MODULE 2

TRANSFORMATION OF POINTS & LINES

Presented by,

VINEETH.V
Asst. Professor
CCET

CAD- SYLABUS

MODULE-2

Transformation of points and line, 2-D rotation, reflection, scaling and combined transformation, homogeneous coordinates, 3-D scaling.
Shearing,rotation, reflection and translation, combined transformations, orthographic and perspective projections, reconstruction of 3-D objects.

CO-ORDINATE TRANSFORMATION

CO-ORDINATE TRANSFORMATION

- It means changing of an image from current position (state) to a new position (state) by applying certain rules.

  Current position (state) $\rightarrow$ New position (state)

- **Geometric transformation** are the transformations or changes in size, shape, location etc are accomplished by altering the coordinate descriptions of an object.

CO-ORDINATE TRANSFORMATION

Types of transformations are:

1. 2D Transformations
2. 3D transformations

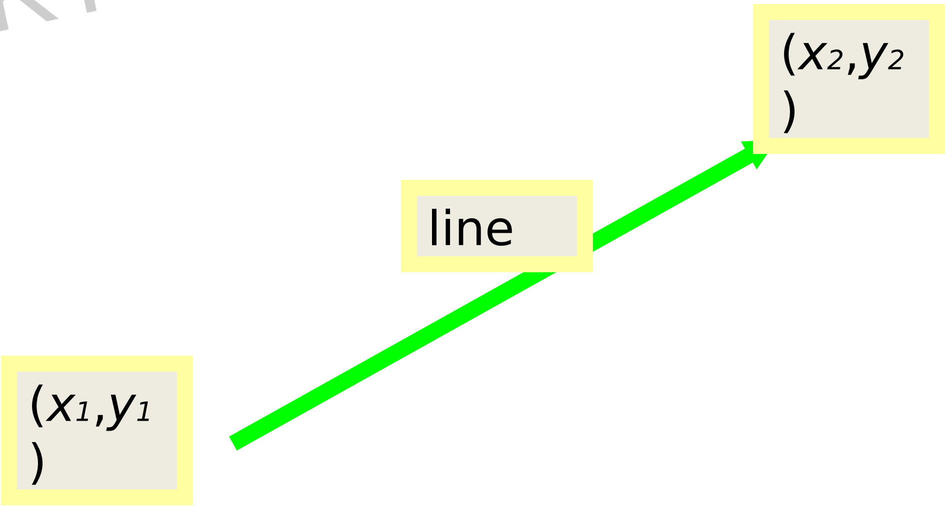
Basic Geometric transformations are:

3. Translation/ Move
4. Scaling
5. Rotation
6. Mirroring/ Reflection/ Flip
7. Shearing

2D TRANSFORMATION

2D TRANSFORMATION

- When transformation of coordinates takesplace on 2D plane or XY plane, it is called as **2D transformation.**



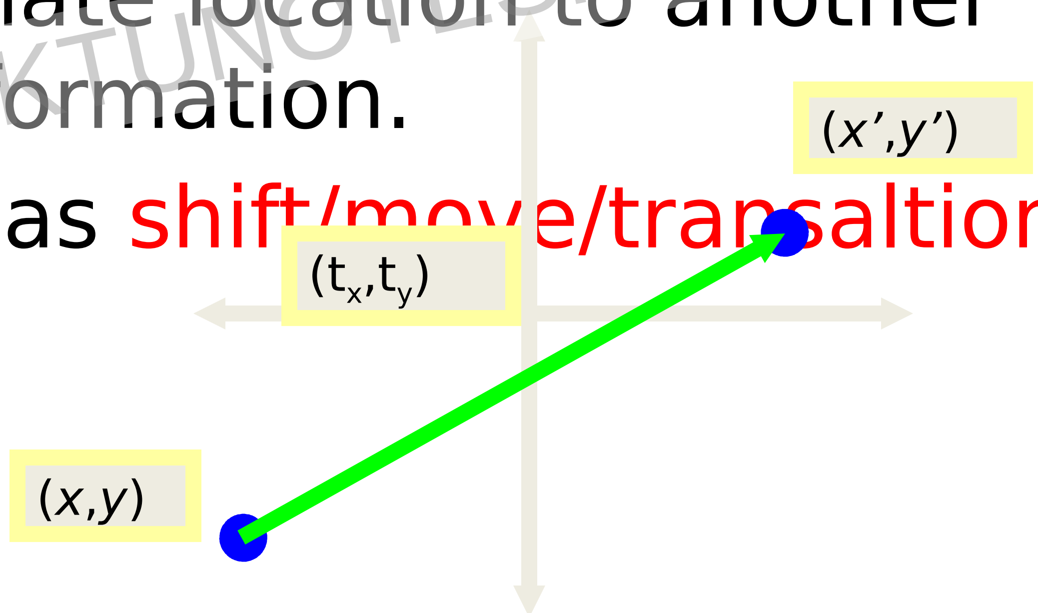
2D TRANSFORMATION

Basic 2D Geometric transformations are:

1. **2D Translation/ Move**
2. **2D Scaling**
3. **2D Rotation**
4. **2D Mirroring/ Reflection/ Flip**
5. **2D Shearing**

1. 2D Translation/ Move/Shift

- It is the repositioning or shifting an object along a straight-line path (**translation distances-** t_x, t_y) from one coordinate location to another without deformation.
- Also called as shift/move/transaltion.



2D Translation/ Move

- We translate a 2D point **by adding a translation distance t_x and t_y ,** to the orginal coordiante position (x,y) to move the point to a new position (*x'*,*y'*)

New position of x,

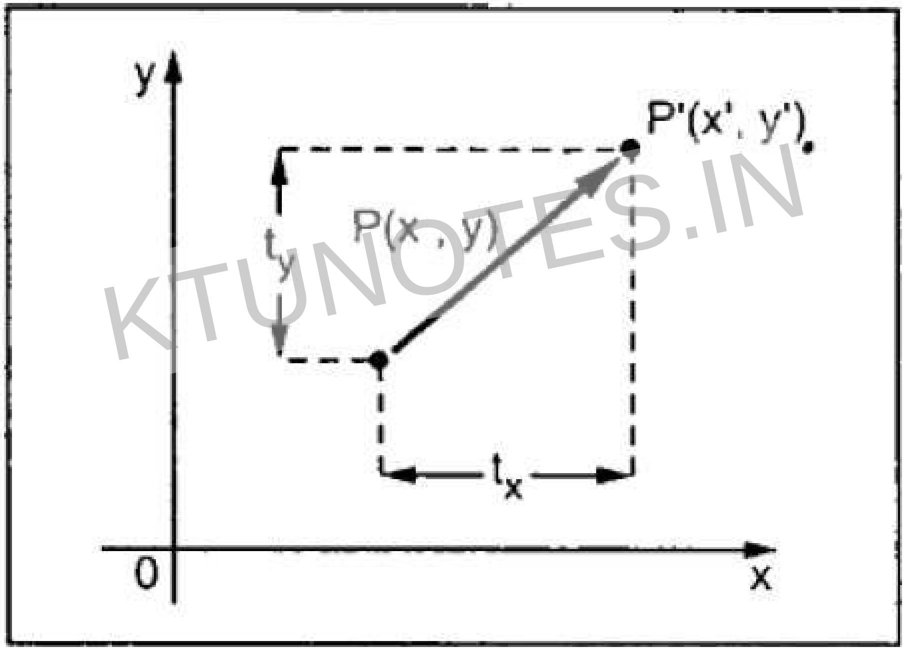
$$x' = x + t_x$$

New position of *y*,

$$y' = y + t_y$$

Where, **t_x and t_y are translation vector or shift vector**

2D Translation/ Move



2D Translations in Homogenised coordinates

- **Transformation matrices for 2D translation in 3x3 column matrix:**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + t_x$$

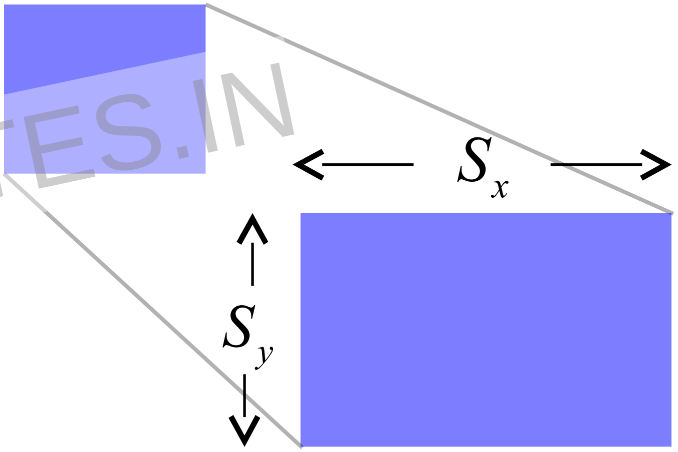
$$y' = y + t_y$$

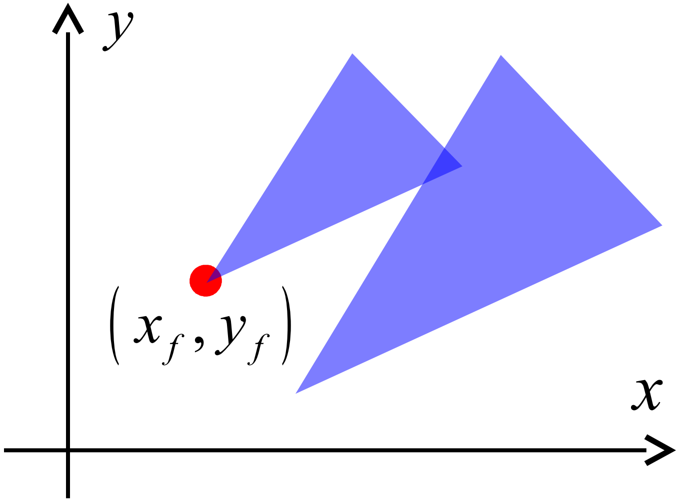
$$1 = 1$$

[P’] =[T] + [P]

2. <u>2D Scaling</u>

- It alters the size of an object (either reduced or enlarged size)





2D Scaling

- It is transformed **by multiplying** the current coordinate values (x,y) of each vertex by **Scaling factors S_x & S_y** to produce the new transformed coordinates (*x'*,*y'*)

New position of x,

$$x' = x \cdot S_x$$

New position of *y*,

$$y' = y \cdot S_y$$

Where, S_x & S_y are scaling factors

<u>2D Scaling-</u> <u>Scale factor [S]</u>

<u>Scale factor [S]</u> <u>value has only positive values</u> :

- Value less than 1 **(<1)** □ **Reduce** the size of object
- Value greater than 1 **(>1)** □ **Enlarge** the size of object
- Same value **(=1)** □ **Uniform scaling**
- Unequal value **(≠1)** □ **Differential scaling**

2D Scaling

- In matrix form,

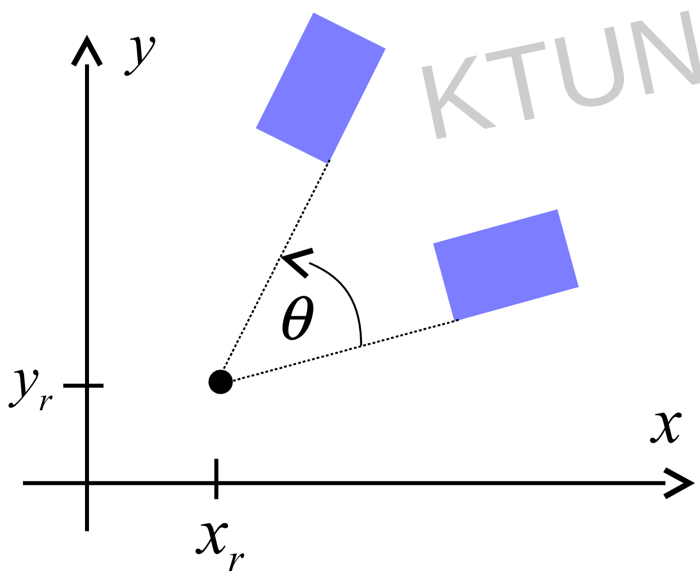
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

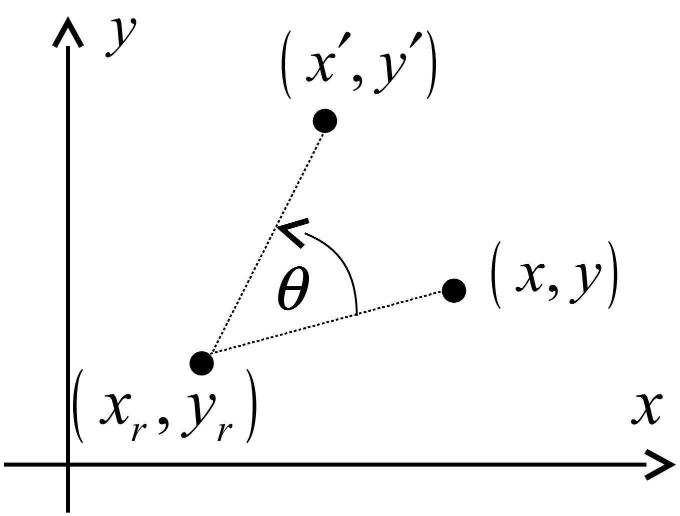
$$x' = x.s_x$$
$$y' = y.s_y$$

[P’] =[S] . [P]

3. 2D Rotation

- It is the repositioning of an object along a circular path in the xy plane.





