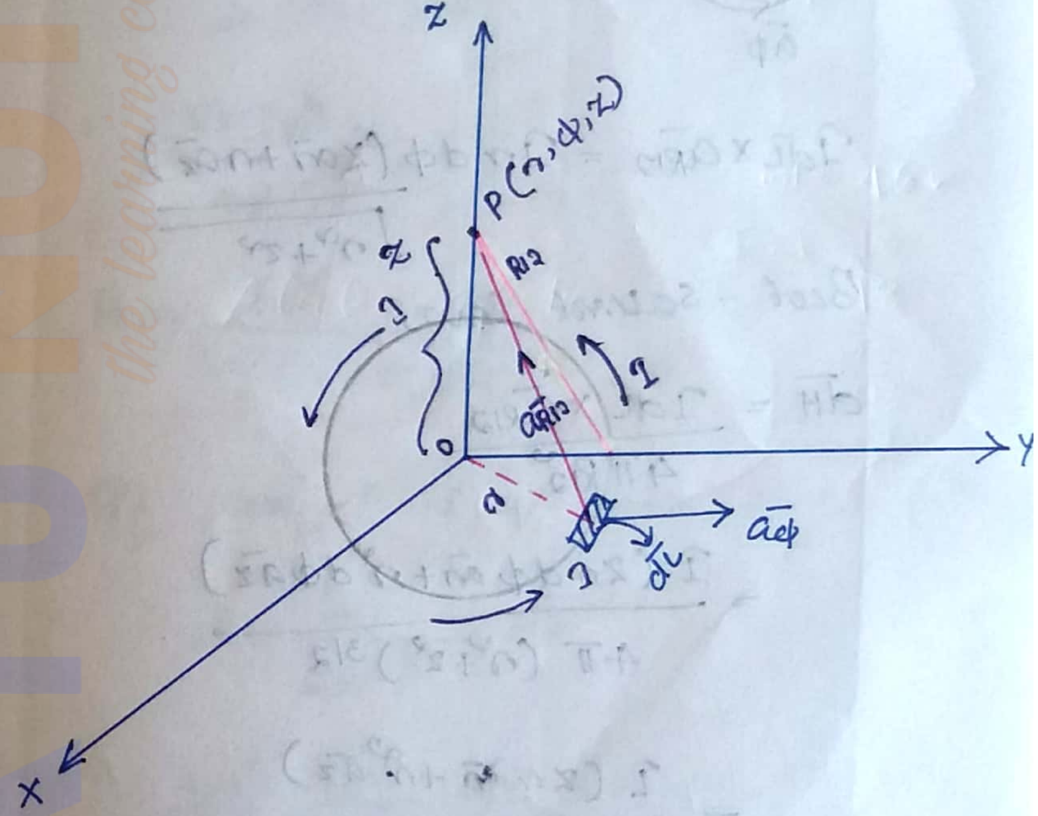
[continuous...]

28/02/18
Wednesday

Module - III

$\bar{H}$ on the axis of a circular loop

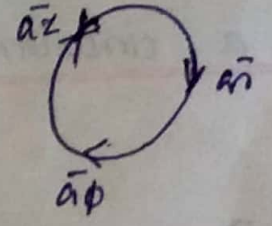


$$\bar{dl} = dr\,\bar{a_r} + a\,d\phi\,\bar{a_\phi} + dz\,\bar{a_z}$$

$$I.\bar{dl} = I.\,s\,d\phi\,\bar{a_\phi}$$

$$\bar{a_{R12}} = \frac{\bar{R_{12}}}{|R_{12}|} = \frac{z\bar{a_z} - r\bar{a_r}}{\sqrt{r^2+z^2}}$$

$$I\,d\bar{l} \times \bar{a}_{R12} = I r \cdot d\phi\, a_\phi \times \left(\frac{z\bar{a}_z - r\bar{a}_r}{\sqrt{r^2+z^2}} \right)$$



$$I\,d\bar{l} \times \bar{a}_{R12} = \frac{I r\, d\phi\, (z\bar{a}_r + r\bar{a}_z)}{\sqrt{r^2+z^2}}$$

Biot - Savart law;

