Chapter 1

STATIC FORCE ANALYSIS

Introduction to force analysis in mechanisms – Static force analysis (four bar linkages only) – Graphical methods
Matrix methods – Method of virtual work – Analysis with sliding and pin friction.

1.1 STATIC FORCE ANALYSIS OF MECHANISMS

For design of machine components, the following forces are considered

- Forces - due to - weight of parts
- Forces of assembly
- Forces from applied loads
- Forces of friction
- Inertia forces
- Spring forces
- Impact forces
- Forces due to temperature changes

For static force analysis, the inertia forces due to acceleration are neglected.

For dynamic force analysis, the inertia forces are taken into account.

In most of the cases, machine component weights are small when compared to other static forces and hence these forces are neglected in static force analysis.

1.2 STATIC EQUILIBRIUM

A body is in static equilibrium, if it is in rest and tends to remain at rest.

A body is in static equilibrium, if it is in motion and tends to keep itself in motion.

The above are true according to Newton's I law.

- The state of equilibrium can be changed by application of external forces (or) moments.

- In a body to be in static equilibrium, the vectorial sum of all the forces and moments about any point is zero.

Mathematically, $\Sigma F_x = 0; \Sigma F_y = 0;$

$$\Sigma M_z = 0 \text{ in two dimensional system}$$

Graphically, the force polygon and couple polygon should be closed.

The following are the conditions for static equilibrium

1. A body under action of two forces will be in equilibrium when the forces F_1 and F_2 are same in magnitude and opposite in direction.

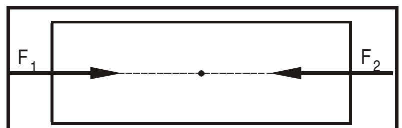


Fig. 1.1.

2. A body under the action of three forces will be in equilibrium, if these forces are concurrent forces and their resultant is zero.

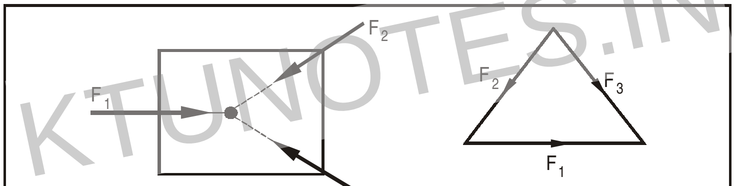


Fig.1.2. (a) Body under forces **Fig.1.2.(b) Force polygon**

3. A body under four forces will be in equilibrium if the vector sum of all forces is zero in such a way that resultant of first two forces. (F_1 and F_2) and remaining two forces (F_3 and F_4) are collinear as shown in **Fig. 1.3.**

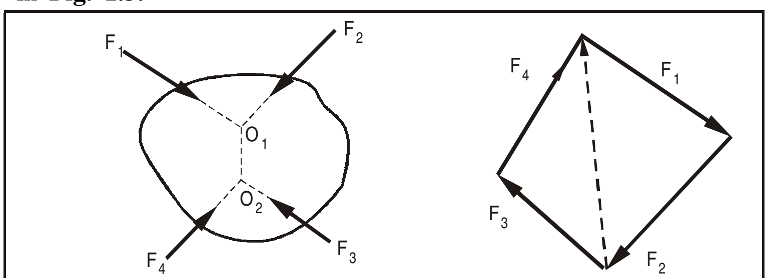


Fig. 1.3 Body in Equilibrium under the Action of Four Forces

4. A body under several forces will be in equilibrium, if the vector sum of all forces is zero and the vector sum of all couples and moments is zero.

5. A beam under three or more parallel forces will be in equilibrium if the algebraic sum of forces and moments is zero **(Fig. 1.4).**

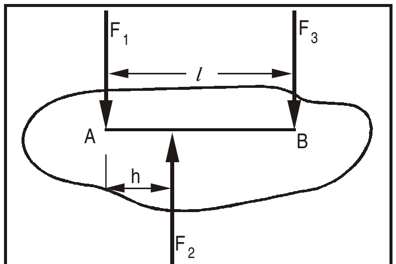


Fig. 1.4 Beam in Equilibrium under Parallel Forces

$$\Sigma F = F_1 - F_2 + F_3 = 0$$

$$\Sigma M_A = F_3 \times l - F_2 \times h = 0$$

6. A link under the action of two forces and an applied couple will be in equilibrium if the forces are (a) equal in magnitude (b) parallel in direction and in opposite sense and (c) the couple formed by them (by these 2 forces F_1 and F_2) should be equal in magnitude and should act opposite to the applied torque **(Fig. 1.5).**

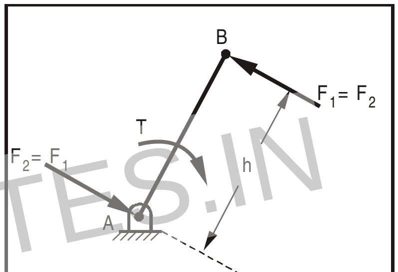


Fig. 1.5 Link with Two Forces & a Couple

Equilibrium conditions are:

$$F_1 = F_2$$

and Couple; $T = F_1 \times h = F_2 \times h$

In static force analysis, the force applied by member 1 on member 2 is represented as $\boldsymbol{F_{12}}$.

1.3 FREE BODY DIAGRAM

A free body diagram is a diagram of the link isolated from the mechanism on which the forces and moments are shown in action.

Fig. 1.6 shows a slider-crank mechanism (4 bar mechanism). The piston is subjected to gas force F_G. This gas force is transmitted to crank shaft to deliver power. The free body diagrams of individual links are shown in **Fig. 1.7.**

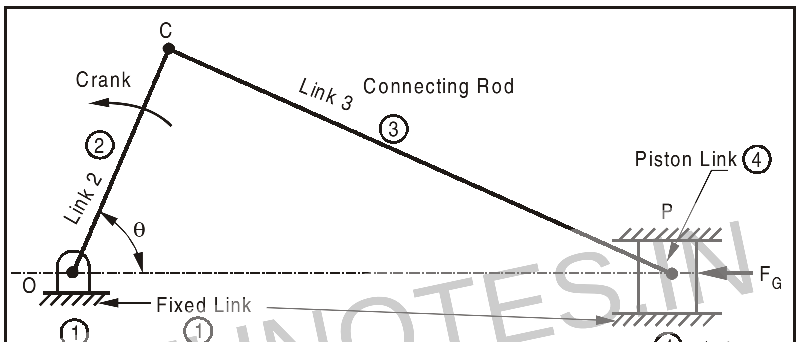


Fig. 1.6 A Slider - Crank Mechanism

1.3.1 Consider Piston - Link 4

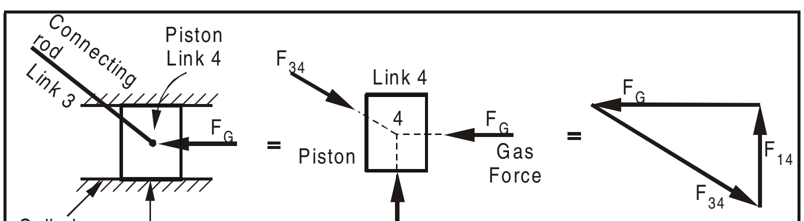


Fig. 1.7 (a) Forces on Piston (Link 4)

Fig. 1.7 (b) Force Polygon Closes here

3 forces are acting on piston.

1. Normal reaction force is acting by cylinder on piston. F_{14}
2. Gas force is acting on piston F_G

3. Force by connecting rod is acting on piston. F_{34}, as these 3 forces are acting on a concurrent point of piston. The concurrent force pass through this concurrent point. Hence, a polygon drawn by these 3 forces should be closed for this piston to be in equilibrium.

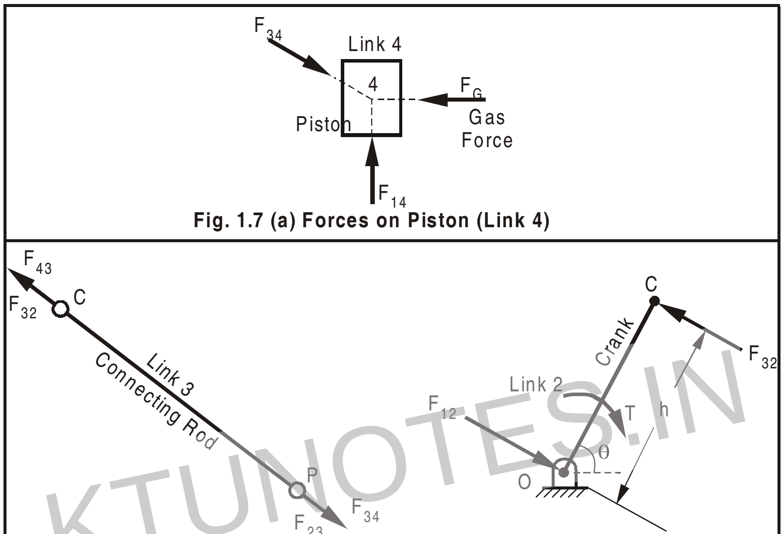


Fig. 1.7 (a) Forces on Piston (Link 4)

(c) Forces on Connecting Rod (Link 3)

(d) Forces on Crank (Link 2)

Fig. 1.7 Free Body Diagrams of Various Links of a Slider-Crank Mechanism

1.3.2 Consider connecting rod - Link 3

Here, the connecting rod is hinged at two ends and hence it is a two force system.

2 forces are acting on connecting rod hinges.

1. $F_{43} \rightarrow$ Force by piston (4) is acting on connecting rod (3)
2. $F_{34} \rightarrow$ Force by connecting rod (3) is acting on piston (4).

