{edges}} = 0.54 \times \text{depth}$$

$$= 0.54 \ [4 \times 4 + 4 \times 3 + 4 \times 2]$$

$$= 19.44 \text{ m}$$

$$S_{\text{corner}} = 0.15(\text{thickness}) \times 8$$

$$= 0.15 \times 8 \times 0.2 = 0.24 \text{ m}$$

$$Q = kS(T_1 - T_2) = 1.6 \times (260 + 19.44 + 0.24)(200 - 100)$$

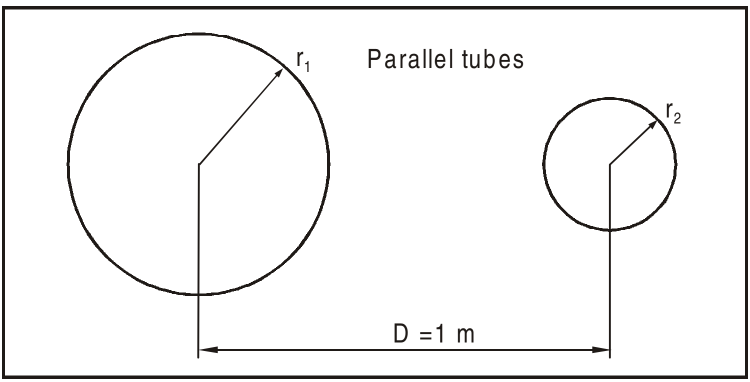
$$= \mathbf{53698 \ W.}$$

Problem 1.71: *One pipe of 50 cm diameter carrying steam at* 160°C *and another pipe 25 cm in diameter carrying water at* 15°C *are buried at a depth with the centres at 1 m.*

Downloaded from Ktunotes.in

Assuming the ground as infinite medium and pipes run parallel to each other. Find the net heat transfer between the pipe/hour. Length of each pipe = 50 m. k_{soil} = 0.4 W/m K. *Neglect the resistance of pipe materials. If the velocity of water is limited to* 1 m/min. *Find out the rise in temperature of water due to the above heat transfer. (April 97, Madras University)*

Solution:



$$L = 50 \text{ m}; \; r_1 = 25 \text{ cm}; \; r_2 = 12.5 \text{ cm}$$

$$S = \frac{2\pi L}{\cosh^{-1}\left[\dfrac{D^2 - r_1^2 - r_2^2}{2r_1 r_2}\right]}$$

$$S = \frac{2\pi \times 50}{\cosh^{-1}\left[\dfrac{1^2 - 0.25^2 - 0.125^2}{2 \times 0.25 \times 0.125}\right]}$$

$$= 92.857 \text{ m}$$

$$Q = kS\,(T_1 - T_2)$$

$$= 0.4 \times 92.857\,(160 - 15)$$

Downloaded from Ktunotes.in

$$= 5385.706 \text{ W}$$

$$= 5385.706 \times 3600$$

$$Q = 19388653.92 \text{ J/hr}$$

$$\text{Velocity} = 1 \text{ m/min} = \frac{1}{60} \text{ m/sec}$$

$$\text{mass rate of flow of water } \dot{m} = \rho A V$$

$$= \rho \pi r_2^2 \text{ V}$$

$$= 1000 \times \pi \times 0.125^2 \times \frac{1}{60} = 0.818123 \text{ kg/sec}$$

$$(C_p)_{water} = 4186 \text{ J/kg K}$$

$$Q = \dot{m} \, (C_p)_w (\Delta T)_w$$

$$5385.7 = 0.818123 \times 4186 \times (\Delta T)$$

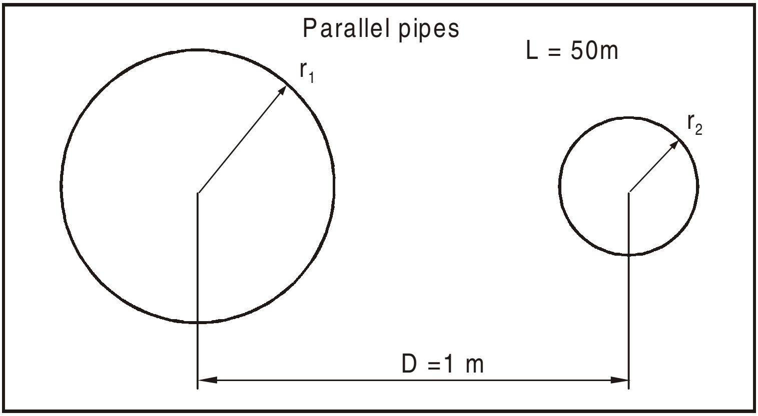
$$\boldsymbol{\Delta T = 1.5726°C.}$$

Problem 1.72: *Two tubes of 600 mm diameter 200 mm diameter are placed parallel to each other with a distance of 1 m between their centres. Their respective temperature are* 150°C, and 10°C. *Length of each cylinder is 50 m.* $k_{soil} = 0.35$ W/m–°C. *Water at a velocity of 1 m/min is flowing through the 200 mm diameter pipe. Determine heat transfer rate and temperature rise of water.*