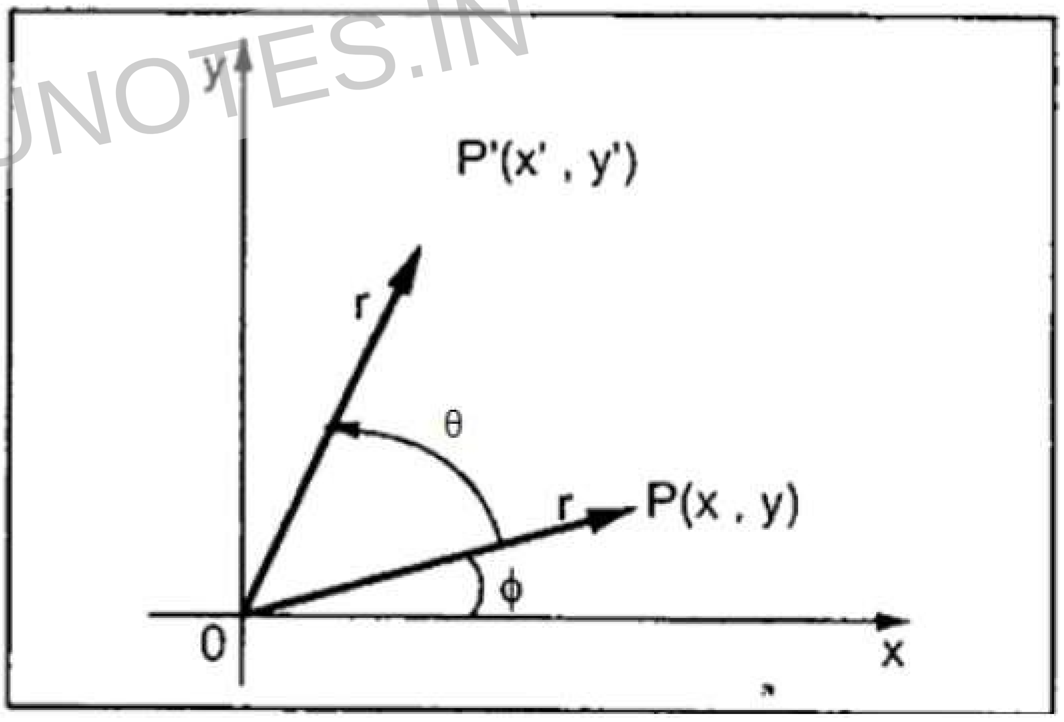
2D Rotation

- To generate a rotation, we specify a rotation angle θ and the position of the rotation point (pivot point) about which the object is to be rotated.
- Positive value for θ □counter clockwise rotation
- Negative value for θ □clockwise rotation

2D Rotation

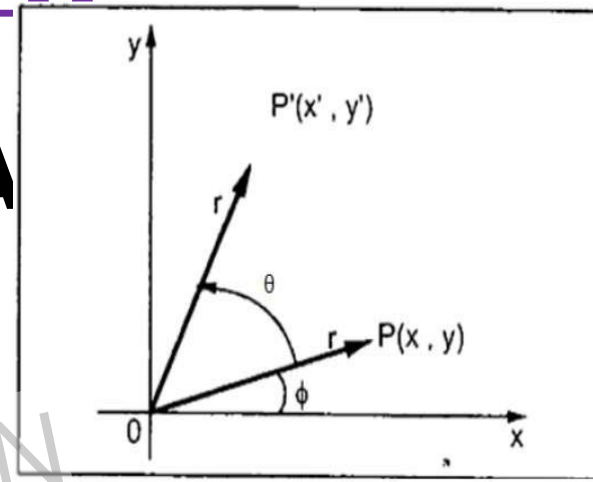
From the figure ,

---we have to find the new position **(x' , y')**



2D Rotation

Considering a **triangle OA**



Cos (θ + Φ) = **x’** / OP’

x’ = OP’ . Cos

= OP’ (Cosθ. CosΦ – Sinθ. SinΦ)

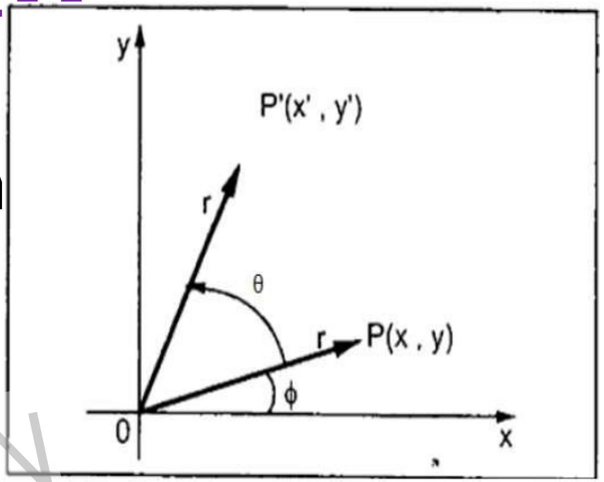
Similarly

Sin(θ + Φ) = **y’** / OP’

y’ = OP’ . Sin (θ + Φ)

= OP’ (Sinθ. CosΦ + Cosθ. SinΦ)

2D Rotation



Considering a **triangle OA**

CosΦ = **x** / OP

x = OP . CosΦ

Similarly

SinΦ = **y** / OP

y = OP .SinΦ

we know **OP = OP'**

So, **x' = x**Cosθ – **y**Sinθ

y' = x Sinθ + **y**Cosθ

2D Rotation

- **In matrix form,**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

[P'] = [R(θ)] . [P]

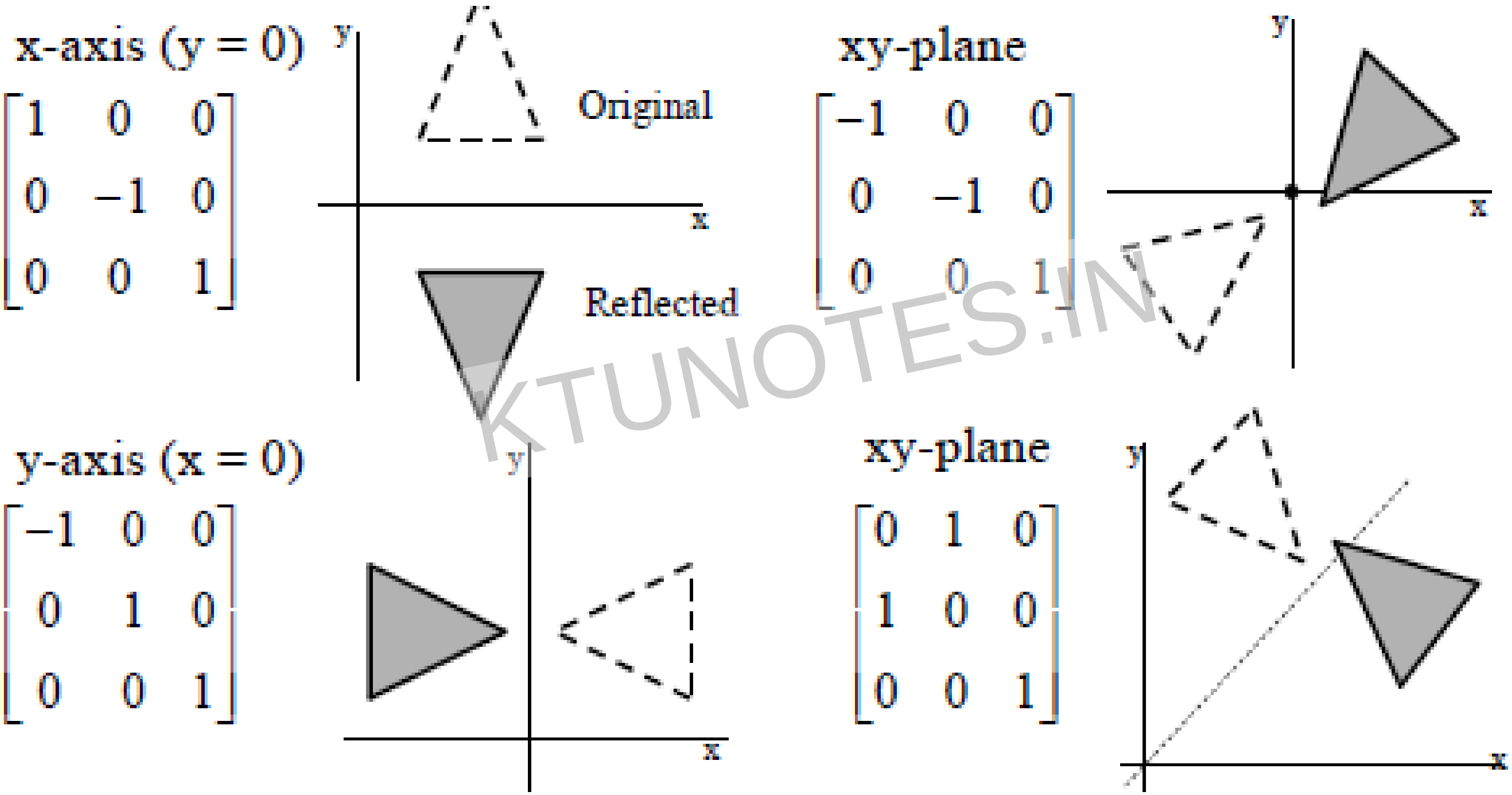
ie...

$$x' = x\text{Cos}\theta - y\text{Sin}\theta$$

$$y' = x\text{Sin}\theta + y\text{Cos}\theta$$

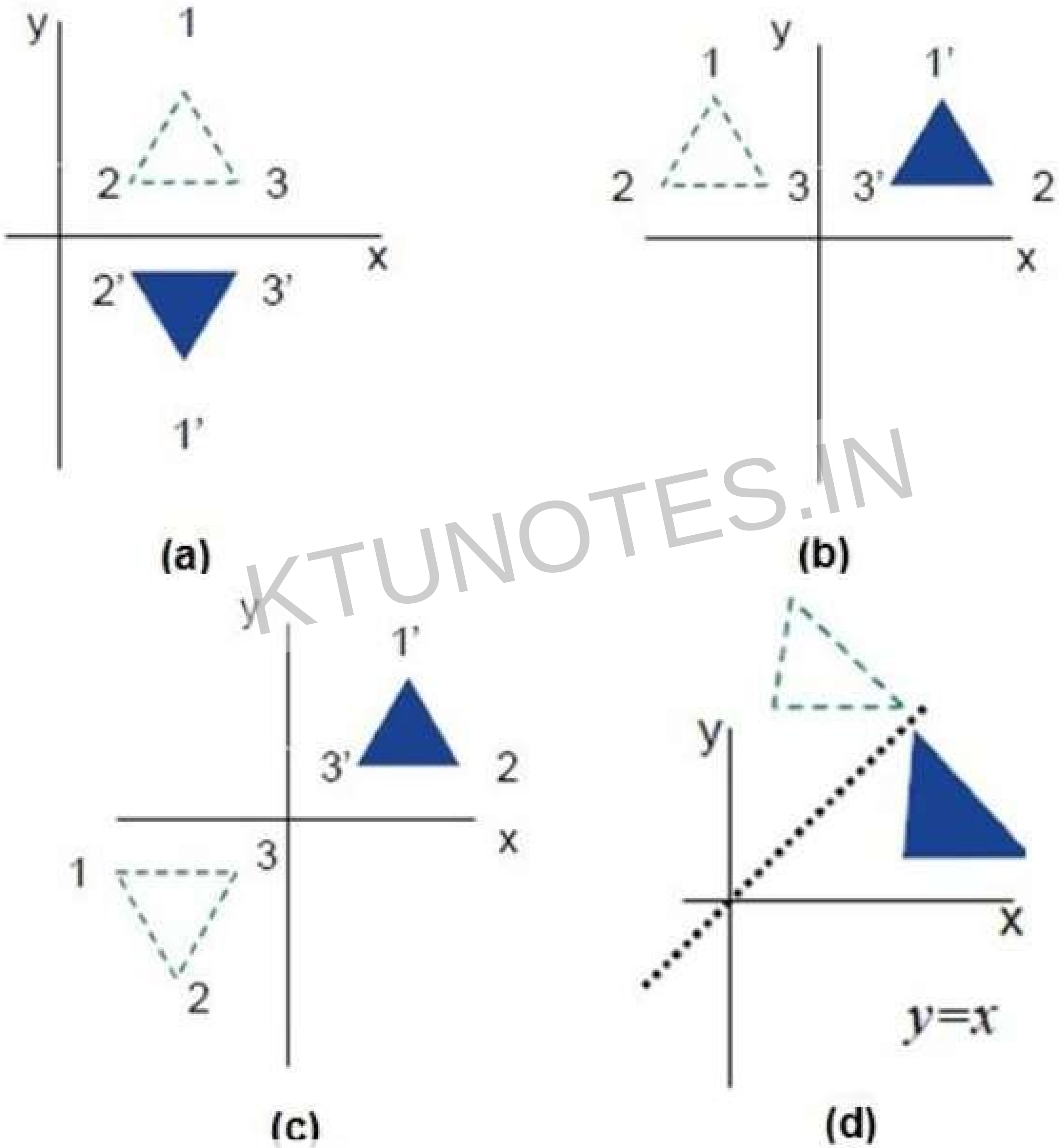
Where, **R(θ)** = **Rotation transformation operator**

4. 2D REFLECTION/ FLIP/MIRRORING



Reflection

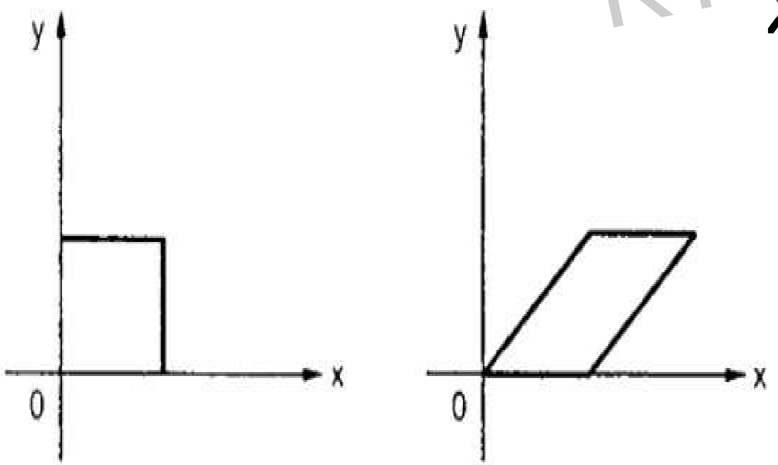
- A transformation produces a mirror image of an object.
- Axis of reflection
  - A line in the xy plane
  - A line perpendicular to the xy plane
  - The mirror image is obtained by rotating the object 180^0 about the reflection axis.
- Rotation path
  - Axis in xy plane: in a plane perpendicular to the xy plane.
  - Axis perpendicular to xy plane: in the xy plane.