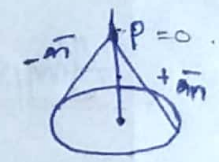
$$d\bar{H} = \frac{I\,dl \times \bar{a}_{R12}}{4\pi R_{12}^2}$$

$$= \frac{I\,(zr\,d\phi\,\bar{a}_r + r^2 d\phi\, \bar{a}_z)}{4\pi\,(r^2+z^2)^{3/2}}$$

$$= \frac{I\,(zr\,\bar{a}_r + r^2\bar{a}_z)}{4\pi\,(r^2+z^2)^{3/2}}$$

$$\bar{H} = \int_{\phi=0}^{2\pi} \frac{I\,(zr\,\bar{a}_r + r^2\bar{a}_z)}{4\pi\,(r^2+z^2)^{3/2}}\, d\phi$$

By Radial symmetry;



$$\bar{a}_r = 0$$

$$\bar{H} = \frac{I r^2 \bar{a}_z}{4\pi\,(r^2+z^2)^{3/2}}\, (\phi)_0^{2\pi}$$

$$= \frac{2\pi I r^2 \bar{a}_z}{4\pi\,(r^2+z^2)^{3/2}}$$

$$\boxed{\bar{H} = \frac{I r^2}{2\,(r^2+z^2)^{3/2}}\, \bar{a}_z}$$

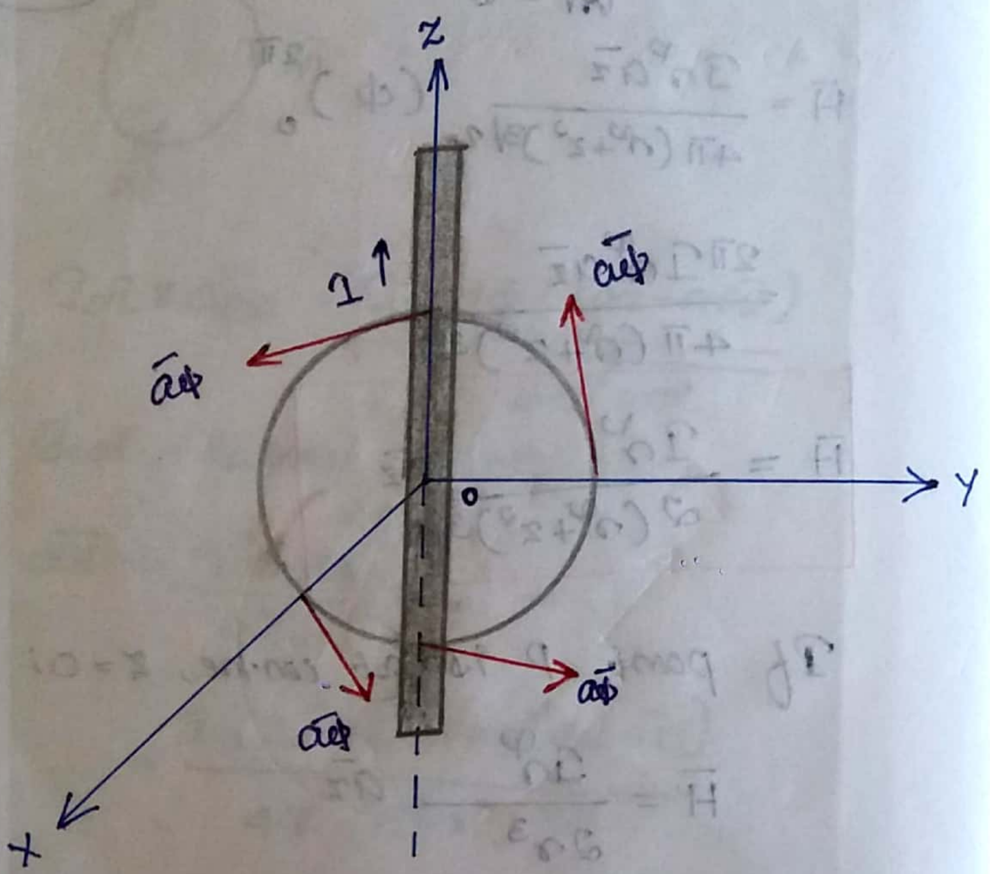
If point P is at centre, $z=0$;

$$\bar{H} = \frac{I r^2}{2 r^3}\, \bar{a}_z$$

$$\bar{H} = \frac{I}{2r}\, \bar{a}_z$$

Scanned by CamScanner

Ampere's Circuital law



The ampere's circuital law states that, the magnetic field intensity around the closed path is exactly equal to the total current enclosed by that path.