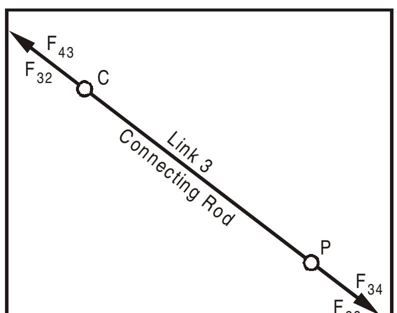


Fig. 1.7 (c) Forces on Connecting Rod (Link 3)

❖ Both forces are equal in magnitude but opposite in direction.

$$F_{34} = F_{43}$$

❖ And force F_{43} is acting on crank as F_{32}. So $F_{43} = F_{32}$

1.3.3 Consider Crank - Link 2

Since crank shaft is acted by F_{32} (ie F_{43}) at C, the fixed end O is acted upon by F_{12} in opposite direction and $F_{32} = F_{12}$. Hence the crank is in equilibrium. But these forces F_{32} and F_{12} are equal and opposite in direction and hence form a couple.

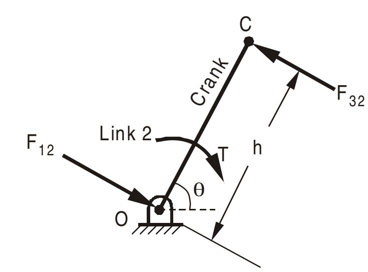


Fig. 1.7 (d) Forces on Crank (Link 2)

❖ And this couple is balanced (cancelled) by the torque transmitted by the crank shaft.

$$\text{Torque} = F_{32} \times h$$

Note:

For a member under the action of 2 forces and applied torque, to be in equilibrium, conditions are:

❖ ***The 2 forces should be equal and opposite.***

❖ ***These 2 forces should form a couple which is equal and opposite to applied torque.***

1.4 ANALYSIS WITH SLIDING AND PIN FRICTION

Friction in machine is classified as

1. Sliding friction
2. Pin friction (Turning friction)

1.4.1 Sliding Friction

Sliding friction is generated when a link (piston) is sliding on another link (cylinder). **(Fig. 1.8)**. In a sliding friction if gas force is acting towards **left**, then friction force (μR_n) will be acting towards **right** side. Refer **Fig. 1.9.**

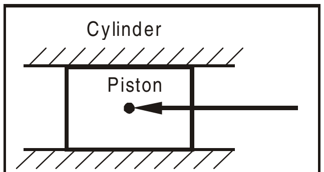


Fig. 1.8

- Normal reaction R_n is acting upward.
- Vectorial sum of μR_n and R_n is F_{14}.
- F_{14} is acting at an angle of ϕ.
- ϕ is called friction angle and

$$F_{14} = \sqrt{(\mu R_n)^2 + R_n^2}$$

and $\tan\phi = \dfrac{\mu R_n}{R_n} = \mu$ = coefficient of friction

- Sliding friction is acting in sliding pair.

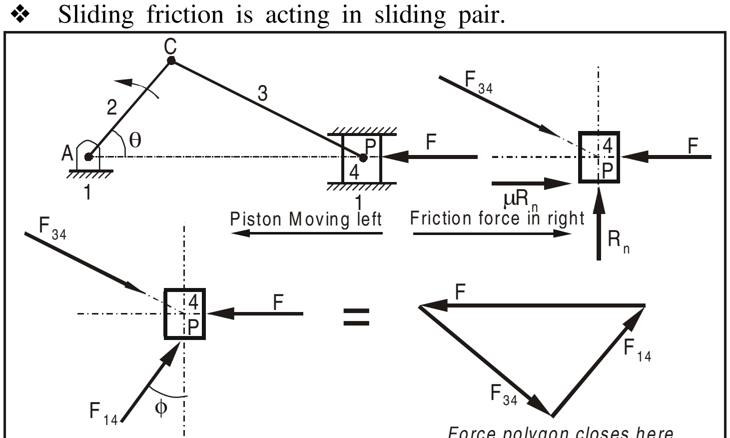


Fig. 1.9 Free Body Diagram of Slider and Force Polygon

1.4.2 Pin Friction (Turning friction)

The turning pair is used to allow turning (or) revolving motion between links.

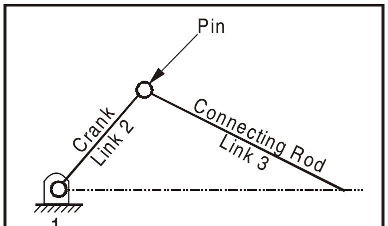


Fig. 1.10

The friction between the pin and the links for revolving (or) rotary motion is called turning friction. In this turning pair, the friction force does not pass through the pin centre but it is tangent to the friction circle of the Pin. The line of action of the force is common tangent to the friction circles of two pins. It is **called friction axis.**

Problem 1.1: *Determine the torque required to be applied at the crank shaft of a slider-crank mechanism to bring it in equilibrium. The slider is subjected to a horizontal force of 8000 N and a force of magnitude 2000 N is applied on the connecting rod as shown in* **Fig**. *The dimensions of various links are as under:*

$OA = 250$ mm, $AB = 750$ mm *and* $AC = 250$ mm, $\angle BOA = 40°$

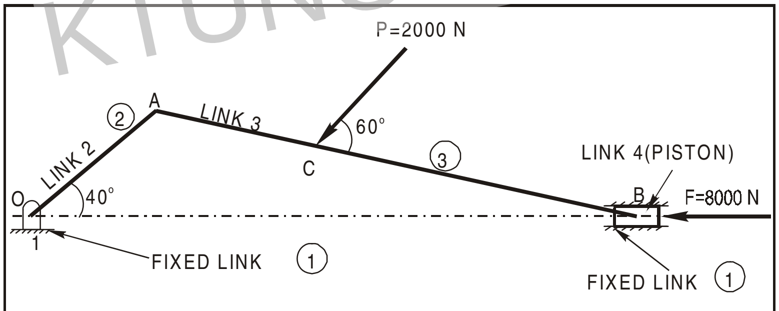


Fig. Slider-Crank Mechanism

Solution: Graphical Method

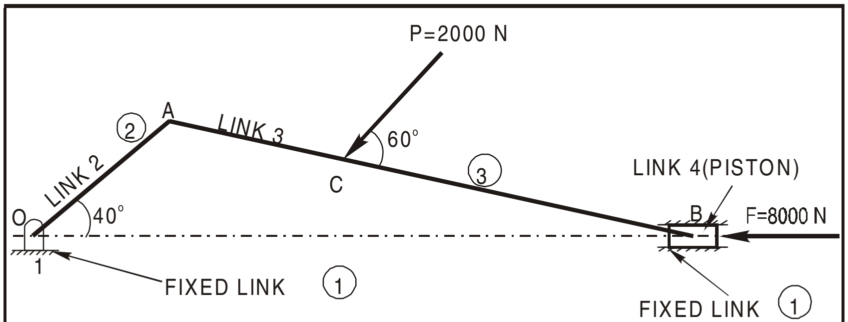
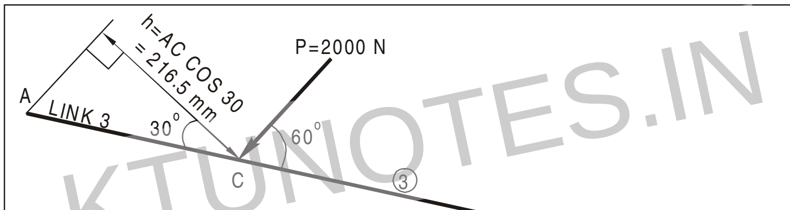


Fig.(a) Configuration diagram

1. Draw configuration diagram of the mechanism as shown in Figure.



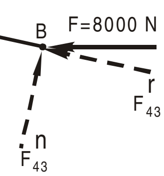


Fig.(b) Free body diagram of LINK3

The force F43 is resolved into 2 components-

F^r_{43}-Radial component of F43 along the Link 3

F^n_{43}-Normal component of F43 perpendicular to the Link 3 ($\perp^r$ to BC)

2. Draw free body diagram of link 3 as shown in **Fig.** The force F_{43} is broken into two components - one along the link F^r_{43} and another perpendicular to the link F^n_{43}.

3. Taking moments about point A,

$$P \times h = F^n_{43} \times AB$$

where $h = AC \cos 30^\circ = 250 \times \cos 30^\circ = 216.5$ mm

$$\text{Force: } F^n_{43} = \frac{2000 \times 216.5}{750} = 577.35\text{N}$$

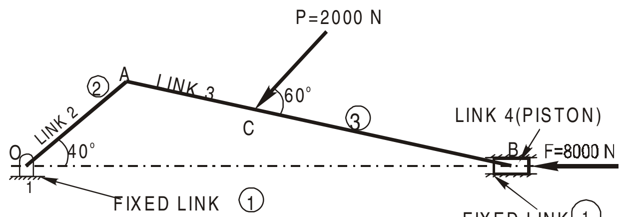


Fig.(a) Configuration diagram

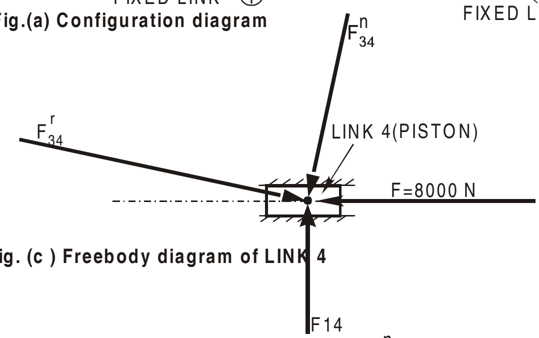


Fig. (c) Freebody diagram of LINK 4

4. Out of forces acting on link 4, forces *F* and F^n_{43} are known completely **(Figure (c)).** Force F_{14} is perpendicular to the sliding surface and force F^r_{34} is along the link 3. Construct a force polygon as shown in **Figure (d)** with suitable scale. (1 mm = 100N) By measurement,

$$F_{34} = 8324 \text{ N}$$

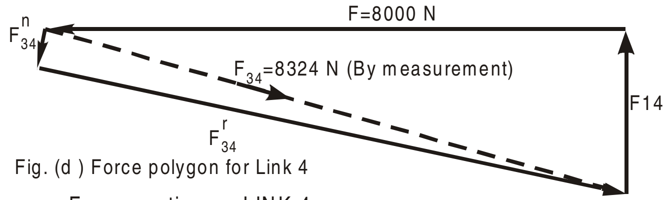


Fig. (d) Force polygon for Link 4

- Forces acting on LINK 4

5. Draw the force polygon of link 3 with forces F_{43}, P and F_{23}.
By measurement F_{23} = 8843 N. [See **Figure (e)**].