y
1
2
3
x
2'
3'
1'
(a)
y
1
1'
2
3
3'
2
x
(b)
y
1'
3'
2
x
3
1
2
(c)
y
X
y=x
(d)

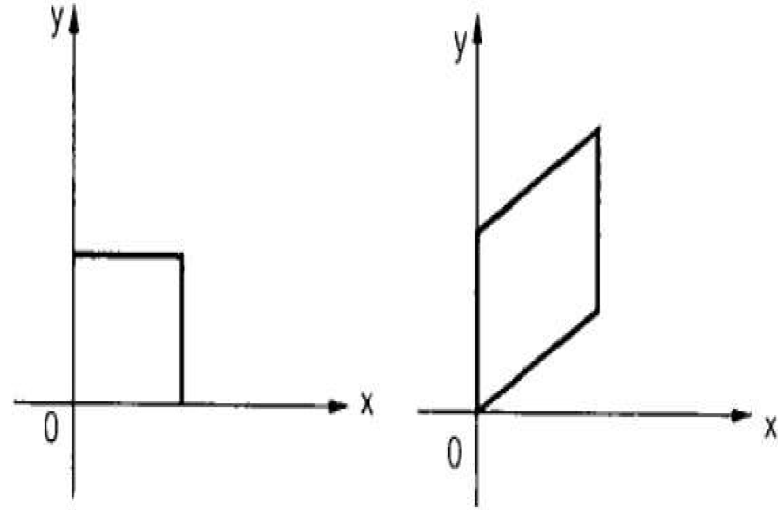
5. 2D SHEARING

- Shearing transformation is the transformation which alters the shape of an object.
- Deformation of shape of object

x and y



(a) Original object (b) Object after x shear



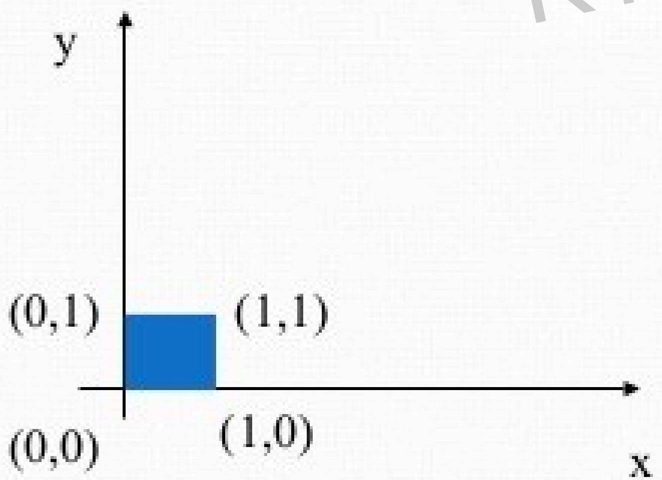
(a) Original object (b) Object after y shear

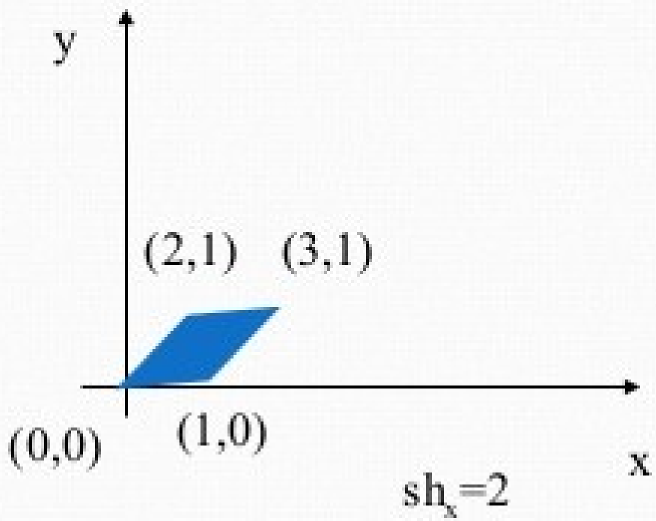
X shear

- Preserve Y coordinates but change the X coordinates values

$$x' = x + sh_x \cdot y$$
$$y' = y$$

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





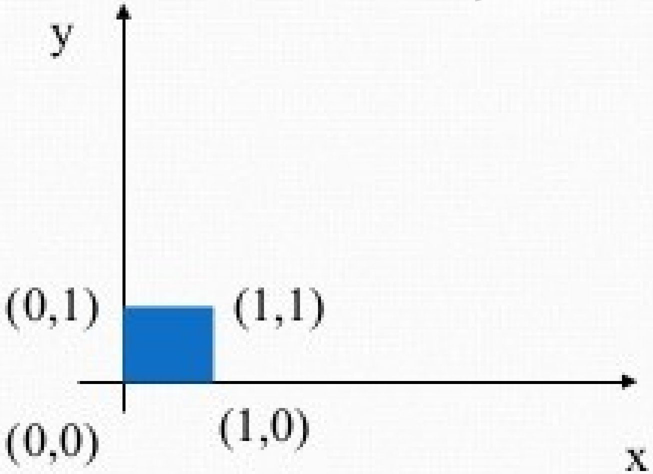
Y shear

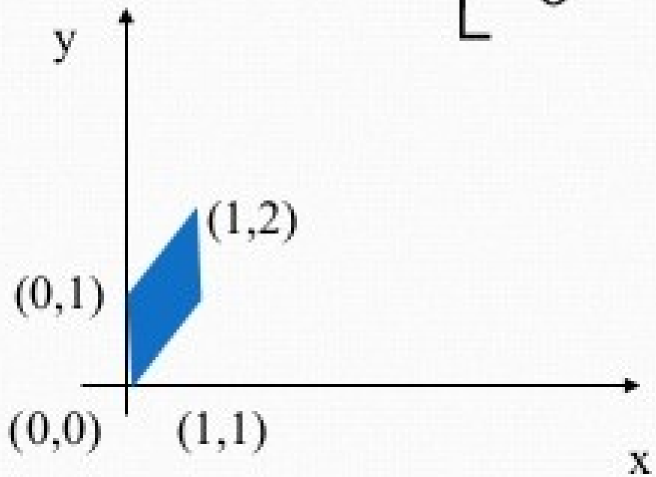
- Preserve X coordinates but change the Y coordinates values

$$x' = x$$

$$y' = y + Sh_y \cdot x$$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





$$SH_x = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad SH_y = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Various 2D TRANSFORMATIONs

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shearing

2D Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{T}(t_x, t_y) \times \mathbf{P}$$

2D Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{R}(\theta) \times \mathbf{P}$$

2D Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{S}(S_x, S_y) \times \mathbf{P}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

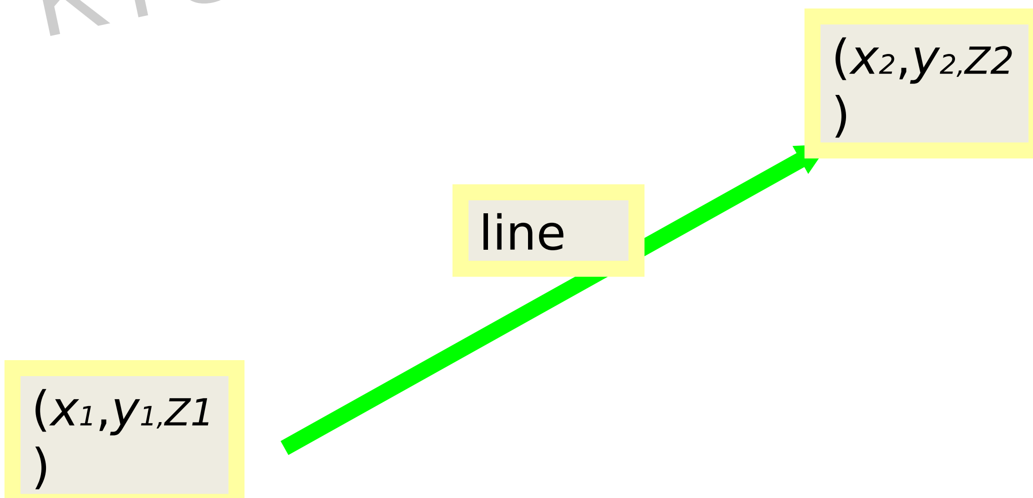
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D TRANSFORMATION

3D TRANSFORMATION

- When transformation of coordinates takesplace on 3D plane or XYZ plane, it is called as **3D transformation.**



3D TRANSFORMATION

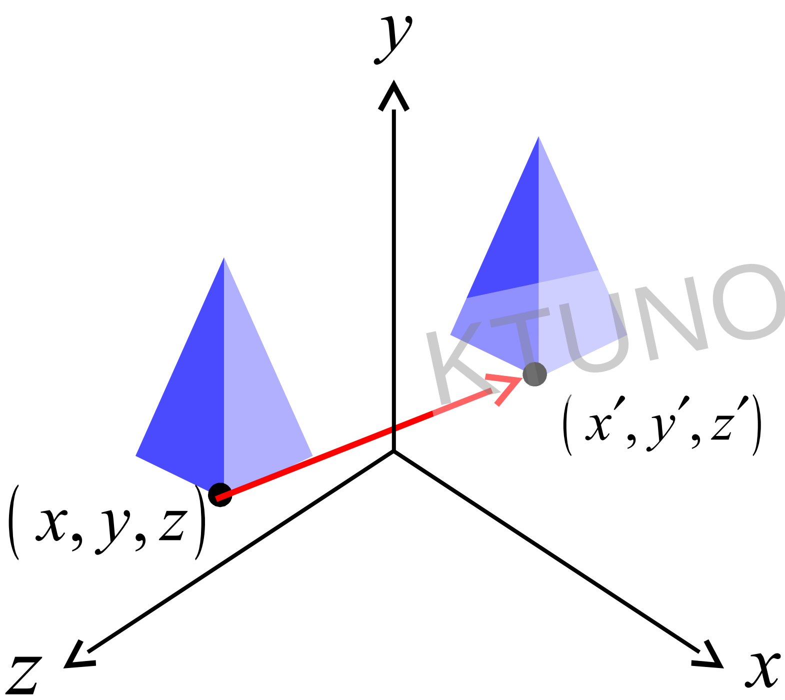
Basic 3D Geometric transformations are:

1. 3D Translation/ Move
2. 3D Scaling
3. 3D Rotation
4. 3D Mirroring/ Reflection/ Flip
5. 3D Shearing

1. 3D TranSlation

.

•**Moving of object in x,y,z direction as translation vector tx,ty,tz respectively**



$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

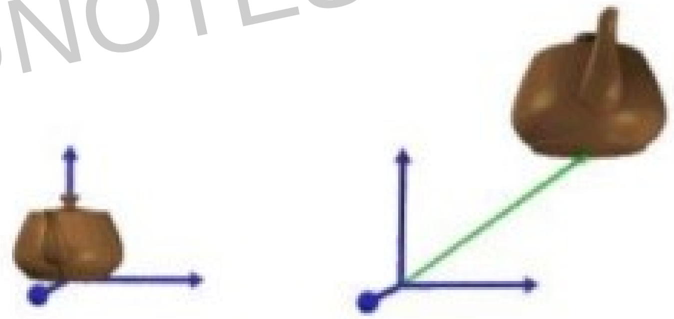
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D TRANSLATION

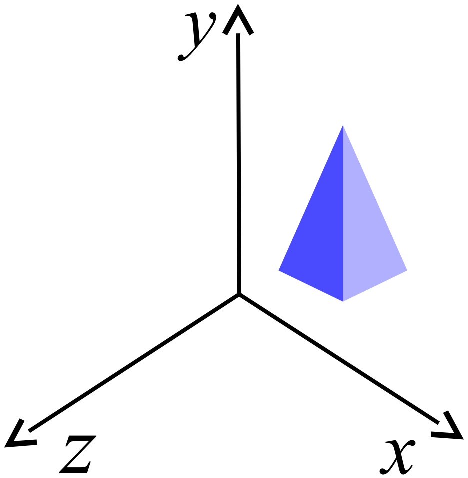
- The matrix representation is equivalent to the three equation.

$$x' = x + t_x\ ,\ \ y' = y + t_y\ ,\ \ z' = z + t_z$$

Where parameter t_x, t_y, t_z are specifying translation distance for the coordinate direction x , y , z are assigned any real value.