$$\boxed{\oint \bar{H}\cdot d\bar{L} = I}$$

Proof :-

$$\bar{H} = \frac{I}{2\pi r}\,\bar{a}_\phi \quad [\text{for infinite}]$$

$$d\bar{L} = r\,d\phi\,\bar{a}_\phi$$

$$\oint \bar{H}\cdot d\bar{L} = \frac{I}{2\pi r}\,\bar{a}_\phi \cdot r\,d\phi\,\bar{a}_\phi$$

$$= \frac{I}{2\pi}\,d\phi$$

$$= \int_{\phi=0}^{2\pi} \frac{I}{2\pi}\,d\phi$$

$$= \frac{I}{2\pi}\,(\phi)_0^{2\pi}$$

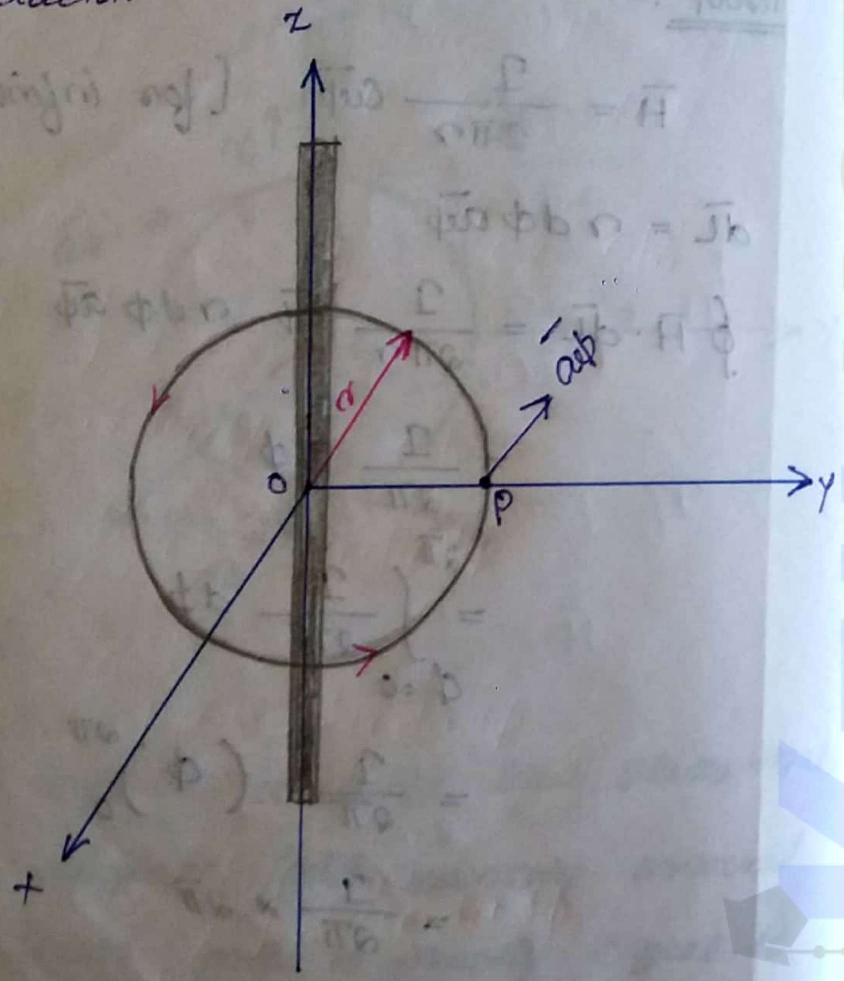
$$= \frac{I}{2\pi}\times 2\pi$$

$$= I$$

06/03/18
Tuesday

Applications of Ampere's circuital law.