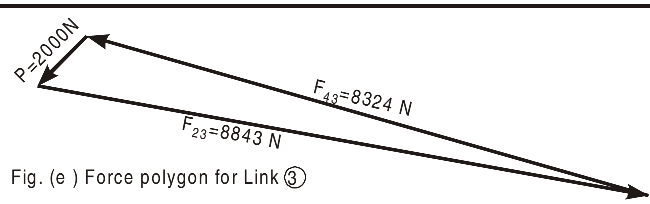


Fig. (e) Force polygon for Link 3

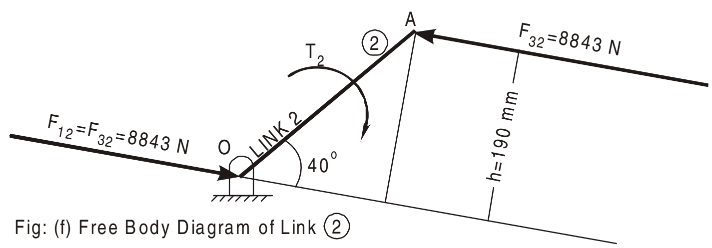


Fig: (f) Free Body Diagram of Link 2

6. Draw the free body diagram of link 2. Measure distance *h* from **[Figure (f)]** and calculate .

$$T_2 = F_{32} x\, h = 8843 \text{ x } 0.19 = \mathbf{1680.2 \text{ Nm clock wise}}$$

PRINCIPLE OF VIRTUAL WORK

The principle of virtual (imaginary) work states that ‘the work done during a virtual displacement from the equilibrium is equal to zero’. Virtual displacement is defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism can be examined as a whole and not as individual links.

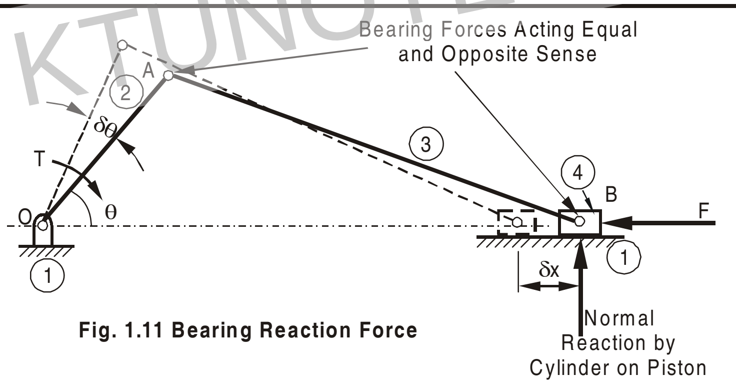


Fig. 1.11 Bearing Reaction Force

Consider a slider-crank mechanism (shown in **Fig.1.11**) acted upon by

- The external piston force ***F***
- The external crankshaft torque ***T*** and
- The force at the bearings.

- If the crank rotates through a small angular displacement $\delta\theta$, then corresponding displacement of the piston is δx and the various forces acting on the system are
- Bearing reaction at O which performs no work.
- Force of cylinder on piston, perpendicular to piston displacement. ie Normal reaction which produces no work.
- Work done by torque $T = T\,\delta\theta$
- Work done by force $F = F\,\delta x$

We know that workdone is **positive** if a force acts in the direction of the displacement and **negative** if it acts in the opposite direction.

According to the Principle of Virtual work,

$$W = T\,\delta\theta + F\,\delta x = 0$$

Since virtual displacement takes place simultaneously during the same interval δt,

$$T\frac{d\theta}{dt} + F\frac{dx}{dt} = 0$$

$$\therefore\ T\,\omega + FV = 0$$

where ω is the angular velocity of the crank and ν, the linear velocity of the piston.

$$T = -\frac{F}{\omega}V$$

The negative sign indicates that for equilibrium, T should be applied in the opposite direction to the angular displacement.

The **Problem 1.1** is solved by using principle of virtual work. First of all, draw the velocity diagram. Refer **Fig.(g)**.

Assume link OB has ω rad/s

Velocity of A with respect to O

$$V_a = \text{radius } OA \times \omega$$

$$\therefore\ V_a = 0.25\,\omega$$

- Draw $V_a = 0.25\ \omega = 250\ \text{mm} = \boldsymbol{oa} \perp^r$ to link OA. Because V_a is perpendicular to OA. [Scale: 100mm $= 0.1\ \omega$]
- From $\boldsymbol{a}$, draw line perpendicular to AB (Length not known to represent V_{ba} (Velocity of b with respect to a).
- From O, draw horizontal line to represent velocity of piston V_b.

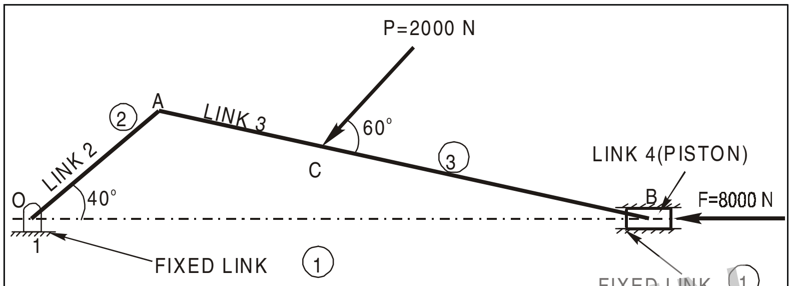


Fig.(a) Configuration diagram

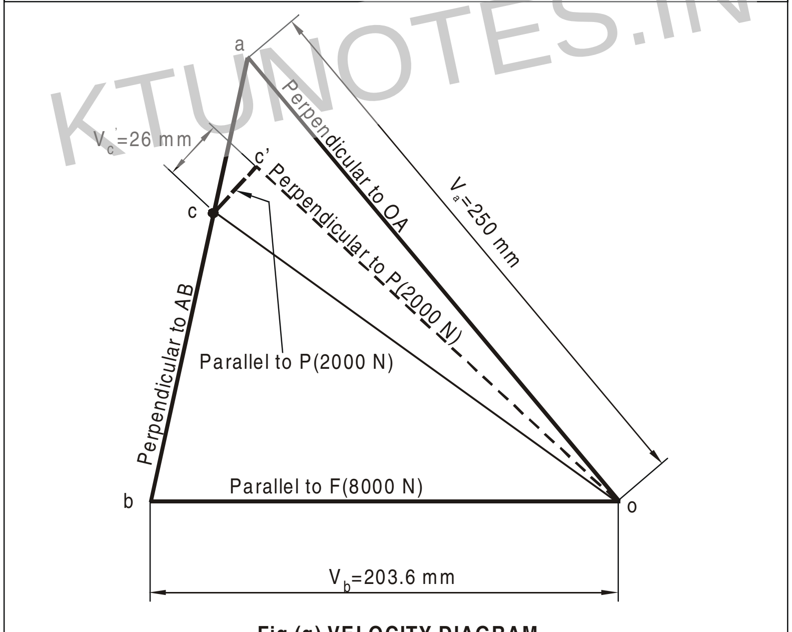


Fig.(g) VELOCITY DIAGRAM

- The above two lines intersect at b.
- Measure $\boldsymbol{ob} = \mathbf{203.6\ mm} = \mathbf{0.2036}\ \omega = V_b$

  $\boldsymbol{ab} = \mathbf{198\ mm} = \mathbf{0.198}\ \omega$
- Mark $\boldsymbol{c}$ on ab in the same ratio as C divides the AB.

$$\text{ie } \frac{ac}{ab} = \frac{AC}{AB} \Rightarrow \frac{ac}{198} = \frac{250}{750} \qquad \therefore \boldsymbol{ac} = 66 \text{ mm}$$

- Join $\boldsymbol{oc}$. Now resolve $\boldsymbol{oc}$ into 2 components as
  - Parallel to force P (2000 N) (V_c') and
  - Perpendicular to force P (2000 N)

Measure $V_c' = \mathbf{26}\text{ mm} = \mathbf{0.026}\ \omega$ (Parallel to $P = 2000$ N)

Using principle of virtual work,

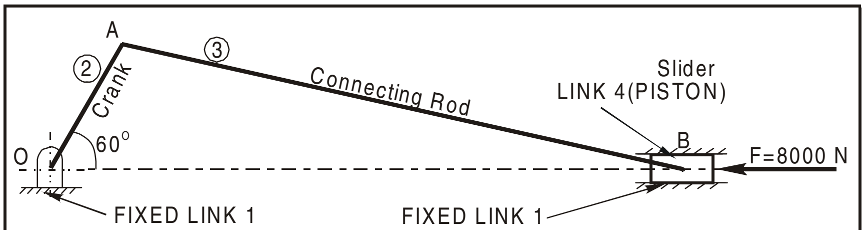
$$T \times \omega - 2000 \times 0.026\ \omega - 8000 \times 0.2036\ \omega = 0$$

$$T = 2000 \times 0.026 + 8000 \times 0.2036$$

$$= 52 + 1629 = \mathbf{1680.8\ N - m}$$

Problem 1.2: *A Slider-crank mechanism as shown in Fig is given below. The force acting on slider is 8000 N and coefficient of friction between all the links is 0.25. Calculate the driving torque if the pin diameters at joints O, A and B are 40 mm, 40 mm and 20 mm respectively. The dimensions of links are:*

$$OA = 200 \text{ mm} ; AB = 800 \text{ mm and } \angle BOA = 60°$$



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Solution:

Friction circle radius:

At point $O = \mu\, r_1 = 0.25 \times \dfrac{40}{2} = 5 \text{ mm}$

At point $A = \mu\, r_2 = 0.25 \times \dfrac{40}{2} = 5 \text{ mm}$

At point $B = \mu\, r_3 = 0.25 \times \dfrac{20}{2} = 2.5 \text{ mm}$

Graphical Method

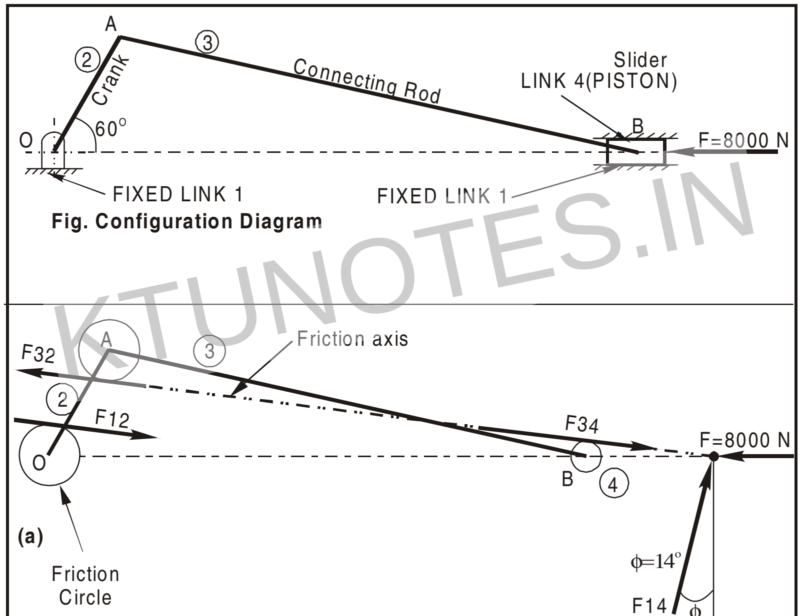
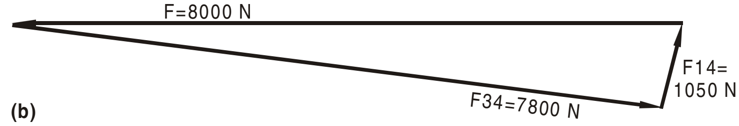


Fig. Configuration Diagram

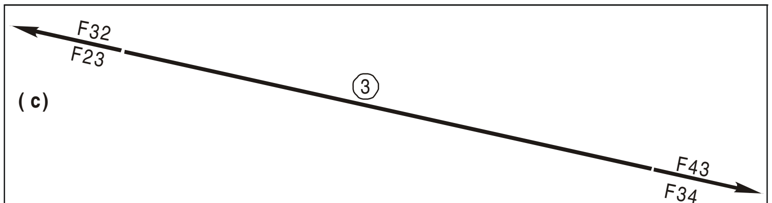
1. Draw configuration diagram of slider-crank mechanism.
2. Draw **friction circle** of different radius at points *O* (5 mm), *A* (5mm) and *B* (2.5mm).
3. Draw tangents and decide **friction axis** of links 2 and 3.
4. Calculate friction angle $\phi = \tan^{-1}(0.25) = 14^\circ$ and draw the resultant side thrust force F_{14} inclined at friction angle.



(b)

5. Consider various forces at the slider and draw the force polygon with suitable scale (100N = 1mm) as shown in **Figure (b)**. Find the forces F_{34} and F_{14} by measurement.

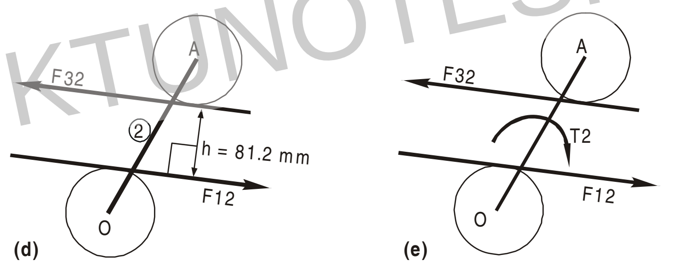
$$F_{34} = 7800 \text{ N and } F_{14} = 1050 \text{ N}$$



(c)

6. Draw the free body diagram of link 3 as shown in **Figure (c)** with

$$F_{34} = F_{43} = F_{32} = F_{23}$$



(d) (e)

7. Mark forces F_{32} and F_{12} at the crank as shown in **Fig. (a)** and **Fig (d)** and **(e).**

8. Measure distance h = 81.2 mm

Torque: $T_2 = F_{32} \times h$

$$= 7800 \times \frac{81.2}{1000}$$

= 633.36 Nm (counter clock wise)

Note: In actual slider-crank mechanism, the coefficient of friction is very low. But in this problem, for easy under standing purpose, an imaginary value of μ (-Higher value-) is given to draw friction circles easily.

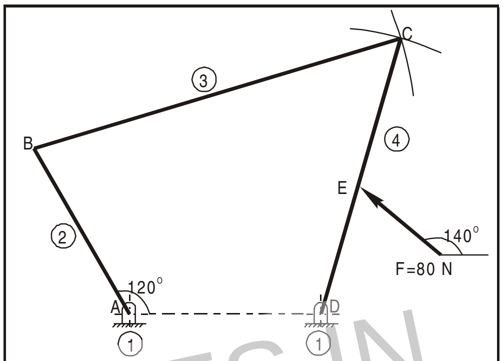
Problem 1.3: *A four-link mechanism with the following dimensions is acted upon by a force 80 N at* $\angle 140°$ *on the link DC [Fig.(a)].* $AD = 250$ mm, $AB = 250$ mm, $BC = 500$ mm, $DC = 375$ mm, $DE = 175$ mm.. *Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.*

Solution: If the mechanism is in static equilibrium, then each of its members should also be in equilibrium individually.

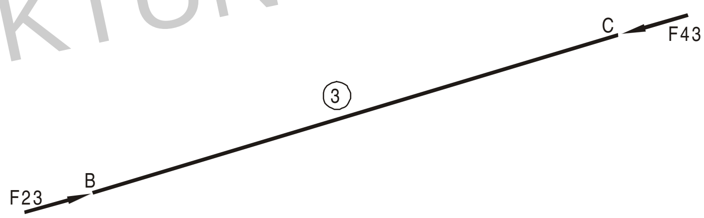
Link 4 is acted upon by three forces **F, F$_{34}$** and **F$_{14}$**.

Link 3 is acted upon by two forces **F$_{23}$** and **F$_{43}$**.

Link 2 is acted upon by two forces **F$_{32}$** and **F$_{12}$** and a torque **T**.

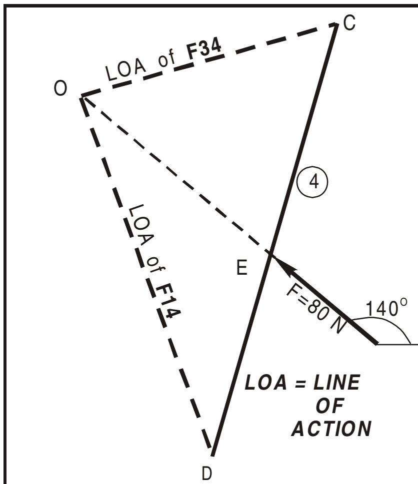


(a) Given Configuration



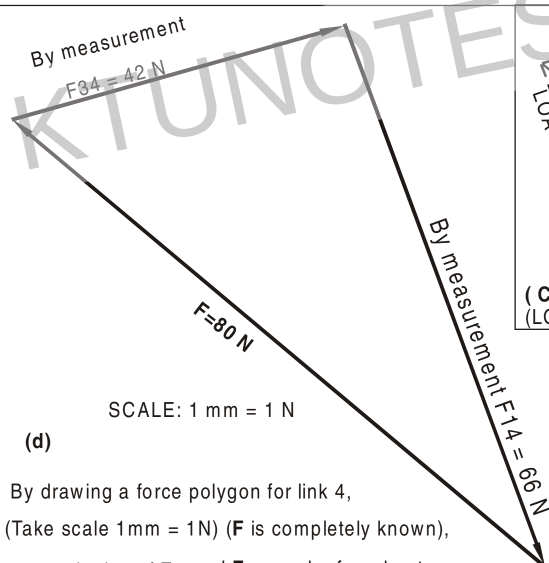
(b)

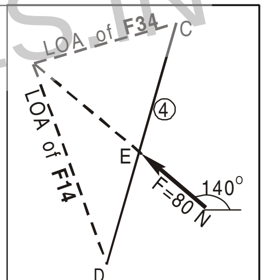
1. Force **F** on the link **4** is known (as 80 N) completely. To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, the **link 3** should be considered first which is a two-force member.
2. As the link 3 is a two-force member **(Fig.(b))**, for its equilibrium, F_{23} and F_{43} must act along BC. Thus, the line of action of F_{34} is also along BC.