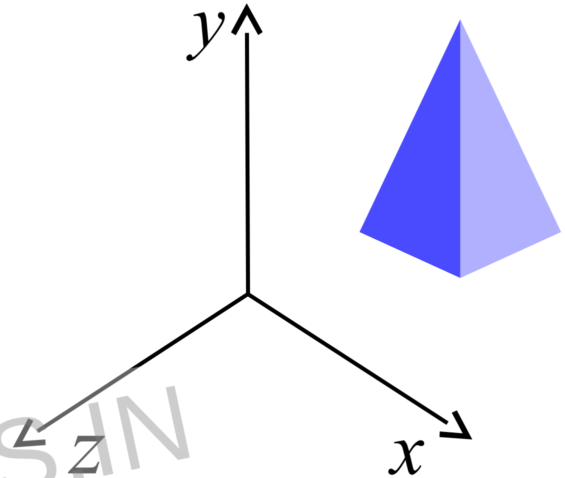
2. 3D Scaling



$$x' = x \times S_x$$

$$y' = y \times S_y$$

$$z' = x \times S_z$$



Enlarging object also moves it from origin

$$\mathbf{P}' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \mathbf{S} \cdot \mathbf{P}$$

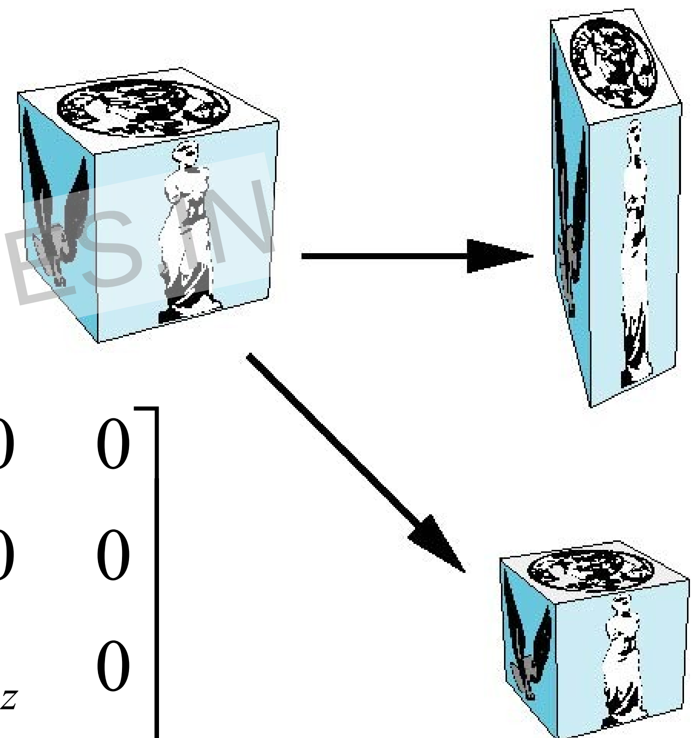
Scaling

g size of object in x,y,z direction as scaling vector sx,sy,sz respectively

$x' = s_x x$
$y' = s_y y$
$z' = s_z z$

p'=Sp

$$\mathbf{S} = \mathbf{S}(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Angel: Interactive Computer Graphics 3E © Addison-Wesley

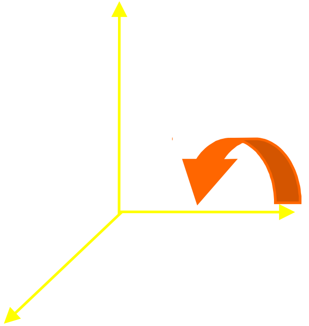
3. 3D Rotation

- ROTATION at x,y,z direction at rotating angle about a fixed pivot point.
- Need to specify which axis the rotation is about.

Rotation about z-axis

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating About the x-axis $R_x(\theta)$



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

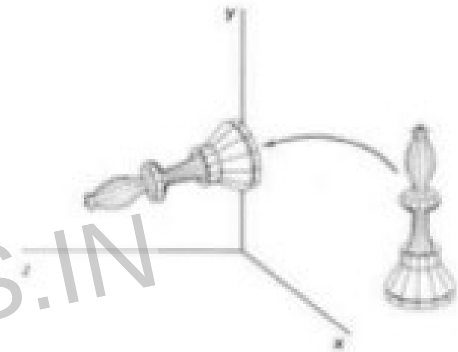
X-AXIS ROTATION

The equation for X-axis rotation

$x' = x$

$y' = y\cos\theta - z\sin\theta$

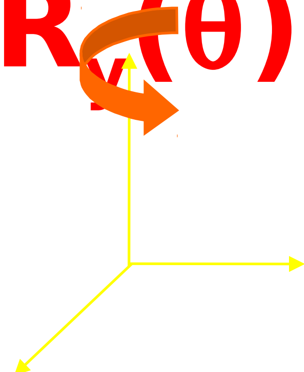
$z' = y\sin\theta + z\cos\theta$



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotating About the y-axis

$R_y(\theta)$



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Y-AXIS ROTATION

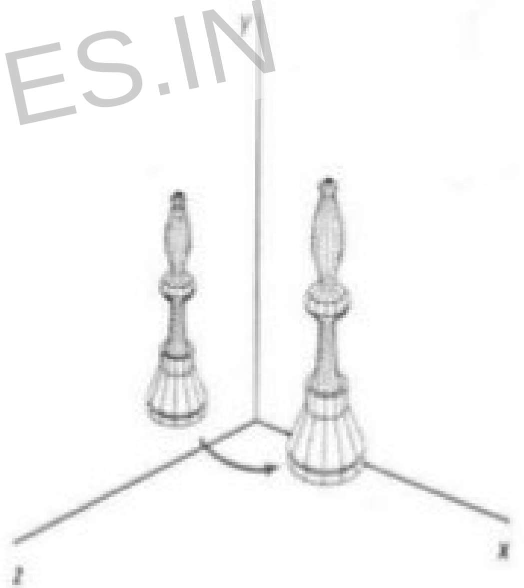
The equation for Y-axis rotaion

$x' = x\cos\theta + z\sin\theta$

$y' = y$

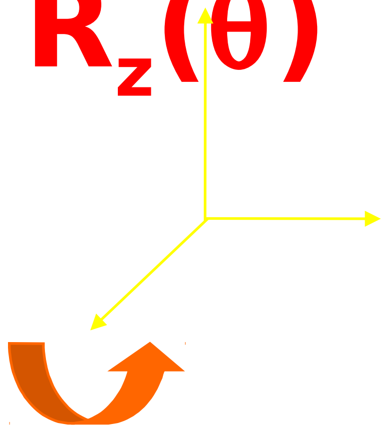
$z' = z\cos\theta - x\sin\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Rotation About the z-axis

$R_z(\theta)$



$$\begin{pmatrix} x' \\ y' \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation in 3D

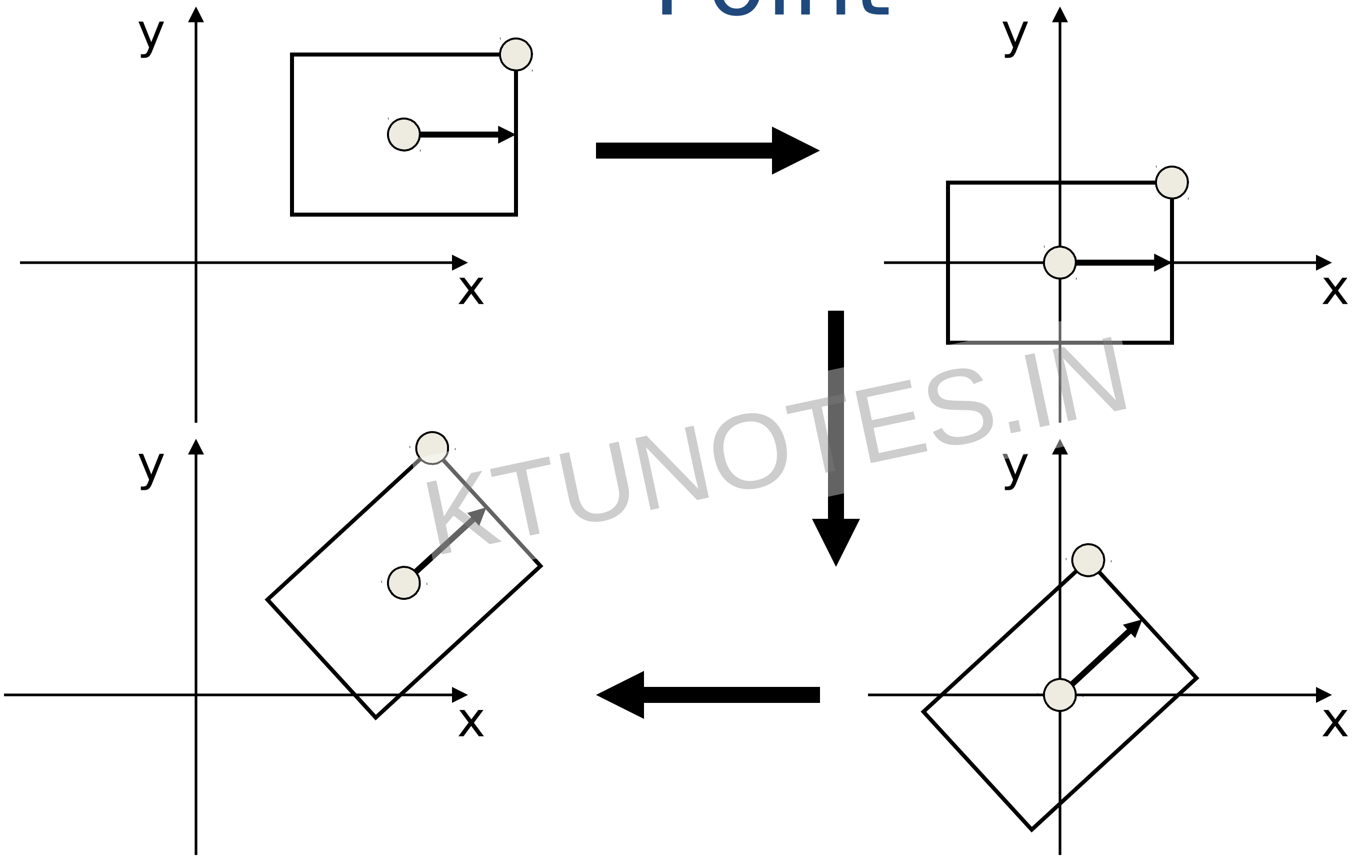
- For rotation about the x and y axes:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating About An Arbitrary Point

- What happens when you apply a rotation transformation to an object that is not at the origin?
- Solution:
  - Translate the center of rotation to the origin
  - Rotate the object
  - Translate back to the original location

Rotating About An Arbitrary Point



Rotation about x and y axes

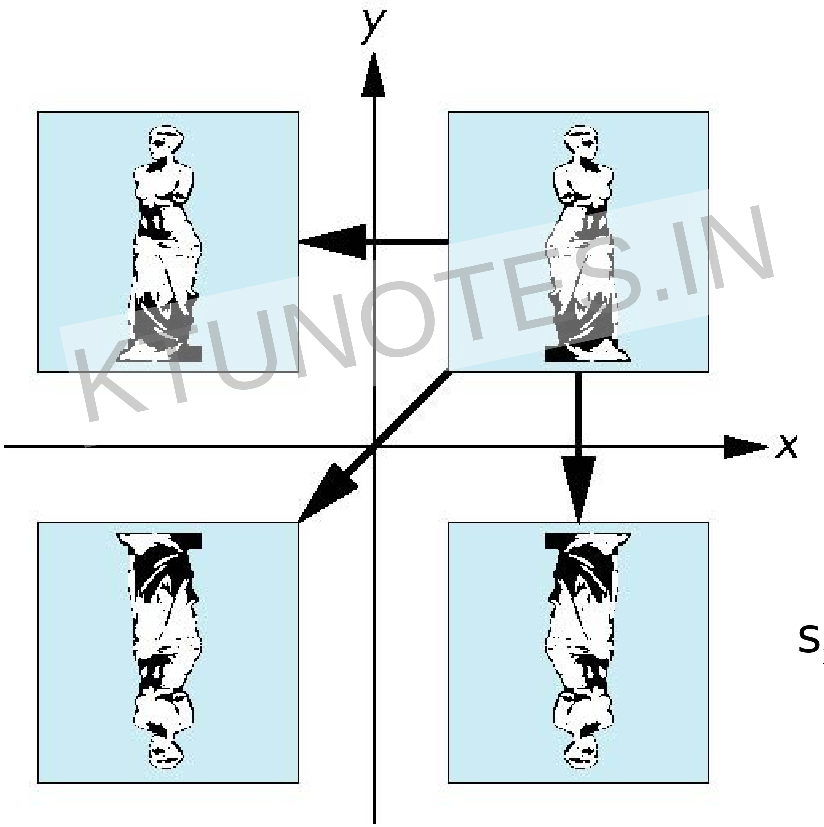
- Same argument as for rotation about *z* axis
  - For rotation about x axis, x is unchanged
  - For rotation about y axis, y is unchanged

$$\mathbf{R} = \mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R} = \mathbf{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. **3D Reflection**

- Mirroring of object along x, y or z axis



$s_x = -1 \; s_y = 1$

original

$s_x = -1 \; s_y = -1$

$s_x = 1 \; s_y = -1$

Angel: Interactive Computer Graphics 3E © Addison-Wesley

3D Reflection

- Reflection or mirror matrix: a scaling matrix where one scaling factor is –1 and two others are 1 *or* all of the three scaling factors are –1.
- If two are –1, it's a 180° rotation.