$\vec{H}$ due to an infinite long straight conductor.



By ampere's circuital law,

$$\oint \bar{H} \cdot dL = I$$

$$\bar{dL} = r\,d\phi\,\bar{a_\phi}$$

$$\bar{H} = H_\phi\,\bar{a_\phi}$$

$$\oint \bar{H} \cdot \bar{dL} = I$$

$$\oint H_\phi\,a_\phi \cdot r\,d\phi\,\bar{a_\phi} = I$$

$$\oint H_\phi\,r\,d\phi = I$$

$$H_\phi r\,[\phi]_0^{2\pi} = I$$

$$H_\phi r\,[2\pi] = I$$

$$H_\phi = \frac{I}{2\pi r}$$

$$\boxed{\bar{H} = \frac{I}{2\pi r}\,\bar{a_\phi}}$$

Ques:- In the region $0 < r < 0.5\,m$ in cylindrical co-ordinates, the current density is $\bar{J} = 4.5 e^{-2r}\, \bar{a_z}$ A/m^2. and $\bar{J} = 0$ else where. Use ampere's circuital law to find $\vec{H}$?

Sol:- Given, $\vec{J} = 4.5 e^{-2r}\, \bar{a_z}$ A/m^2.

$$I = \oint \bar{J} \cdot \bar{ds}$$

$$\bar{ds} = r\,dr\,d\phi\, \bar{a_z}$$

$$I = \oint 4.5 e^{-2r} \cdot r\,dr\,d\phi$$

$$= \int_0^{2\pi} \int_0^{0.5} 4.5 e^{-2r} \cdot r\,dr\,d\phi$$

$$= 4.5 \int_0^{2\pi} \int_0^{0.5} e^{-2r} \cdot r\,dr\,d\phi$$

$$= 4.5 \times 2\pi \int_0^{0.5} \cancel{e^{2r}}\, r\, e^{-2r}\, dr$$

$$= 2\pi \times 4.5 \left[r \cdot \frac{e^{-2r}}{-2} - \int 1 \cdot \frac{e^{-2r}}{-2} \right]_0^{0.5}$$

$$= 2\pi \times 4.5 \left[r \cdot \frac{e^{-2r}}{-2} - \frac{1}{-2} \left(e^{-2r}/_{-2} \right) \right]_0^{0.5}$$

$$= 2\pi \times 4.5 \left[\frac{r e^{-2r}}{-2} - \frac{e^{-2r}}{4} \right]_0^{0.5}$$

$$= 2\pi \times 4.5 \left(\left[\frac{0.5 e^{-1}}{-2} - \frac{e^{-1}}{4} \right] - \left[0 - \frac{1}{4} \right] \right)$$

$$= 2\pi \times 4.5 \left(\left\{ -0.25 e^{-1} - 0.25 e^{-1} \right\} + \frac{1}{4} \right)$$

$$\cancel{2\pi \times 4.5}$$

$$= 9\pi \left[\cancel{-0} \; -0.092 - 0.0919 \right\} + 1/4 \Big)$$

$$= \underline{\underline{1.8689}}$$

$\cancel{H}$ $\oint H \cdot \bar{dl} = I$

$$\oint H_\phi\, \bar{a_\phi} \cdot r\, d\phi\, \bar{a_\phi} = I$$

$$\oint H_\phi \cdot r\, d\phi = I$$

$$\int_0^{2\pi} H_\phi \, r \, d\phi = I$$

$$H_\phi \, r \, 2\pi = I$$

$$\frac{I}{2\pi r} = H_\phi$$

$$H_\phi = \frac{1.8689}{2\pi r}$$

$$\vec{H} = \frac{0.297}{r} \bar{a}_\phi \text{ A/m.}$$

Scalar and vector magnetic potentials.

There are two types of potentials in magnetic fields.

1) Scalar magnetic potential denoted as V_m.
2) The vector magnetic potential denoted as $\bar{A}$

To define scalar and vector magnetic potential they must satisfy the following identities.

$$\nabla \times \nabla V = 0 ,$$

V = scalar potential.

$$\nabla \cdot (\nabla \times \bar{A}) = 0 .$$

$\bar{A}$ = vector.