(C)To find LOA of F34 and F14

3. .Draw a link *DC* Refer **Fig.(c)** As the force $\mathbf{F_{34}}$ acts through the point *C on* the link 4, draw a line parallel to *BC* through *C*. Extend force *F* to intersect O. Now, as the link 4 is a three-force at member, the third force $\mathbf{F_{14}}$ should pass through the intersection O of **F** and $\mathbf{F_{34}}$ as the three forces are to be concurrent for equilibrium of the link **[Fig. (c)].**





(C) To find Line Of Action (LOA) of F34 and F14

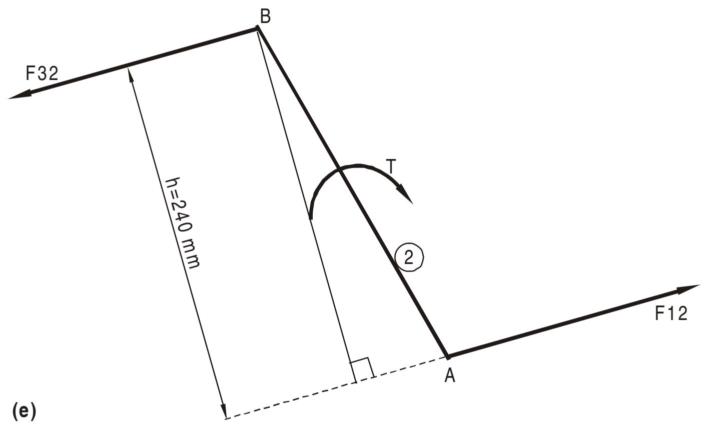
SCALE: 1 mm = 1 N

(d)

By drawing a force polygon for link 4, (Take scale 1mm = 1N) (**F** is completely known), magnitudes of $\mathbf{F_{14}}$ and $\mathbf{F_{34}}$ can be found out.

From force polygon, $F_{34} = 42$ N

$$F_{34} = F_{43} = F_{23} = F_{32} = 42 \text{ N}$$



(e)

Link 2 will be in equilibrium **[Fig. (e)]** if $\mathbf{F_{12}}$ is equal, parallel and opposite to $\mathbf{F_{32}}$ and

$$T = F_{32} \times h = 42 \times 0.240 = 10 \text{ Nm anticlock wise}$$

The input torque has to be equal and opposite to this couple, i.e.,

$$\mathbf{T = 10 \text{ Nm (clockwise)}}$$

Matrix Method:

First of all, the angular inclinations of the links *BC* and *DC* i.e., angles β and ϕ are to be determined by drawing the configuration diagram **(Fig.(a))**

Position vectors:

AB = 0.25 at $\angle$ 120°, BC = 0.5 at $\angle$ 17°, DC = 0.375 at $\angle$ 72°, DE = 0.175 at $\angle$ 72°

The direction of $\boldsymbol{F_{34}}$ is along *BC* since it is a two-force member,

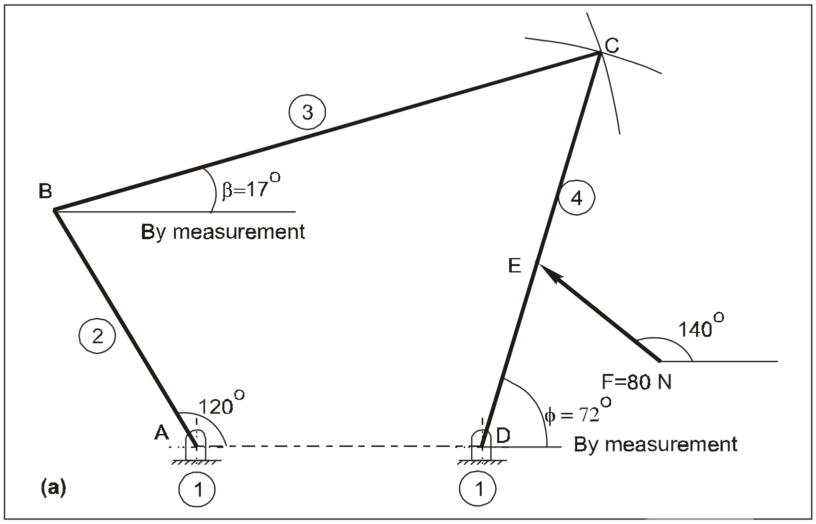
$$\mathbf{F_{34}} = F_{34} \text{ at } \angle\, 17°$$

Since the link *DC* is in static equilibrium, there are no resultant forces and summation of moments acting on it is zero. Taking moments of the forces about point D.

$$M_D = \mathbf{F_4} \times \mathbf{DE} + \mathbf{F_{34}} \times \mathbf{DC} = 0 \qquad \text{...(i)}$$

Moments are the cross-multiplication of the vector, so it is done in rectangular coordinates.

$$\mathbf{F_4} = 80 \cos 140\, i + 80 \sin 40\, j = -61.28\, i + 51.42\, j$$



$$\mathbf{DE} = 0.175 \cos 72\, i + 0.175 \sin 72\, j = 0.054\, i + 0.166\, j$$

$$\mathbf{F_{34}} = F_{34} \cos 17\, i + F_{34} \sin 17\, j = F_{34}\,(0.956\, i + 0.292\, j)$$

$$\mathbf{DC} = 0.375 \cos 72\, i + 0.375 \sin 72\, j = 0.1159\, i + 0.357\, j$$

Inserting the values of vectors in equation (i), we get

$$M_D = (-61.28\, i + 51.42\, j) \times (0.054\, i + 0.166\, j) + F_{34}\,(0.956\, i + 0.292\, j) \times (0.1159\, i + 0.357\, j) = 0$$

Assembling in matrix form,

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -61.28 & 51.42 & 0 \\ 0.054 & 0.166 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.956\, F_{34} & 0.292\, F_{34} & 0 \\ 0.1159 & 0.357 & 0 \end{vmatrix} = 0$$

$$(-61.28 \times 0.166 - 51.42 \times 0.054)$$

$$+ (0.956\, F_{34} \times 0.357 - 0.292\, F_{34} \times 0.1159) = 0$$

$$-12.95 + 0.307\, F_{34} = 0 \text{ (or) } F_{34} = 42 \text{ N}$$

Thus, $\mathbf{F_{34}} = 42$ at $\angle\, 17°$

Now,

$$\mathbf{F_{32}} = \mathbf{F_{23}} = \mathbf{F_{43}} = \mathbf{F_{34}} = 42 \text{ at } \angle\,(180° + 17°) = 42 \cos 197\, i + 42 \sin 197\, j$$

$$= -40.16\, i - 12.28\, j$$

$$\mathbf{AB} = 0.25 \cos 120\, i + 0.25 \sin 120\, j$$

$$= -0.125\, i + 0.2165\, j$$

$$\mathbf{T_{2c}} = \mathbf{F_{32}} \times \mathbf{AB}$$

$$\mathbf{T_{2c}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -40.16 & -12.28 & 0 \\ -0.125 & 0.2165 & 0 \end{vmatrix}$$

$$= (-40.16 \times 0.2165) - (12.28 \times 0.125) = -10.22 \text{ N} - \text{m (counterclockwise)}$$

Thus input torque $= \mathbf{10.22\ N - m\ (clockwise)}$

Principle of Virtual Work

Assume link AB has ω rad/s

Velocity $= \text{radius} \times \omega$

$\therefore V_b = 0.25\, \omega$

❖ Draw $V_b = 0.25\, \omega = 25 \text{ mm} = ab \perp^r$, to link AB.

Refer **Fig.(V)** [Scale 10 mm = 0.1 ω]

(Because V_b will be perpendicular to link AB)

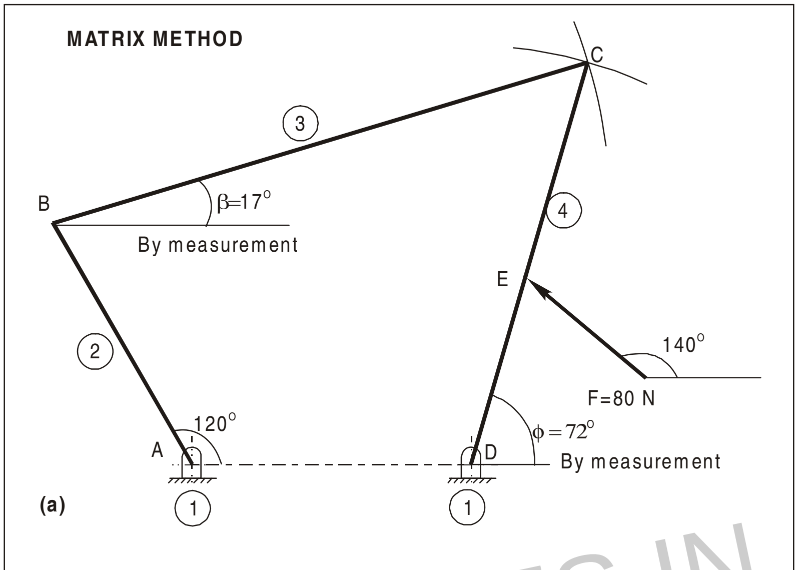
❖ Also mark ***d*** nearer to ***a*** (as both are fixed link (1))

❖ From ***a*** draw line, $\perp^r$ to link ***DC*** from ***d***. (Length not known)

❖ And draw another line, $\perp^r$ to link ***BC*** from ***b*** (Length not known).

❖ Both of the above lines intersect at ***c***.

❖ Locate ***e*** in ***dc*** in the same ratio as ***E*** divides DC in configuration diagram.