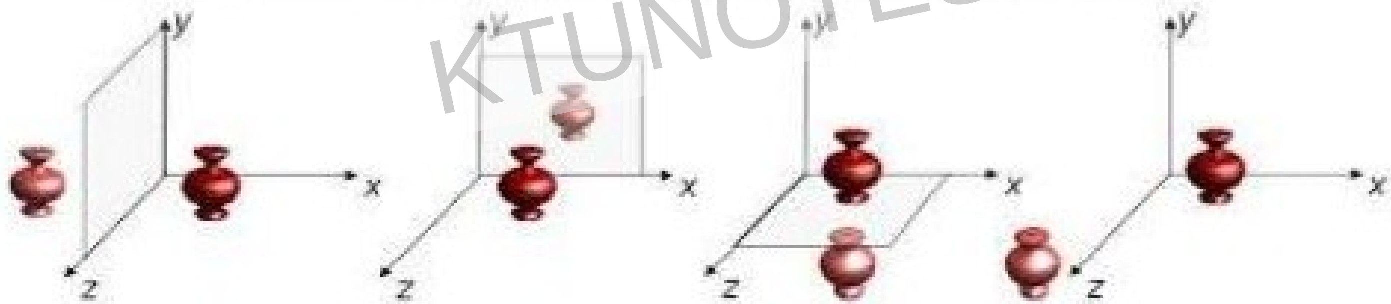
$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

reflection wrt yz-plane	reflection wrt xz-plane	reflection wrt xy-plane	reflection wrt the origin

3D REFLECTION

Reflection

- Reflection over planes, lines or points



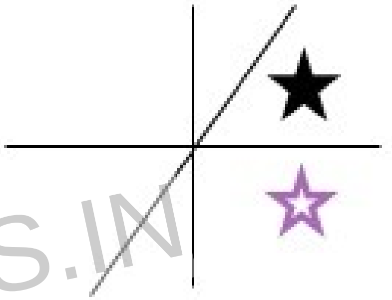
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D REFLECTION

- Reflection about x-axis:-

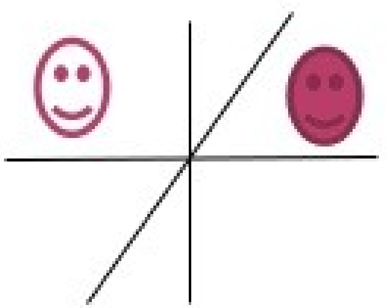
x'=x y'=-y z'=-z

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$



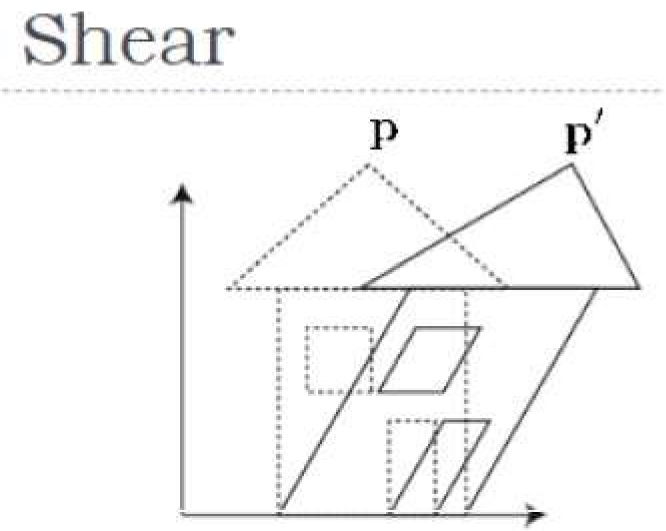
Reflection about y-axis:-

y'=y x'=-x z'=-z



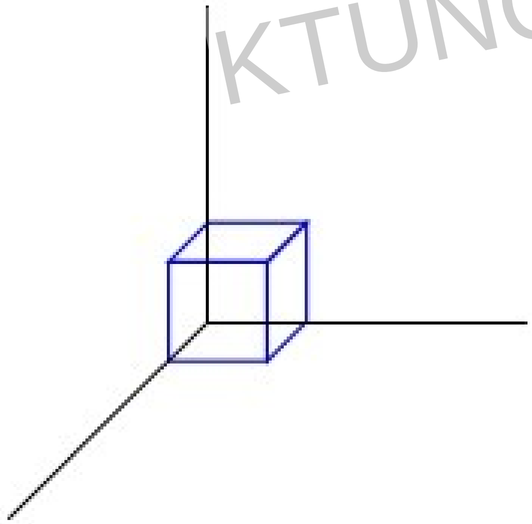
5. **3D Shearing**

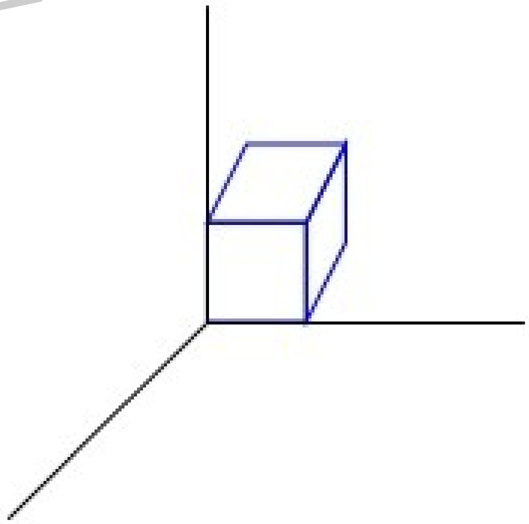
- Shearing transformation is the transformation which alters the shape of an object.
- Deformation of shape of object takesplace in x,y and z direction



3D SHEARING

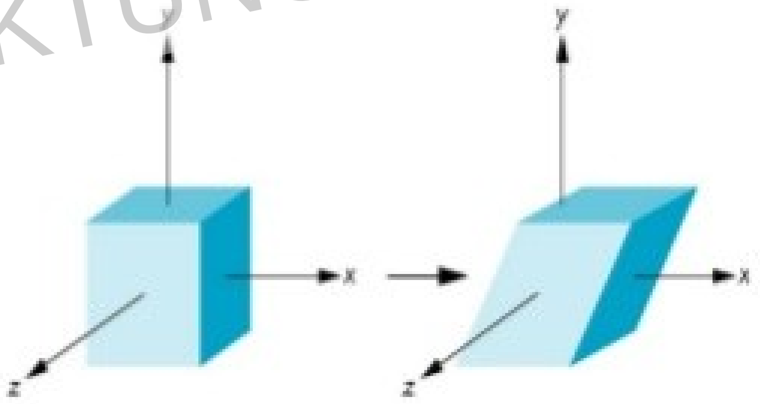
E.g. draw a cube (3D) on a screen (2D) Alter the values for **x** and **y** by an amount proportional to the distance from z_{ref}





Shear

- Let pull in top right edge and bottom left edge
  - Neither y nor z are changed
  - Call *x* shear direction



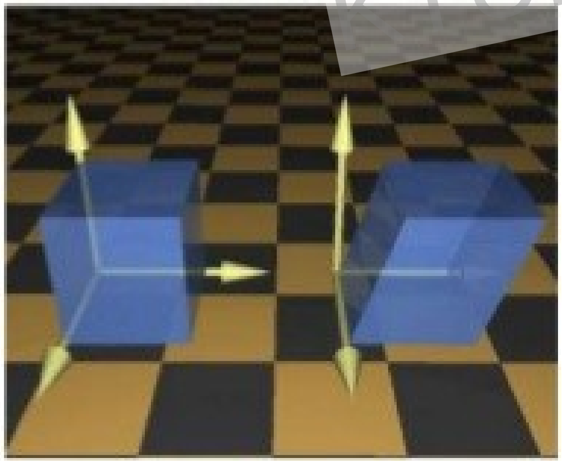
Shear

Surivong L.

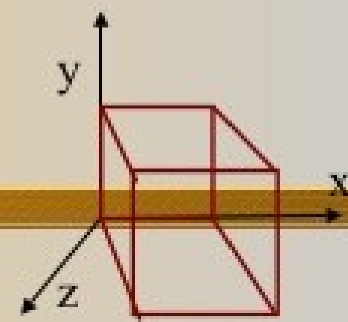
3D SHEARING

- Matrix for 3d shearing
- Where a and b can
  Be assigned any real
  Value.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Shear along Z-axis



$$SH_{xy}(sh_x, sh_y) \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + z * sh_x \\ y + z * sh_y \\ z \\ 1 \end{bmatrix}$$

Other Transformations : SHEARING

X-axis 3-D Shear transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-axis 3-D Shear transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Combined transformation

- It is the transformation of different types of geometric transformation like:
  1. Translate or move object by 2 units in x,y and z direction
  2. Scaling it into 2 units in x,y and z direction
  3. Rotating the points at rotating angle.

Combined transformation

- 2 methods are:

1. **Direct method**
   - **Step by step method of combined transformation**

   1. $[\bar{x}] = [T].[X]$
   2. $[\bar{x}] = [S].[X]$
   3. $[\bar{x}] = [R(\theta)].[x]$

2. **Concatenation method**

2. Concatenation transformation

- We can form arbitrary affine transformation matrices by multiplying together translation,scaling & rotation matrices

$$[\bar{x}] = [Tc]. [X]$$

Where, Tc= concatenation transformation= **[T]. [S]. [R(**θ)**].**

Matrix Composition

- Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$p' \quad = \quad \mathrm{T}(tx,\ ty) \qquad \mathrm{R}(\theta) \qquad \mathrm{S}(sx,\ sy) \qquad p$$

– Matrix multiplication is associative

$$\mathbf{p}' = (\mathbf{T} \times (\mathbf{R} \times (\mathbf{S} \times \mathbf{p}))) \Longrightarrow \mathbf{p}' = (\mathbf{T} \times \mathbf{R} \times \mathbf{S}) \times \mathbf{p}$$

Matrix concatenation properties

- Multiplication is associative

$$\mathbf{M}_3 \cdot \mathbf{M}_2 \cdot \mathbf{M}_1 = (\mathbf{M}_3 \cdot \mathbf{M}_2) \cdot \mathbf{M}_1 = \mathbf{M}_3 \cdot (\mathbf{M}_2 \cdot \mathbf{M}_1)$$

- Multiplication is NOT commutative
  - Unless the sequence of transformations are all of the same kind
  - M_2M_1 is not equal to M_1M_2 in general

Homogeneous transformation (w or h)

2D

- It is the conversion of 2x2 matrices of points P(x,y) to 3x3 matrices of point P(x,y,1) in 2D

3D

- It is the conversion of 3x3 matrices of points P(x,y,z) to 4x4 matrices of point P(x,y,z,1)

2D Translations in Homogenised coordinates

- **Transformation matrices for 2D translation in 3x3 column matrix:**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + t_x$$
$$y' = y + t_y$$
$$1 = 1$$

[P’] =[T] + [P]

Matrix representations and homogeneous coordinates

- Multiplicative and translational terms for a 2D transformation can be combined into a single matrix
- This expands representations to 3x3 matrices
  - Third column is used for translation terms
- Result: All transformation equations can be expressed as matrix multiplications
- Homogeneous coordinates: (x_h, y_h, h)
  - Carry out operations on points and vectors "homogeneously"
  - h: Non-zero homogeneous parameter such that

$$x = \frac{x_h}{h}, \qquad y = \frac{y_h}{h}$$

  - We can also write: (hx, hy, h)
  - h=1 is a convenient choice so that we have (x, y, 1)
  - Other values of h are useful in 3D viewing transformations

Homogeneous Coordinates

The homogeneous coordinates form for a three dimensional point [x y z] is given as

$\mathbf{p} = [x' \; y' \; z' \; w]^T = [wx \; wy \; wz \; w]^T$

We return to a three dimensional point (for $w \neq 0$) by

$x \leftarrow x'/w$

$y \leftarrow y'/w$

$z \leftarrow z'/w$

$$P = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x/w \\ y/w \\ z/w \\ 1 \end{pmatrix}$$

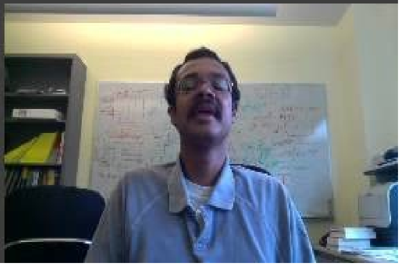
If $w=0$, the representation is that of a vector

Note that homogeneous coordinates replaces points in three dimensions by lines through the origin in four dimensions

For w=1, the representation of a point is [x y z 1]

Advantages of Homogeneous Coords

- Unified framework for translation, viewing, rot…
- Can concatenate any set of transforms to 4x4 matrix
- No division (as for perspective viewing) till end
- Simpler formulas, no special cases
- Standard in graphics software, hardware



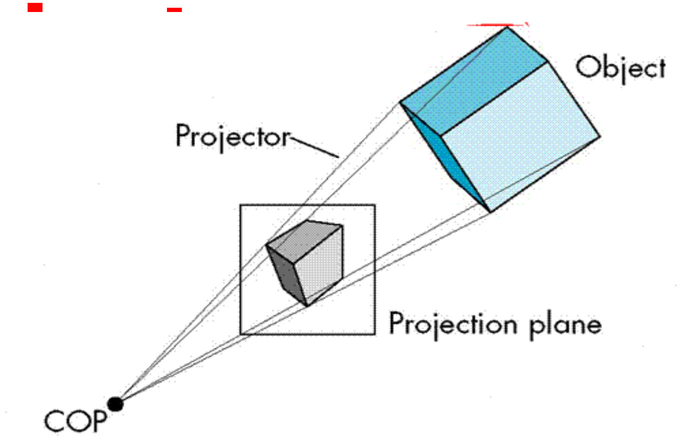
PROJECTIONS

PROJECTIONS

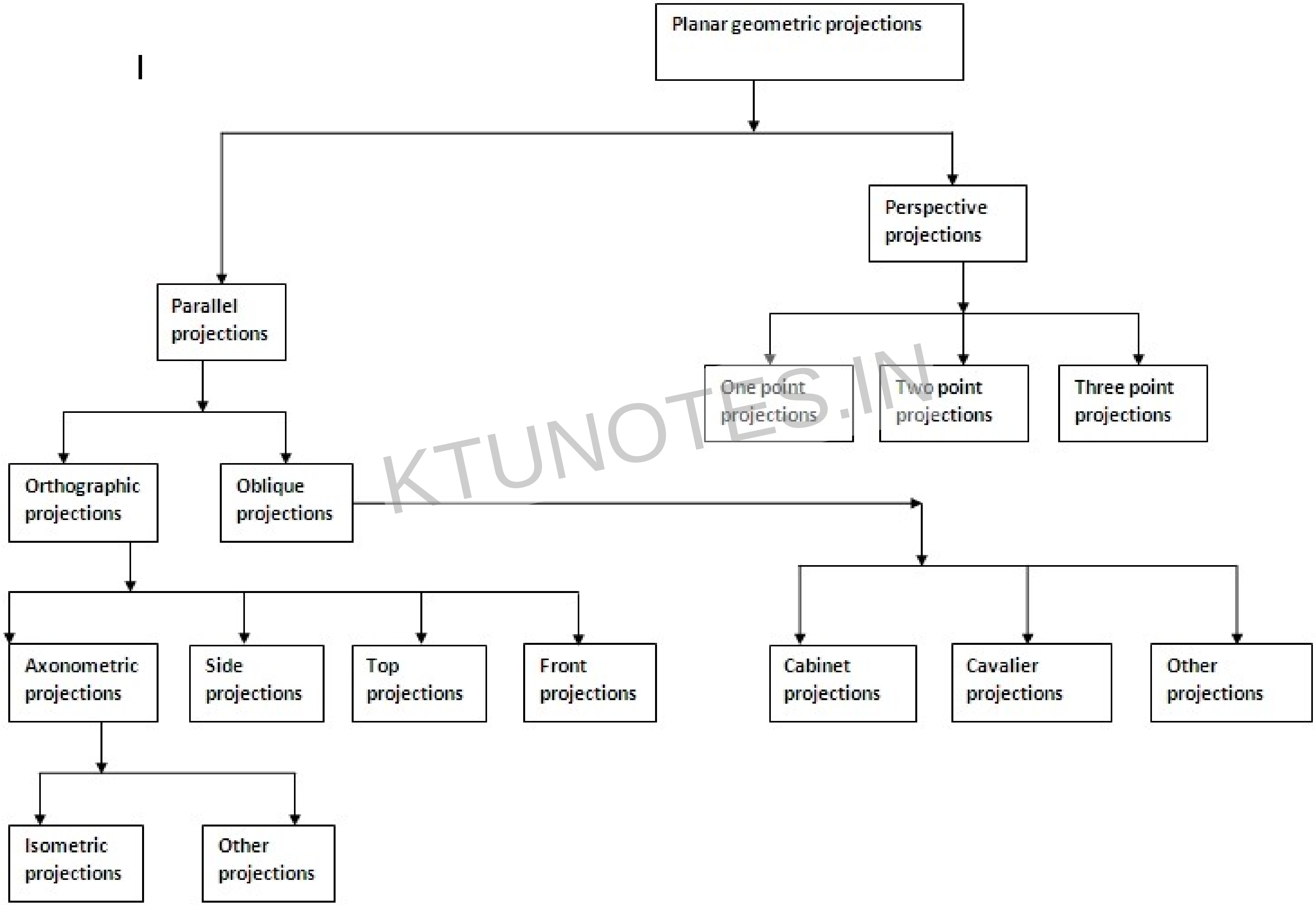
- Once the world coordinate description of the object in a scene are converted to viewing coordinates, we can project 3D object onto 2D view plane.