Scalar magnetic potential (V_m)

$$\nabla \times \nabla V_m = 0 \quad \text{———} (1)$$

$$\bar{H} = - \nabla V_m . \quad \text{———} (2)$$

(1) ⇒ $\nabla \times (-\bar{H}) = 0$.

$$\nabla \times \bar{H} = 0 \quad \text{———} (3)$$

$$\boxed{\nabla \times \bar{H} = \bar{J}} \quad \text{———} (4)$$

from (3) & (4) ⇒

$$\bar{J} = 0$$

Vector magnetic potential.

Vector magnetic potential is denoted as $\bar{A}$, and it should satisfy the following eq^2, that is the divergence of curl of a vector is always zero.

ie; $\nabla \cdot (\nabla \times \bar{A}) = 0$ ——— (1)

from the div theorem,

$\nabla \cdot \bar{B} = 0$ ——— (2)

On comparing (1) and (2) ⇒

$\boxed{\bar{B} = \nabla \times \bar{A}}$ ——— (3)

from the scalar magnetic potential,

$\nabla \times \bar{H} = \bar{J}$

$\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J}$

$\nabla \times \bar{B} = \vec{J}\mu_0$ ——— (4)

(3) in (4) ⇒

$\nabla \times (\nabla \times \bar{A}) = \vec{J}\mu_0$

$\therefore \quad \bar{J} = \frac{1}{\mu_0} \nabla \times \nabla \times \bar{A}$ ——— (5)

using vector identity; the RHS of (5) can be expressed as;

$\nabla \times (\nabla \times \bar{A}) = \nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$ ——— (6)

(6) in (5) ⇒

$$\boxed{\vec{J} = \frac{1}{\mu_0} \nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A}}$$

Qn:- In cylindrical co-ordinates $\bar{A}$ =

$\bar{A} = 50 r^2 \bar{a}_z$ Wb/m

is a vector magnetic potential, in a certain region of free space. Find $\bar{H}$, $\bar{B}$, $\bar{J}$ and using $\bar{J}$ find the total current I crossing the surface $0 \leq r \leq 1$, $0 \leq \phi \leq 2\pi$

Vector magnetic potential.

Vector magnetic potential is denoted as $\bar{A}$, and it should satisfy the following eq^n, that is the divergence of curl of a vector is always zero.

ie; $\nabla \cdot (\nabla \times \bar{A}) = 0$ ——— (1)

from the div theorem,

$\nabla \cdot \bar{B} = 0$ ——— (2)

On comparing (1) and (2) ⇒

$\boxed{\bar{B} = \nabla \times \bar{A}}$ ——— (3)

from the scalar magnetic potential,

$\nabla \times \bar{H} = \bar{J}$

$\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J}$

$\nabla \times \bar{B} = \vec{J} \mu_0$ ——— (4)

(3) in (4) ⇒

$\nabla \times (\nabla \times \bar{A}) = \vec{J} \mu_0$

$\therefore \quad \bar{J} = \frac{1}{\mu_0} \nabla \times \nabla \times \bar{A}$ ——— (5)

using vector identity; the RHS of (5) can be expressed as;

$\nabla \times (\nabla \times \bar{A}) = \nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$ ——— (6)

(6) in (5) ⇒

$$\boxed{\vec{J} = \frac{1}{\mu_0} \nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A}}$$

Qus:- In cylindrical co-ordinates $\bar{A} =$

$\bar{A} = 50 r^2 \bar{a}_z$ wb/m

is a vector magnetic potential, in a certain region of free space. Find $\bar{H}$, $\bar{B}$, $\bar{J}$ and using $\bar{J}$ find the total current I crossing the surface $0 \leq r \leq 1$, $0 \leq \phi \leq 2\pi$

and $z=0$.