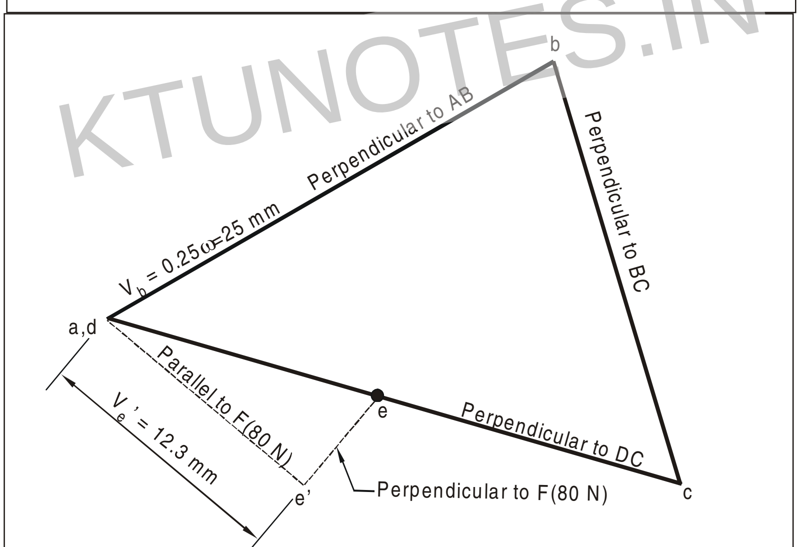


Fig. (V) VELOCITY DIAGRAM

$$\text{ie } \frac{de}{dc} = \frac{DE}{DC} \Rightarrow \frac{de}{29} = \frac{175}{375} \Rightarrow de = 13.53 \text{ mm}$$

❖ Join ***ae*** – Now resolve ***ae*** into 2 components as

– parallel to force *F* (80 N) and

– perpendicular to force *F* (80 N)

❖ Now measure

$$V_e' = \mathbf{12.3\ mm} = 0.123\ \omega \text{ (Parallel to } F \text{ (80 N))}$$

Assume *T* as counterclockwise (+), we can apply

Principle of Virtual work

$$T \times \omega + 80 \times 0.123\ \omega = 0$$

$$T = -80 \times 0.128 = -10.24 \text{ N} - \text{m}$$

$$T = 10.24 \text{ N} - \text{m clockwise}$$

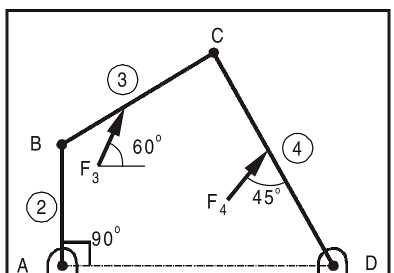
Problem 1.4: *A four bar mechanism as shown in Fig. is subjected to two forces,* $F_3 = 2000$ N *at* 60° *from horizontal at mid point of link 3 and* $F_4 = 4000$ N *at* 45° *from link 4 at mid point of link 4. The dimensions of links are as under:*

$$AB = 0.3 \text{ m}, \quad BC = 0.4 \text{ m}, \quad CD = 0.45 \text{ m} \quad \text{and} \quad AD = 0.6 \text{ m}$$

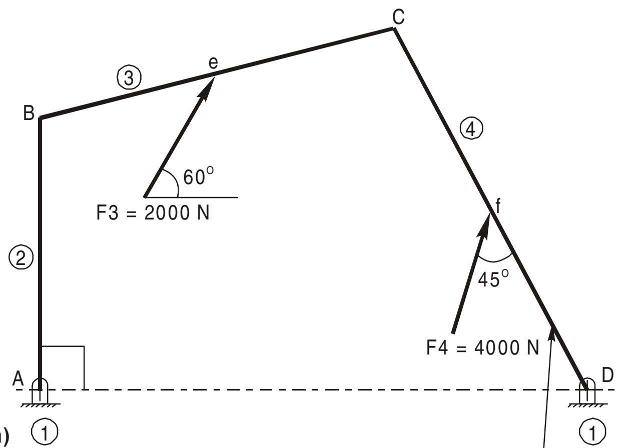
Perform static force analysis and determine resisting torque on link 2.

Solution:

This type of problem can be solved by Principle of superposition. ie, Net effect is equal to superposition of the effect of individual loads taken one at a time.



Given Figure

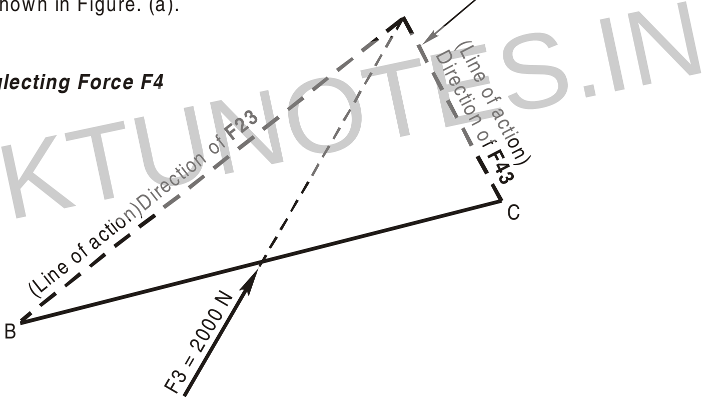


(a)

1. Draw the configuration diagram of mechanism as shown in Figure. (a).

Both are parallel

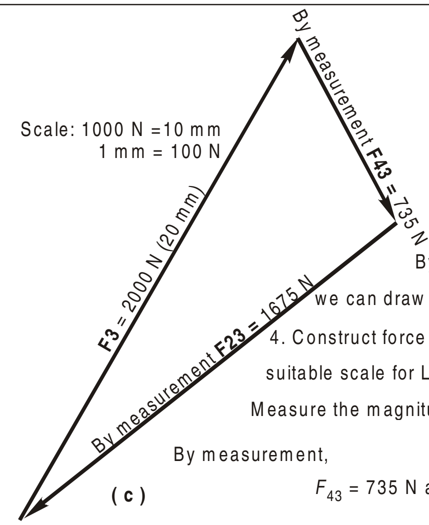
Neglecting Force F4



(b)

2. **Consider the effect of force F_3, neglecting force F_4. .**

3. Draw the free body diagram of link 3 and find the (line of action (LOA)) direction of forces F_{23} and F_{43} **[Figure (b)].**



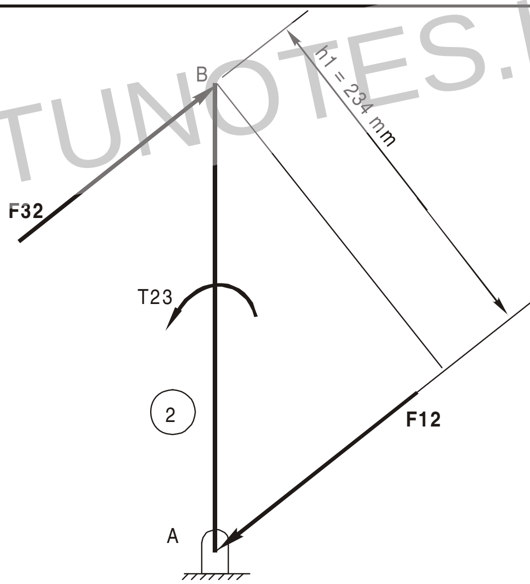
By using these directions, we can draw force polygon for Link (3)

4. Construct force polygon [Figure (c)] with suitable scale for Link 3 (1000 N = 10 mm) Measure the magnitude of forces F_{43} and F_{23}.

By measurement,

F_{43} = 735 N and F_{23} = 1675 N

(c)

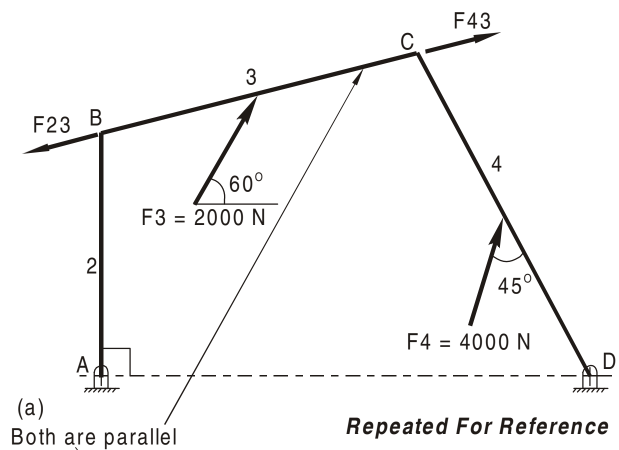


(d)

5. Draw free body diagram of link 2 **[Figure.(d)]**. Measure distance $h1$ = 234 mm.

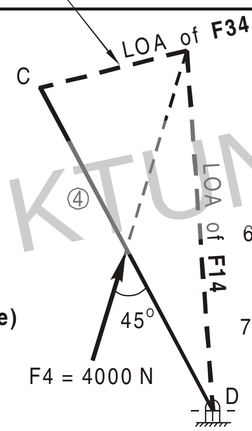
. Torque due to force F_3,

$$T_{23} = F_{23} \times h_1 = 1675 \times \frac{234}{1000} = 392 \text{ Nm (counterclockwise)}.$$



(a)

Repeated For Reference

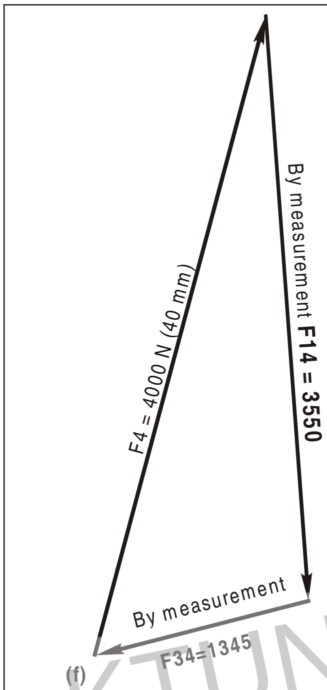


(e)

Neglecting Force F3

6. Now consider the effect of force F_4 neglecting the effect of force F_3.
7. Draw the free body diagram of link 4 and find the LOA of forces F_{34} and F_{14} **[Figure.(e)].**

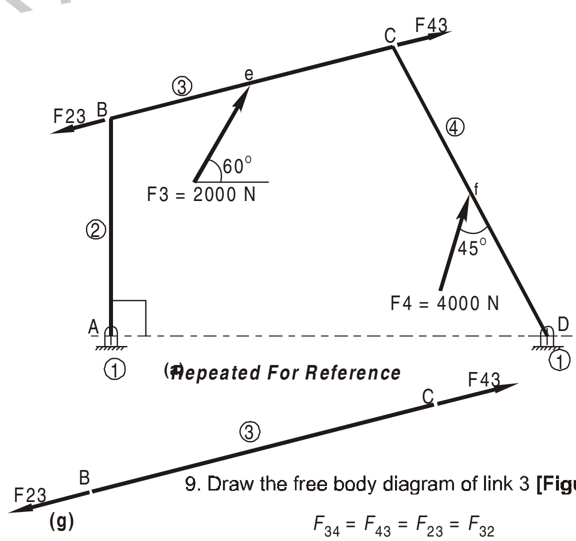
LOA = LINE OF ACTION



(f)

8. Construct force polygon for link 4 **[Figure.(f)]** with suitable scale (1 mm = 100 N). Measure the magnitude of forces F_{14} and F_{34}.