project

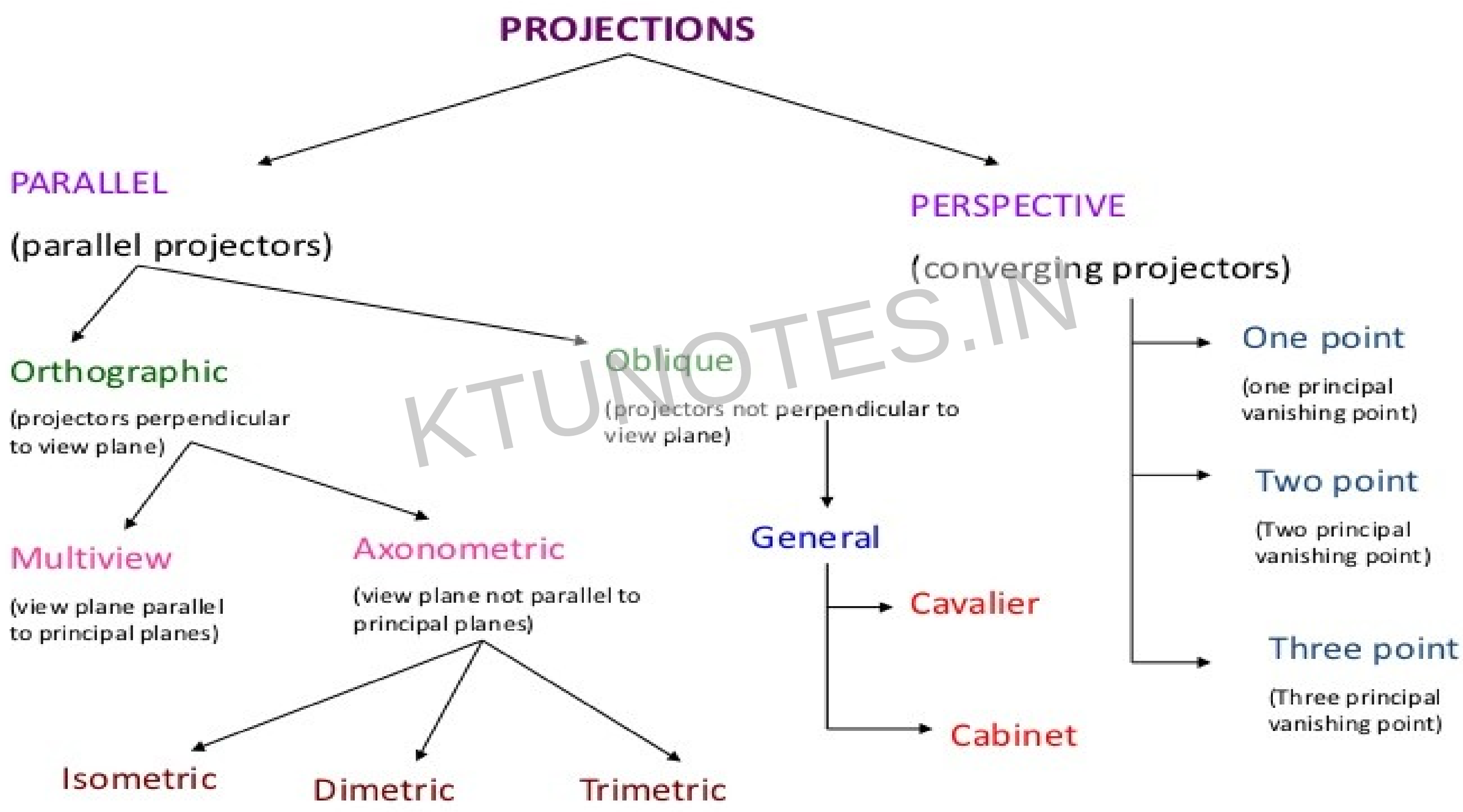
3D ob **> 2D view plane**



TYPES OF PROJECTIONS



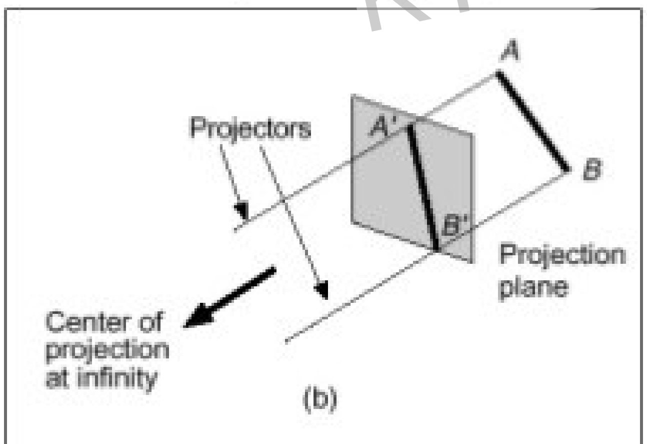
TYPES OF PROJECTIONS



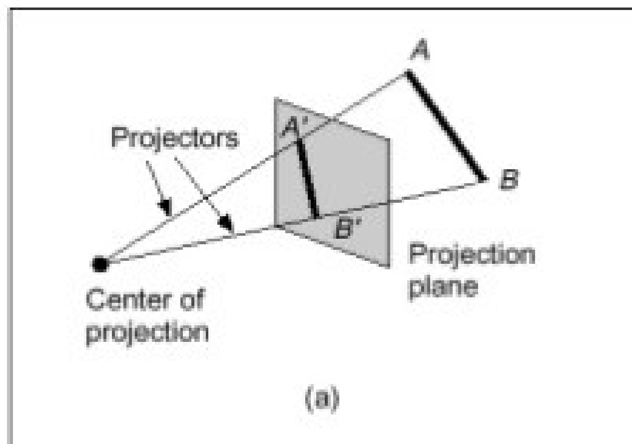
Types Of Projections

There are two broad classes of projection:

- Parallel: Typically used for architectural and engineering drawings
- Perspective: Realistic looking and used in computer graphics



Parallel Projection

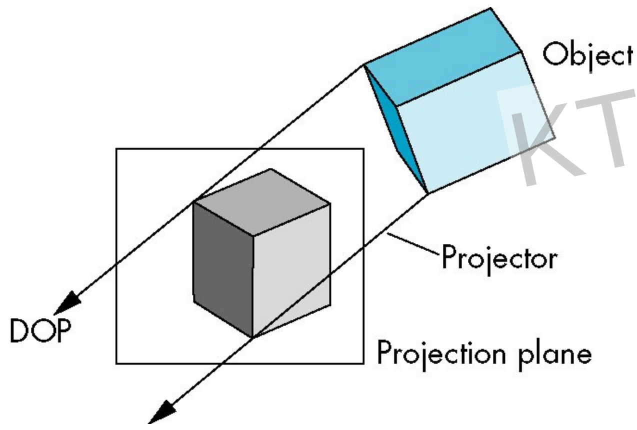


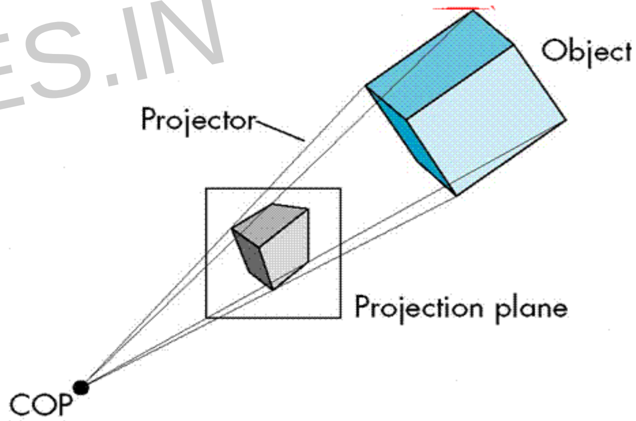
Perspective Projection

PARALLEL PROJECTION VS PERSPECTIVE PROJECTION

PARALLEL PROJECTION

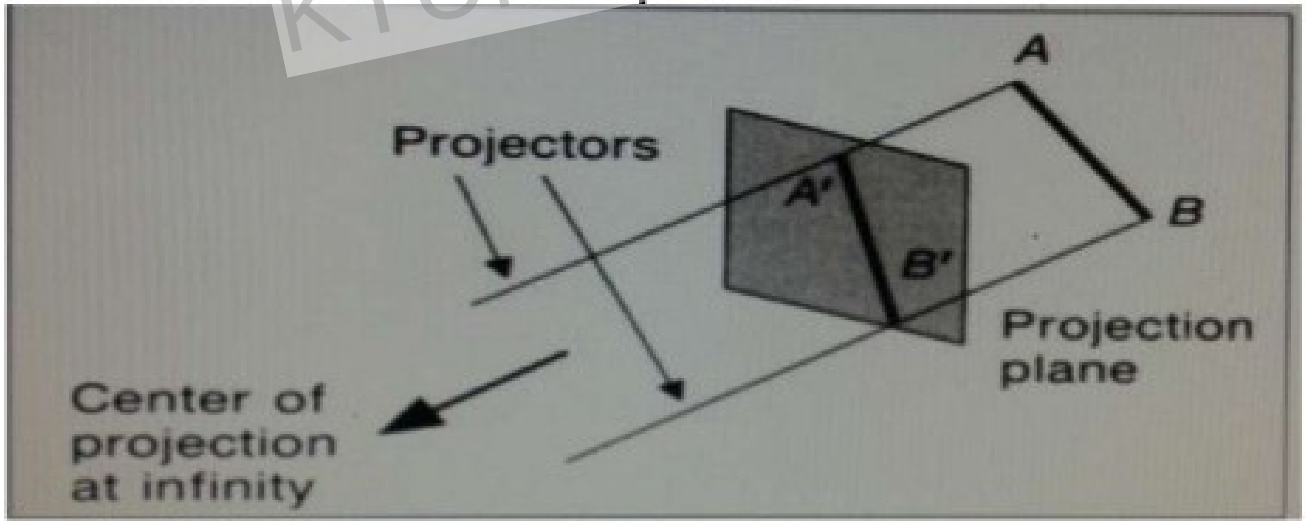
PERSPECTIVE PROJECTION



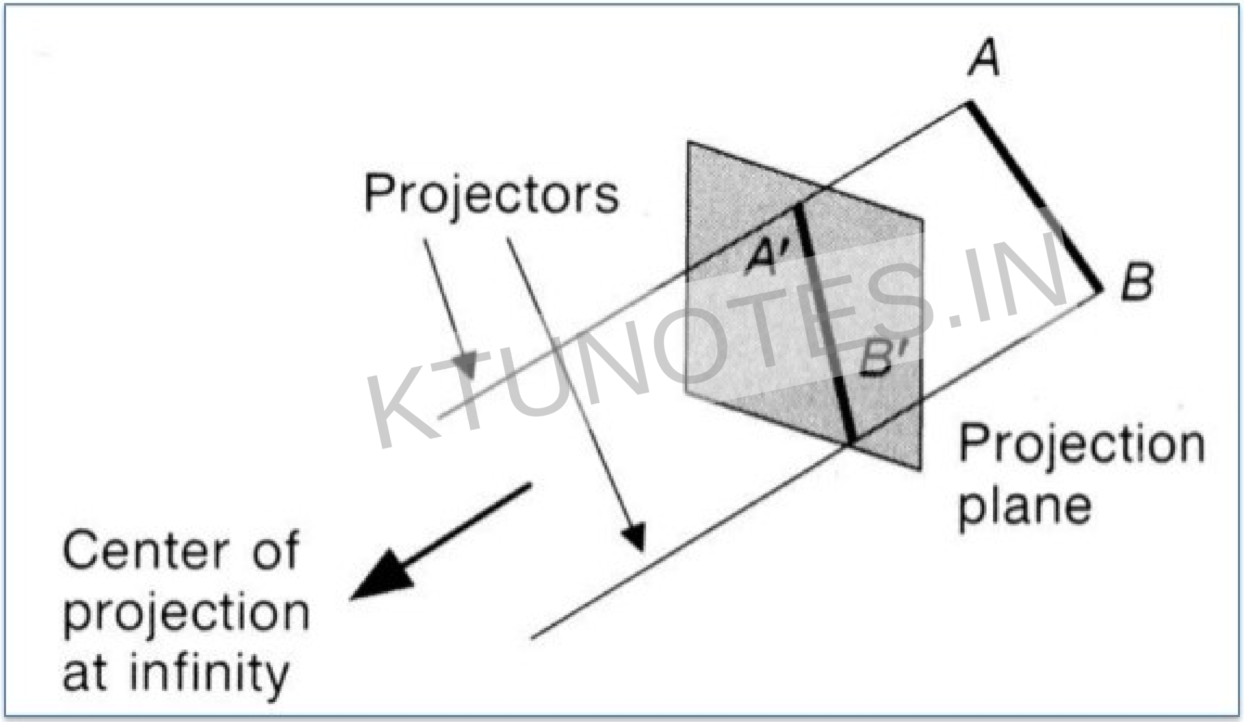


PARALLEL PROJECTION

- In this,coordinate positions are transformed to the view plane along parallel lines.
- Projection lines are parallel to each other.
- Projection lines are extended from the object and intersect the view plane.