Solution

$r_1 = 300$ mm; $r_2 = 100$ mm

Downloaded from Ktunotes.in



$$S = \frac{2\pi L}{\cosh^{-1}\left[\dfrac{D^2 - r_1^2 - r_2^2}{2r_1 r_2}\right]}$$

$$= \frac{2\pi \times 50}{\cosh^{-1}\left[\dfrac{1^2 - 0.3^2 - 0.1^2}{2 \times 0.3 \times 0.1}\right]} = 92.397 \text{ m}$$

$$Q = kS\,(T_1 - T_2)$$

$$= 0.35 \times 92.397\,(140)$$

$$= 4527.477 \text{ W}$$

$$A_2 = \pi r_2^2 = \pi \times 0.1^2 = 0.031416 \text{ m}^2$$

$$1 \text{ m/min} = \frac{1}{60} \text{ m/sec}$$

$$\dot{m} = \rho A_2 V_2$$

$$= 1000 \times 0.031416 \times \frac{1}{60}$$

$$= 0.5235 \text{ kg/sec}$$

Downloaded from Ktunotes.in

$$Q = m(C_p)_w\ (\Delta T)_w$$

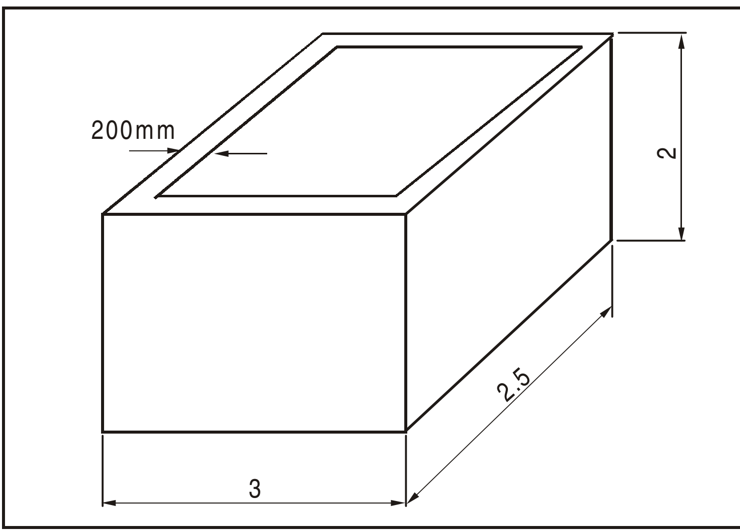
$$4527.277 = 0.5235 \times 4186 \times (\Delta T)_w$$

$$(\Delta T)_w = 2.0655\,°\text{C}.$$

$$\boldsymbol{Q = 4527.477\ \text{W}}$$

$$\boldsymbol{(\Delta T)_w = 2.0656\,°\text{C}.}$$

Problem 1.73: *A furnace of* $3\text{ m} \times 2.5\text{ m} \times 2\text{ m}$ *having a thickness of 200 mm with* $k = 1.3$ W/m–°C. *Determine heat loss, if the surface temperature are* 250°C, and50°C.



Solution

$$S_{\text{surface}} = \frac{\text{Area}}{\text{Width (or) thickness}}$$

$$= \frac{2}{0.2}\,[\,3 \times 2.5 + 2.5 \times 2 + 2 \times 3\,]$$

$$= 185\ \text{m}$$

Downloaded from Ktunotes.in

$$S_{\text{edges}} = 0.54 \times \text{depth}$$

$$= 0.5 + (3 \times 3 + 2.5 \times 3 + 2 \times 3)$$

$$= 12.15 \text{ m}$$

$$S_{\text{corner}} = 0.15(\text{thickness}) \times 8$$

$$= 0.15 \times 8 \times 0.2 = 0.24 \text{ m}$$

$$Q = kS(T_1 - T_2)$$

$$= 1.3 \times (185 + 12.15 + 0.24)\,(200) = \mathbf{51321.4 \ W.}$$

Downloaded from Ktunotes.in