$\bar{B} = \nabla \times \bar{A}$

$\bar{H} = \bar{B}/\mu_0$

$\nabla \times \bar{H} = \bar{J}$

$I = \oint \bar{J} \cdot \bar{ds}$

$\bar{B} = \nabla \times \bar{A}$

$$\nabla \times \bar{A} = \frac{1}{r} \begin{vmatrix} \bar{a_r} & r\bar{a_\phi} & \bar{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$\bar{A} = 50r^2 \bar{a_z}$, $A_r = 0$ $A_\phi = 0$

$$\nabla \times \bar{A} = \frac{1}{r} \begin{vmatrix} \bar{a_r} & r\bar{a_\phi} & \bar{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & 50r^2 \end{vmatrix}$$

$$\nabla \times \bar{A} = 1/r \left[\bar{a_r} \left(\frac{\partial}{\partial \phi} 50r^2 - 0 \right) - r\bar{a_\phi} \left(\frac{\partial}{\partial r} 50r^2 \right) + \bar{a_z}(0) \right]$$

$$= \frac{1}{r} \left[0 - r\bar{a_\phi} (50 \times 2r) \right]$$

$$= -100 r \bar{a_\phi}$$

$\bar{B} = -100 r \bar{a_\phi}$ wb/m^2.

$\bar{H} = \bar{B}/\mu_0 = \frac{-100 r a_\phi}{\mu_0}$ A/m.

$\nabla \times \bar{H} = \bar{J}$

$$\nabla \times \bar{H} = \frac{1}{r} \begin{vmatrix} \bar{a_r} & r\bar{a_\phi} & \bar{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \frac{-100r^2}{\mu_0} & 0 \end{vmatrix}$$

$$= \frac{1}{r} \left[\bar{a_r} \left(0 + \frac{\partial}{\partial z} (100r/\mu_0) \right) - r\bar{a_\phi} (0-0) + \bar{a_z} \left(\frac{\partial}{\partial r} \frac{-100r^2}{\mu_0} \right) \right]$$

$$= \frac{1}{r} \left[\bar{a_z} \left(-100 \times 2r/\mu_0 \right) \right] =$$

$\cancel{\bar{J} = \frac{-100}{r/\mu_0}\,\bar{a_z}\ A/m^2}$.

$J = \frac{-200}{\mu_0}\,\bar{a_z}\ A/m^2$

$I = \oint \bar{J}\cdot \bar{ds}$

$= \oint \frac{-200}{\mu_0}\,\bar{a_z}\cdot r\,dr\,d\phi\,\bar{a_z}$

$= \oint \frac{-200}{\mu_0}\,r\,dr\,d\phi$

$= \int_0^1\int_0^{2\pi} \frac{-200}{\mu_0}\,r\,d\phi\,dr$

$= \int_0^1 \frac{r\cdot -200 \times 2\pi}{\mu_0}\,dr$

$= \frac{-400\pi}{\mu_0}\left[\frac{r^2}{2}\right]_0^1$

$= \frac{-400\pi}{\mu_0}\left(\frac{1}{2}\right)$

$= \frac{-200\pi}{\mu_0}$

$= \frac{-200\pi}{4\pi\times10^{-7}}$

$= -500\times10^{6}\ A$

assignment ①

Qn:- At a pnt $P\ (x,y,z)$ the components of vector magnetic potential, $\bar{A}$ are given as;

$A_x = 4x+3y+2z$,

$A_y = 5x+6y+3z$ and

$A_z = 2x+3y+5z$.

Determine $\bar{B}$ at pnt P ?

$\bar{B} = \nabla\times\bar{A}$

Qus:- The magnetic field intensity is given in a region of space as,

② $$\bar{H} = \frac{x+2y}{z^2}\,\bar{a_y} + \frac{2}{z}\,\bar{a_z} \ A/m.$$

a) Find $\nabla \times \bar{H}$

b) Find $\bar{J}$

c) Use $\bar{J}$ to find the total current passing through the surface $z=4$, $1<x<2$, $3<y<5$ in the $\bar{a_z}$.

Qus:- If the vector magnetic potential is

③ given by $\frac{10}{x^2+y^2+z^2}\,\bar{a_x}$,

Obtain the magnetic flux density in vector form.

Qus:- Let $\bar{A} = (3y-z)\,\bar{a_x} + 2xz\,\bar{a_y}$ Wb/m in a certain region of free space.

④ a) Show that $\nabla \cdot \bar{A} = 0$

b) at $P(2,-1,3)$.

Find $\bar{A}$, $\bar{B}$, $\bar{H}$ and $\bar{J}$?