Parallel Projection



PARALLEL PROJECTION

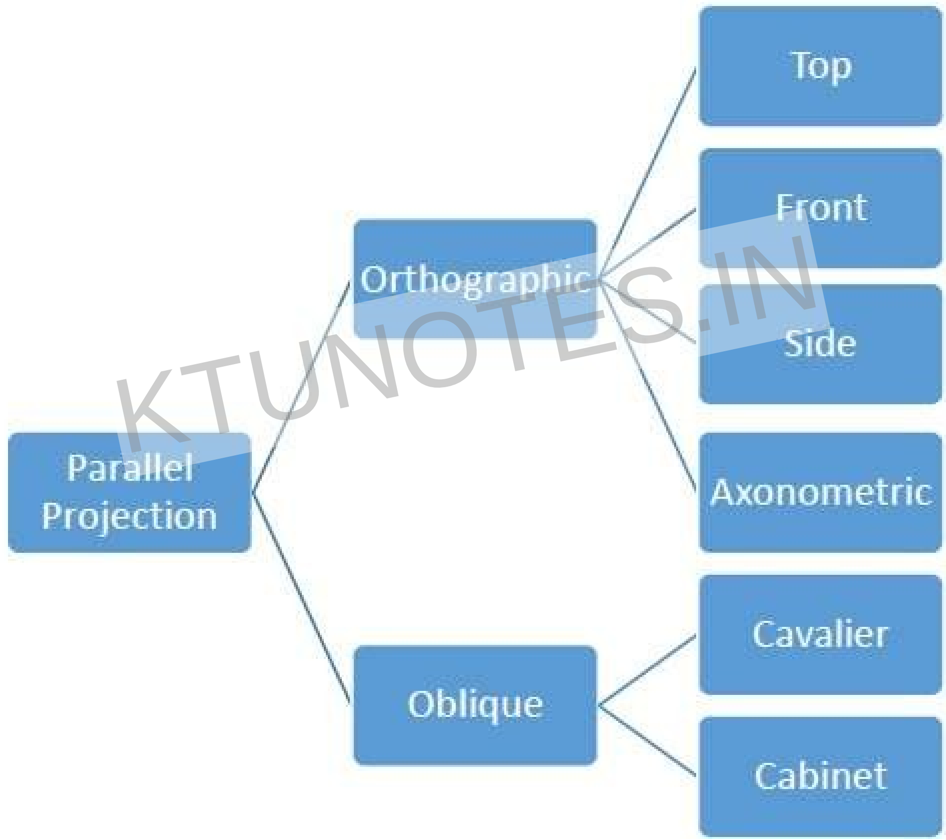
ADVANTAGE

- Accurate views of various sides of object are obtained.

DISADVANTAGE

- Does not give a realistic representation of appearance of 3D object.

TYPES OF PARALLEL PROJECTION



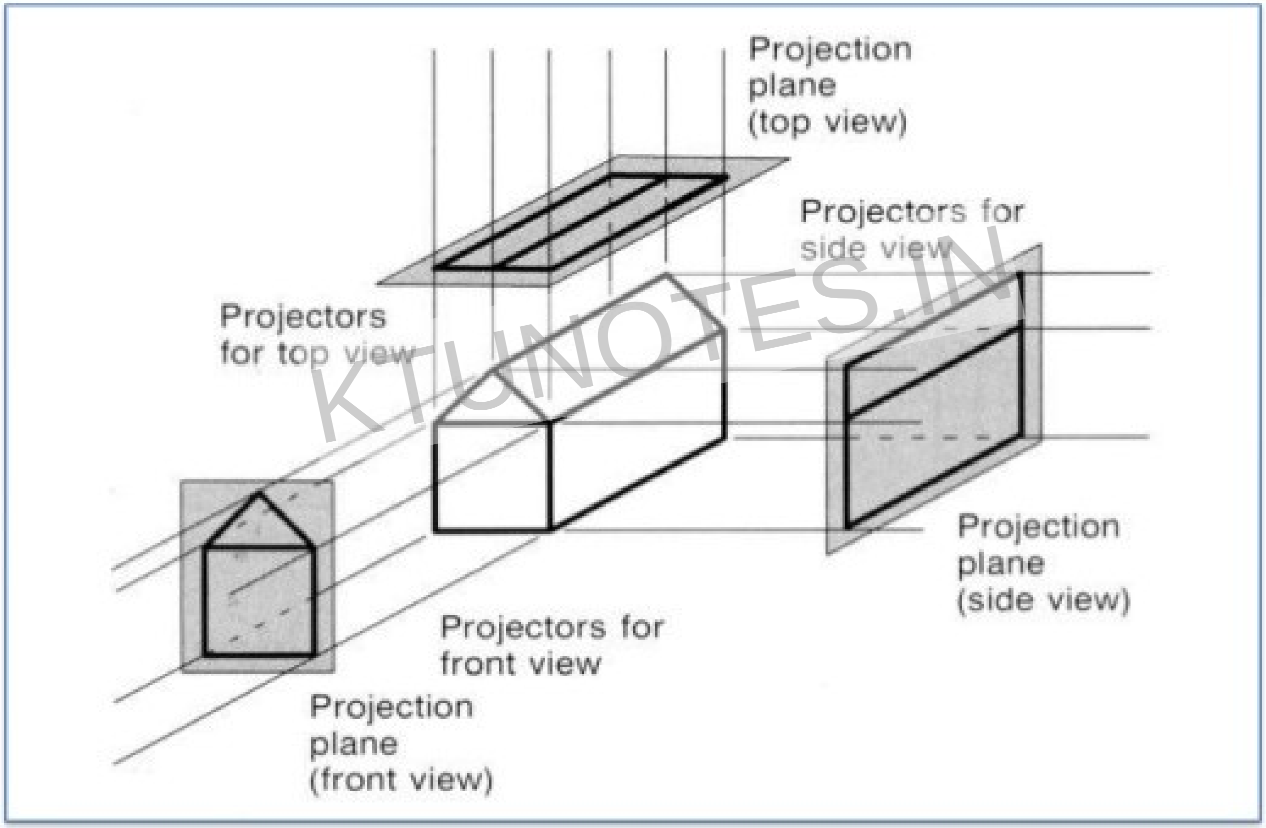
Types of Parallel Projection

There are two types of parallel projection:

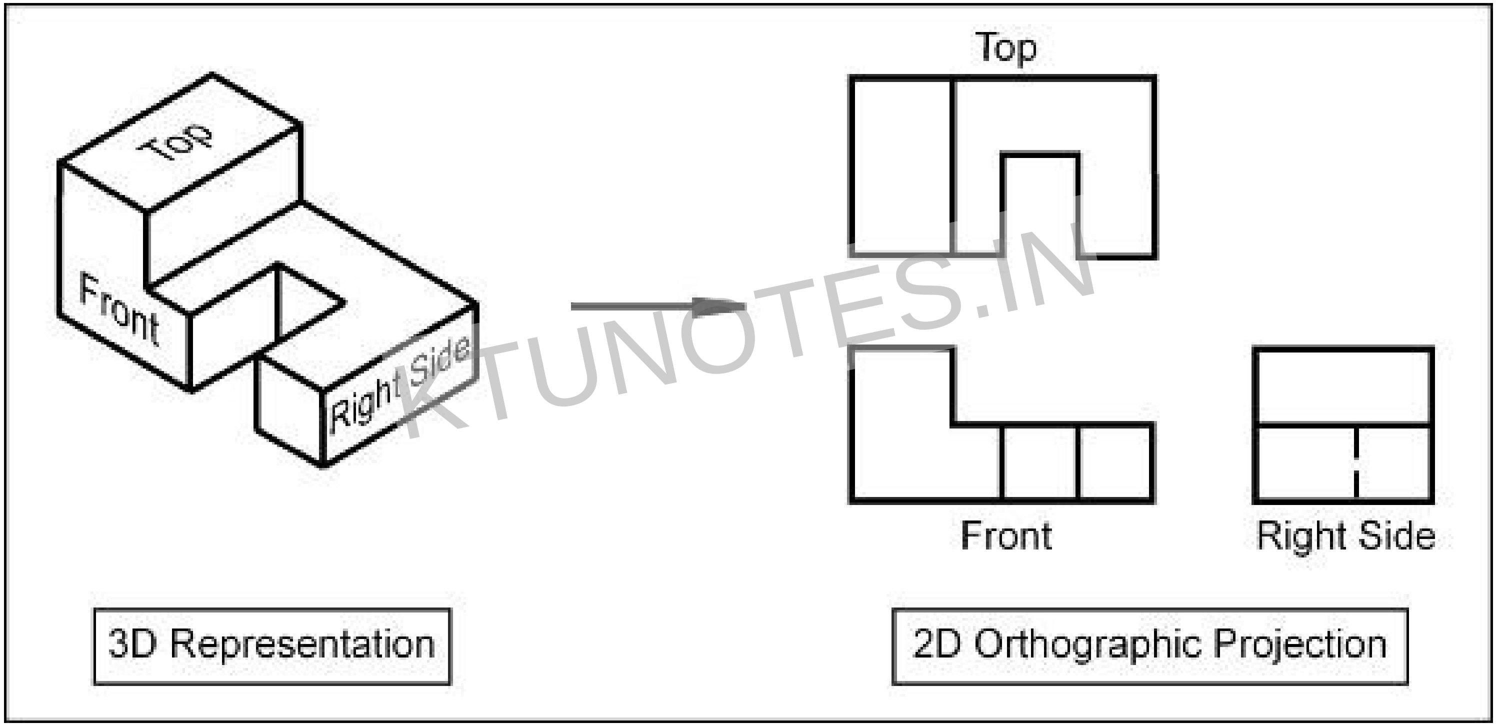
1. ORTHOGRAPHIC:

- ✓ Projection lines are parallel to each other & also perpendicular to the plane.
- ✓ It used to create different views of given object.
- ✓ There are 3 views:
    - i. Front view
    - ii. Side view
    - iii. Top view

Orthogonal projections:



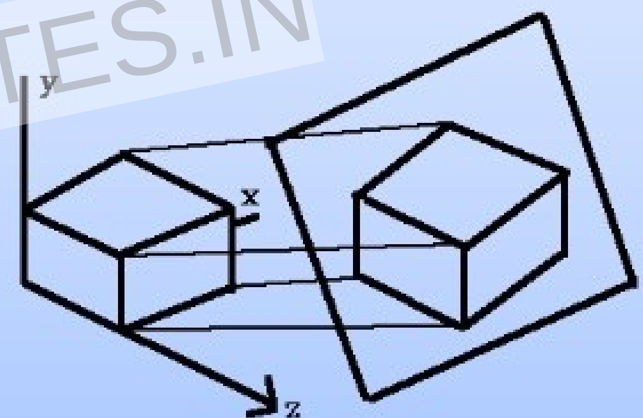
1. MULTIPLE VIEWS



Axonometric orthographic projections

- Orthographic projections that *show more than one face of an object* are called **axonometric** orthographic projections.
-

- The most common axonometric projection is an **isometric** projection where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance.



ORTHOGRAPHIC PROJECTION- Advantage & Application

- Engineering and architectural drawing commonly employ these orthographic projections, because length and angles are accurately depicted and can measure from drawing itself.

OBLIQUE PROJECTION

- It is obtained by projecting points along the parallel lines that are not perpendicular to projection plane.

TYPES:

1. **CAVALIER PROJECTION**
2. **CABINET PROJECTION**

CAVALIER PROJECTION

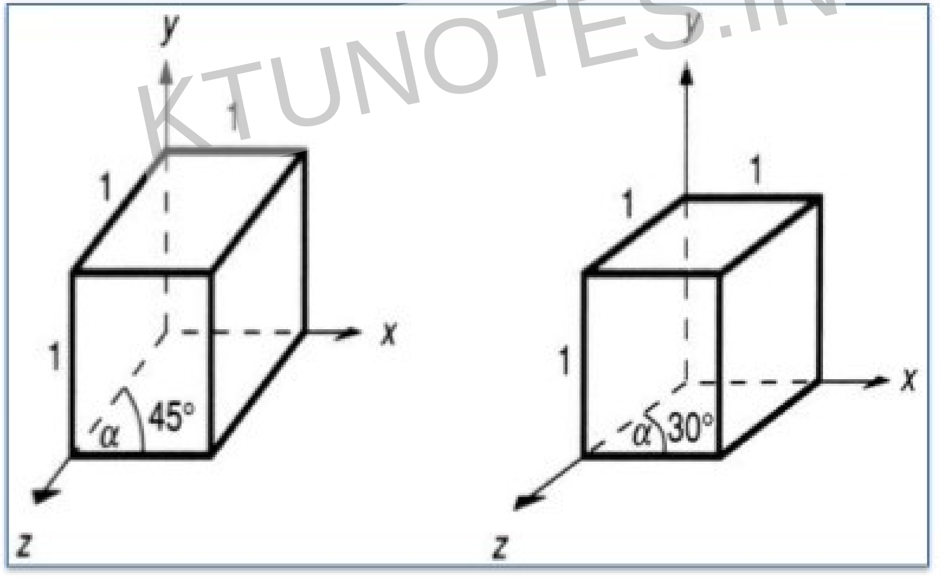
- It makes 45° with projection plane, **α =45°,** the view obtained are called **cavalier projections.**
- All the lines perpendicular to projection plane are projected with **no change in length**

➢ **Length L_1 depends on angle and z coordinate of point.**

- 2 common oblique parallel projections: *Cavalier* and *Cabinet*

Cavalier projection:

All lines perpendicular to the projection plane are projected with no change in length.