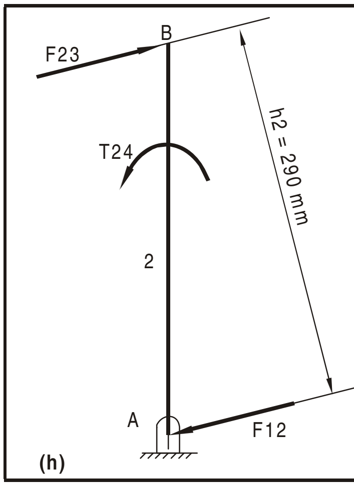
F_{14} = 3550 and F_{34} = 1345 N



(g)

9. Draw the free body diagram of link 3 **[Figure.(g)]**.

$F_{34} = F_{43} = F_{23} = F_{32}$



10. Draw the free body diagram of link 2 **[Figure.(h)]**. Measure the distance h_2 = 290 mm.

Torque: $T_{24} = F_{32} \times h_2$

$$= 1675 \times \frac{290}{1000} = 486 \text{ Nm (counterclockwise)}$$

Total resisting torque:

$$T_2 = T_{23} + T_{24}$$

$$= 392 + 486 = 878 \text{ N - m}$$

$$= 878 \text{ Nm (counterclockwise)}$$

Principle of Virtual Work

Assume link *AB* has an instantaneous angular velocity of ω rad/s counter clockwise.

Velocity = radius × ω

Hence $V_b = 0.3\,\omega$ [∴ $AB = 0.3$ m]

By knowing magnitude and direction of V_b and knowing the direction of velocity V_{cb} and V_{cd}, velocity diagram can be drawn.

❖ Draw $V_b = 0.3\,\omega = 30$ mm horizontally as ***ab***. (Perpendicular to link *AB*). Refer Fig. (V).

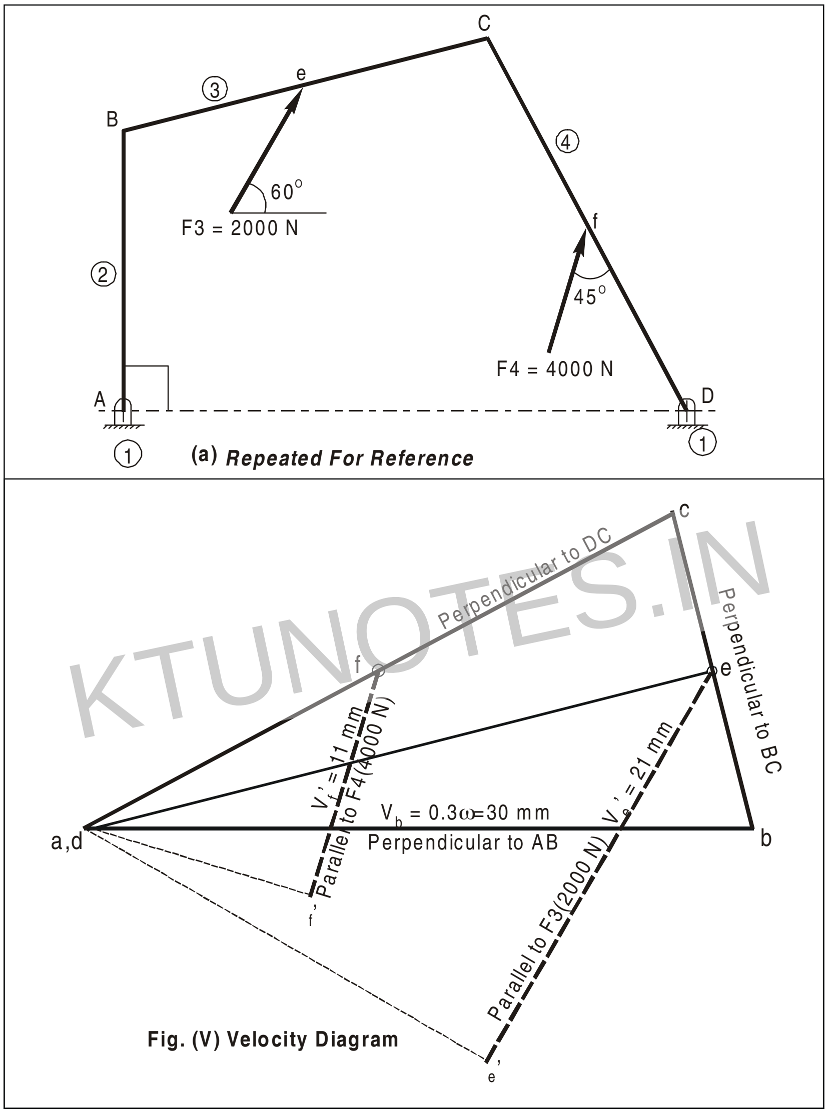
❖ Mark ***d*** nearer to ***a*** (as both are fixed link 1)

❖ Draw line $\perp^r$ to link *DC* from ***d*** and draw another line $\perp^r$ to Link ***BC*** from ***b***. Both lines intersect at ***c***.

❖ Now mark point ***e*** as mid point of ***bc*** and mark ***f*** as mid point of ***dc***.

❖ Join ***ae***.

❖ Then joint ***df***



(a) ***Repeated For Reference***

Fig. (V) Velocity Diagram

- Now resolve ***ae*** into 2 components parallel to force $F_3\,(V_e')$ and perpendicular to F_3
- Measure $V_e' = 0.21\,\omega$ (= 21 mm by measurement)

- Similarly, resolve ***df*** into 2 components.
- Parallel to force $F_4\,(V_f')$ and perpendicular to F_4
- Measure $V_f' = 0.11\ \omega = 11$ mm by measurement. ∴ From velocity diagram

Parallel to $F_3 \rightarrow V_e' = 0.21\ \omega$ (ie 21 mm) and

Parallel to $F_4 \rightarrow V_f' = 0.11\ \omega$ (ie 11 mm)

Assume T as counterclockwise (positive) and apply **principle of virtual work.**

$$T \times \omega - F_3 \times 0.02\ \omega - F_4 \times 0.11\ \omega = 0$$

$$T - 2000 \times 0.21 - 4000 \times 0.11 = 0$$

$$\boldsymbol{T = 860\ \mathrm{N-m}}$$

Problem 1.5: *For the mechanism shown in Fig., determine the torque on the link AB for the static equilibrium of the mechanism.*

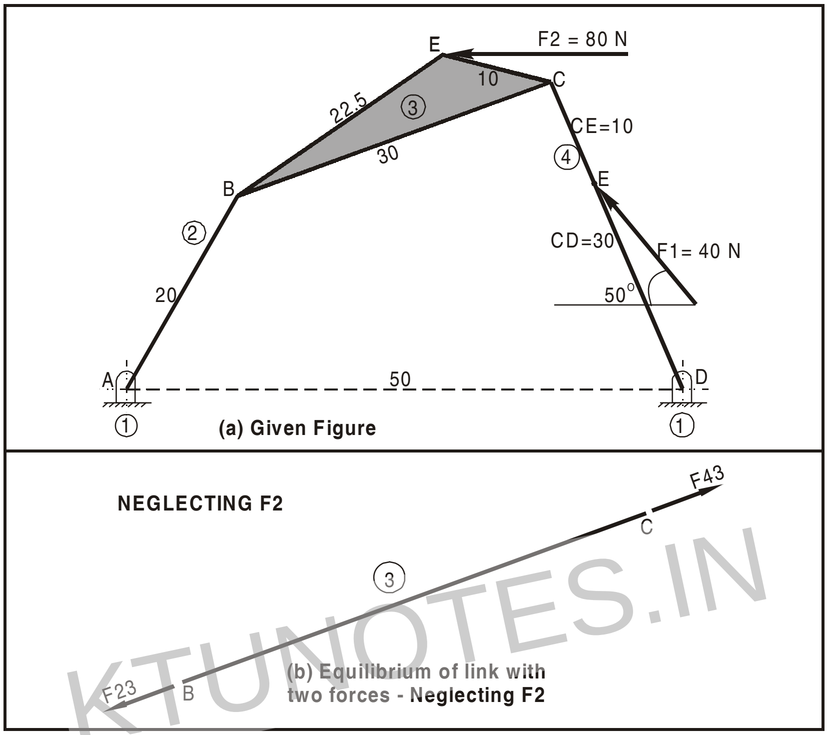
Solution:

If the mechanism is in static equilibrium, each of its members should also be in equilibrium individually.

- Member 4 is acted upon by three forces $\mathbf{F_1}$, $\mathbf{F_{34}}$ and $\mathbf{F_{14}}$
- Member 3 is acted upon by three forces $\mathbf{F_2}$, $\mathbf{F_{23}}$ and $\mathbf{F_{43}}$.
- Member 2 is acted upon by two forces $\mathbf{F_{32}}$ and $\mathbf{F_{12}}$ and a torque **T**.

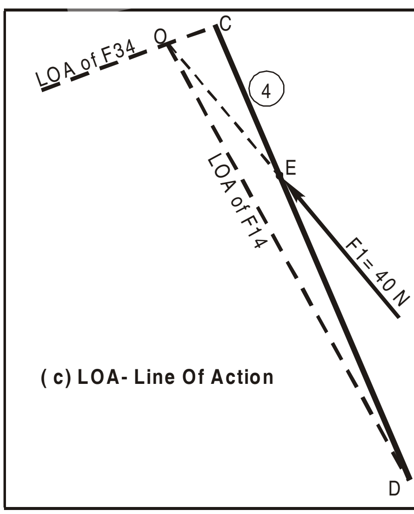
Graphical Solution by Superposition Method

(Fig. b and c) Neglecting force $\mathbf{F_2}$



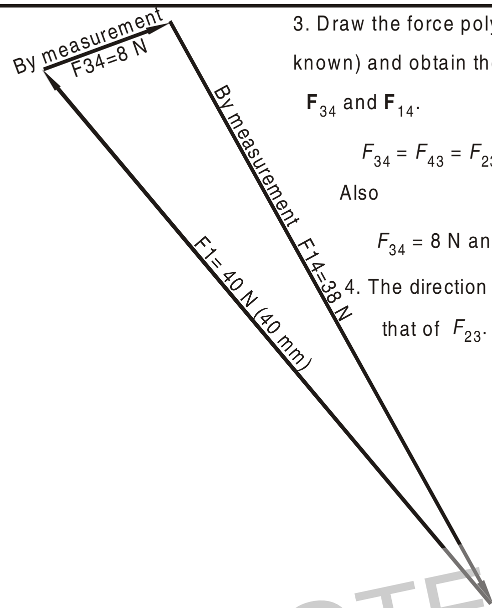
(a) Given Figure

(b) Equilibrium of link with two forces - Neglecting F2



(c) LOA- Line Of Action

1. Link 4 is a three-force member in which only one force $\mathbf{F_1}$ is known.
2. However, the line of action (LOA) of $\mathbf{F_{34}}$ can be obtained from the equilibrium of the link 3 which is a two-force member and is acted upon by forces $\mathbf{F_{23}}$ and $\mathbf{F_{43}}$. Thus, lines of action of forces $\mathbf{F_{43}}$ and $\mathbf{F_{34}}$ are along *BC*. If $\mathbf{F_1}$ and $\mathbf{F_{34}}$ intersect at *O*, then line of action of $\mathbf{F_{14}}$ will be along *OD* since the three forces are to be concurrent.



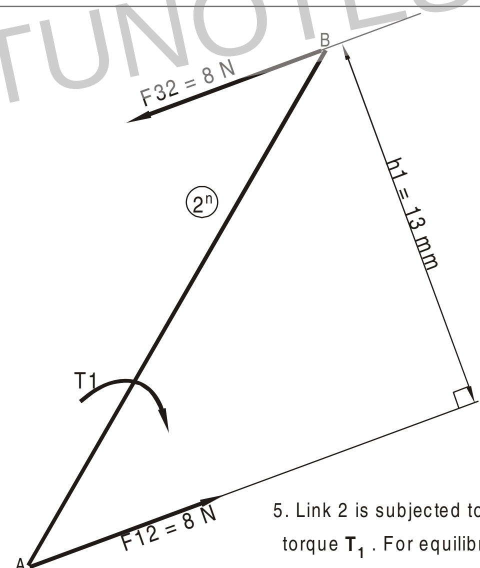
3. Draw the force polygon (**F**$_1$ is completely known) and obtain the magnitudes of forces **F**$_{34}$ and **F**$_{14}$.

$$F_{34} = F_{43} = F_{23} = F_{32} = 8 \text{ N}$$

Also

$$F_{34} = 8 \text{ N and } F_{14} = 38 \text{ N}$$

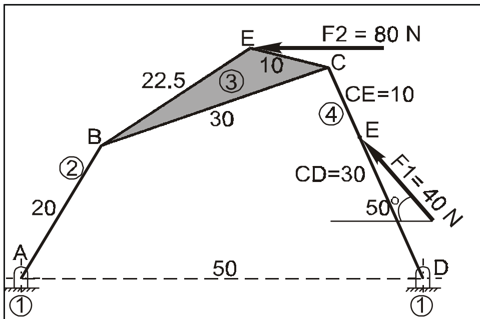
4. The direction of F_{32} is opposite to that of F_{23}.



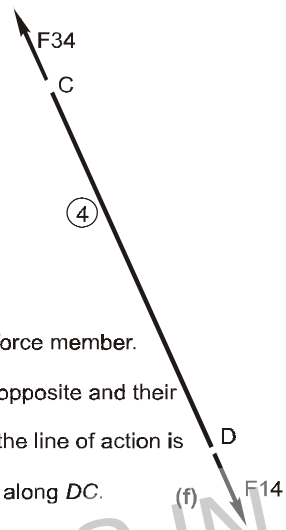
(e)

5. Link 2 is subjected to two forces and a torque **T**$_1$. For equilibrium, **F**$_{12}$ is equal, parallel and opposite to **F**$_{32}$.

$$T_1 = F_{32} \times h_1 = 8 \times 13 = 104 \text{ N.mm}$$
(clockwise)

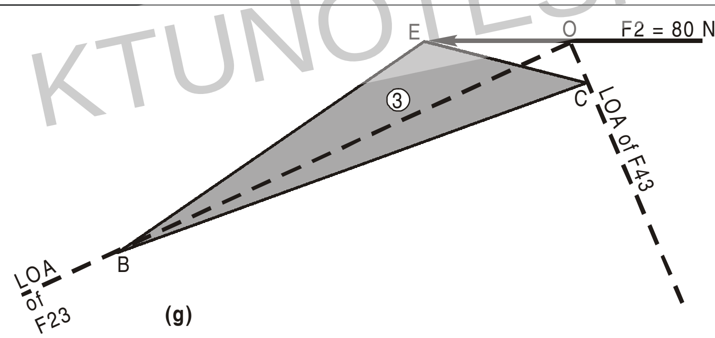


NEGLECTING F1



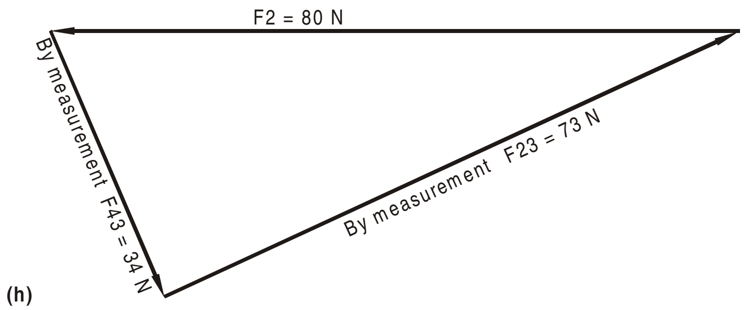
6. **(Fig. f) Neglecting force F_1:** Link 4 is a two-force member. The two forces $\mathbf{F_{14}}$ and $\mathbf{F_{34}}$ are to be equal and opposite and their line of action is to be the same which shows that the line of action is along *DC*. Thus, the line of action of $\mathbf{F_{43}}$ is also along *DC*.

(f)



(g)

7. **Fig.(g).** Link 3 is a three-force member in which $\mathbf{F_2}$ is completely known, only the direction of $\mathbf{F_{43}}$ is known (parallel to *DC*) and $\mathbf{F_{23}}$ is completely unknown. If the line of action of $\mathbf{F_2}$ and $\mathbf{F_{43}}$ meet at *O*, then the line of action of $\mathbf{F_{23}}$ will be along *OB* as the three forces are to be concurrent.



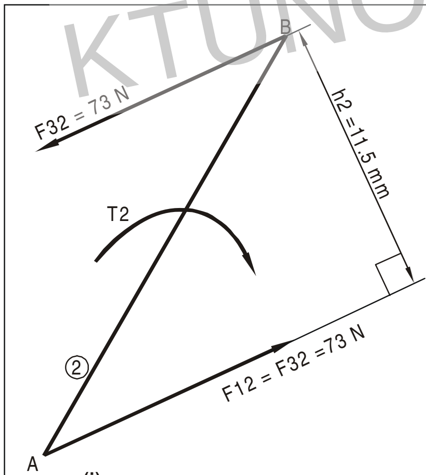
(h)

Draw the force polygon ($\mathbf{F_2}$ is completely known) by taking $\mathbf{F_2}$ to a suitable scale and two lines parallel to lines of action of $\mathbf{F_{23}}$ and $\mathbf{F_{43}}$. Mark arrowheads on $\mathbf{F_{23}}$ and $\mathbf{F_{43}}$ to know the directions. Refer Fig.(h)

$$F_{23} = 73 \text{ N and } F_{43} = 34 \text{ N}$$

and hence $\boldsymbol{F_{23} = F_{32} = 73}$ **N**

The direction of F_{32} is opposite to that of F_{23}.



(i)

Link 2 is subjected to two forces ($\mathbf{F_{12}}$ and $\mathbf{F_{32}}$) and a Torque T_2.

Refer Fig.(i): $\mathbf{F_{12}} = \mathbf{F_{32}} = 73$ N

For equilibrium, $\mathbf{F_{12}}$ is equal, parallel and opposite to $\mathbf{F_{32}}$.

$T_2 = F_{32} \times h_2 = 73 \times 11.5 = 839.5$ N.mm clockwise

Total torque

$= 104 + 839.5 = 943.5$ N.mm