CABINET PROJECTION

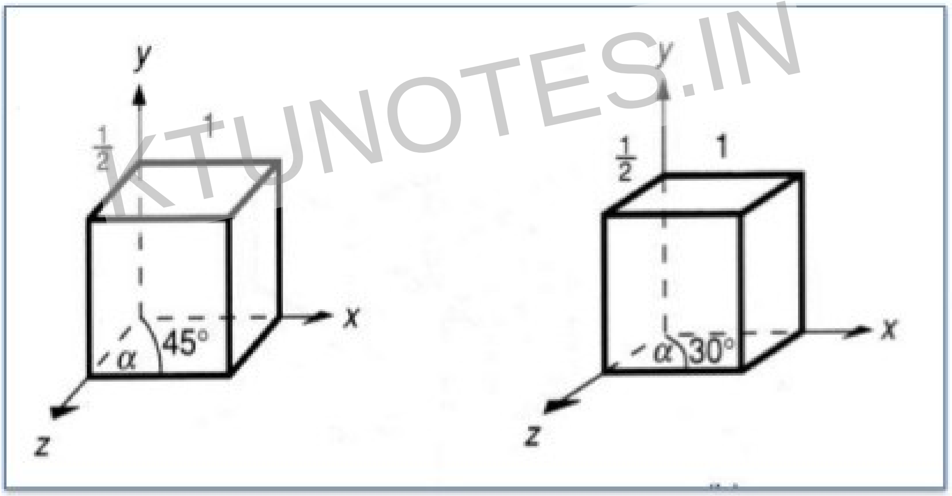
- It makes 63.4° angle with projection plane, **α =63.4°,** the view obtained are called **cavalier projections.**
- The lines perpendicular to projection plane are projected with **½ of the length.**

ADVANTAGE

- It appears more realistics than cavalier, because of reduction in length of perpendiculars.

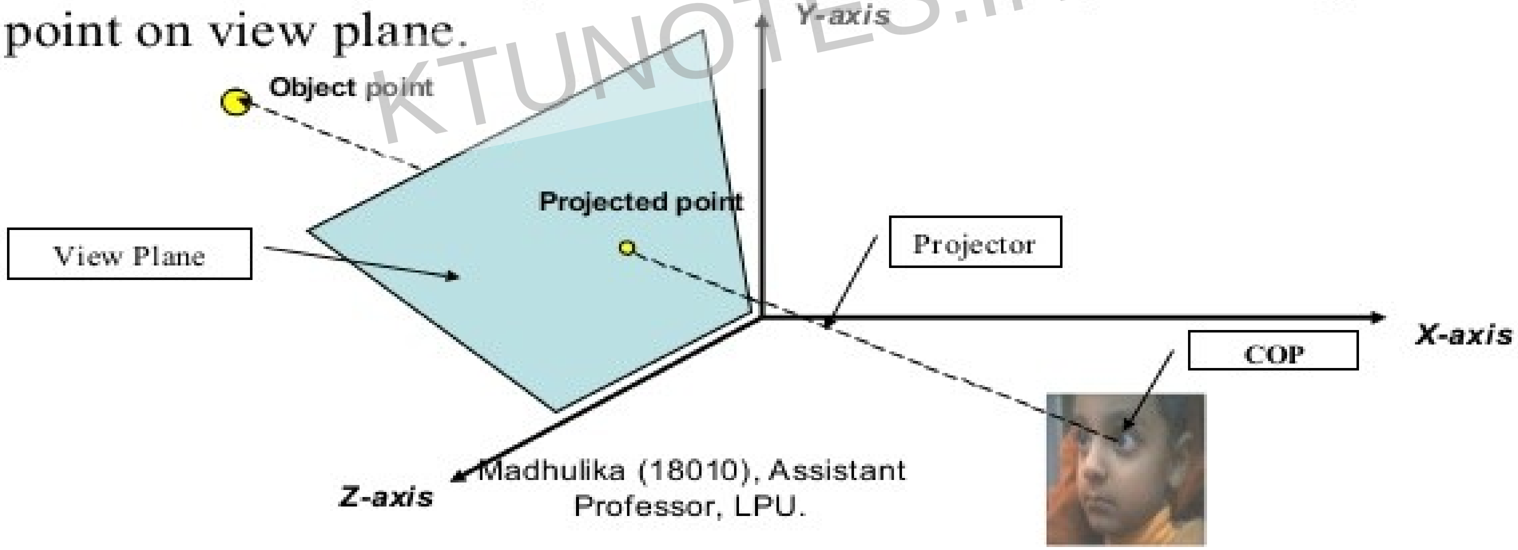
Cabinet projection:

- Lines which are perpendicular to the projection plane (viewing surface) are projected at 1 / 2 the length .
- This results in foreshortening of the z axis, and provides a more “realistic” view.



Perspective Projections

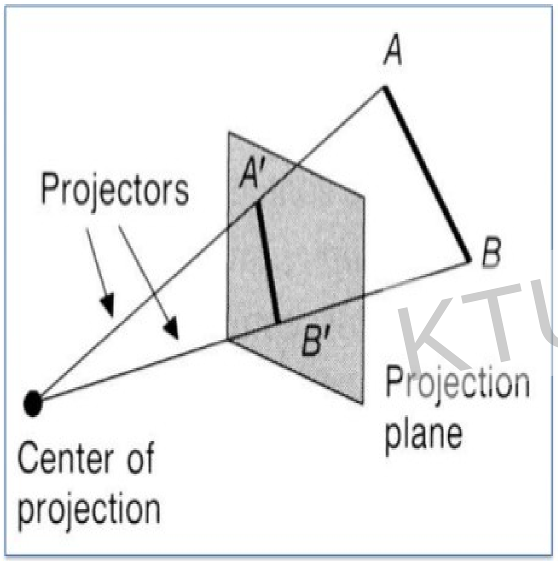
- Perspective projections are described by
  - **Centre of projection**: Eye of artists or lens of camera
  - **View Plane**: Plane containing canvas or film strip or frame buffer
- A ray called ***projector*** is drawn from COP to object point, its intersection with view plane determines the projected image point on view plane.

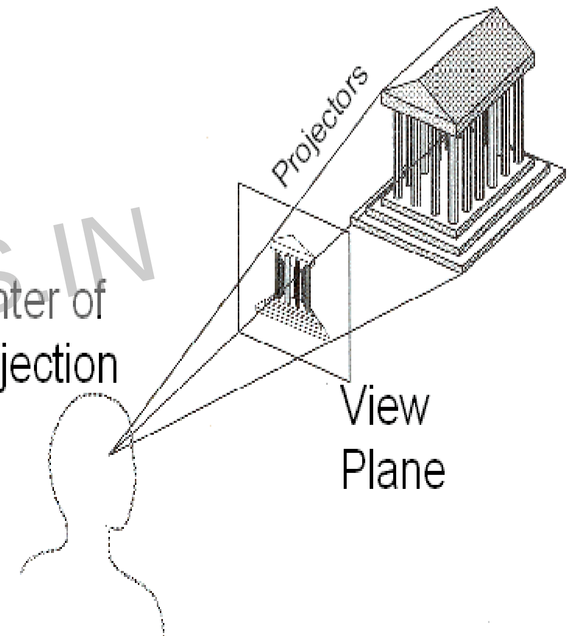


Madhulika (18010), Assistant Professor, LPU.

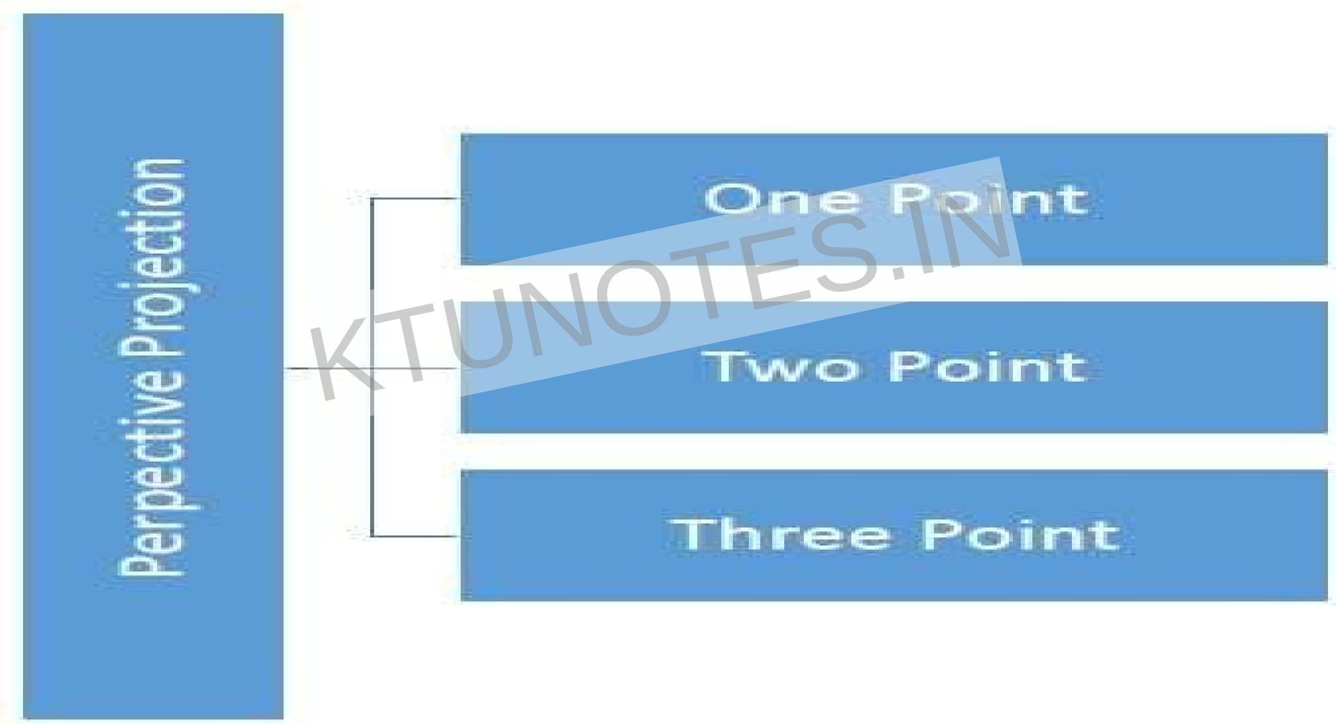
Perspective Projection

Perspective Projection



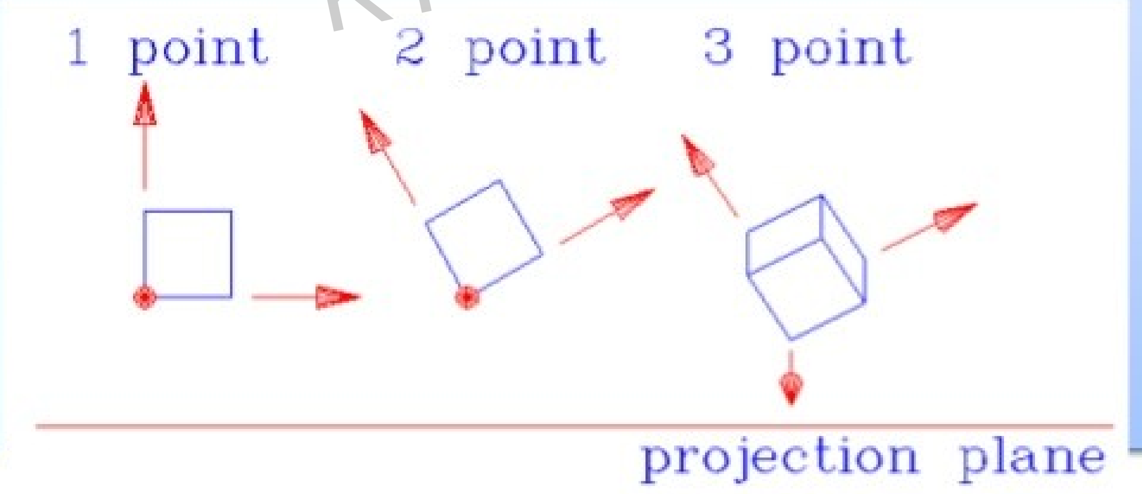


Types of Perspective Projection

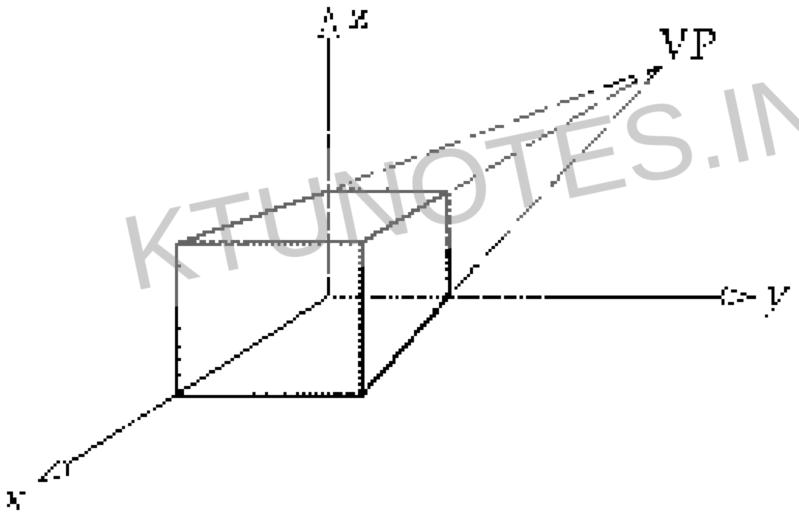


Classes of Perspective Projection

- One-Point Perspective
- Two-Point Perspective
- Three-Point Perspective



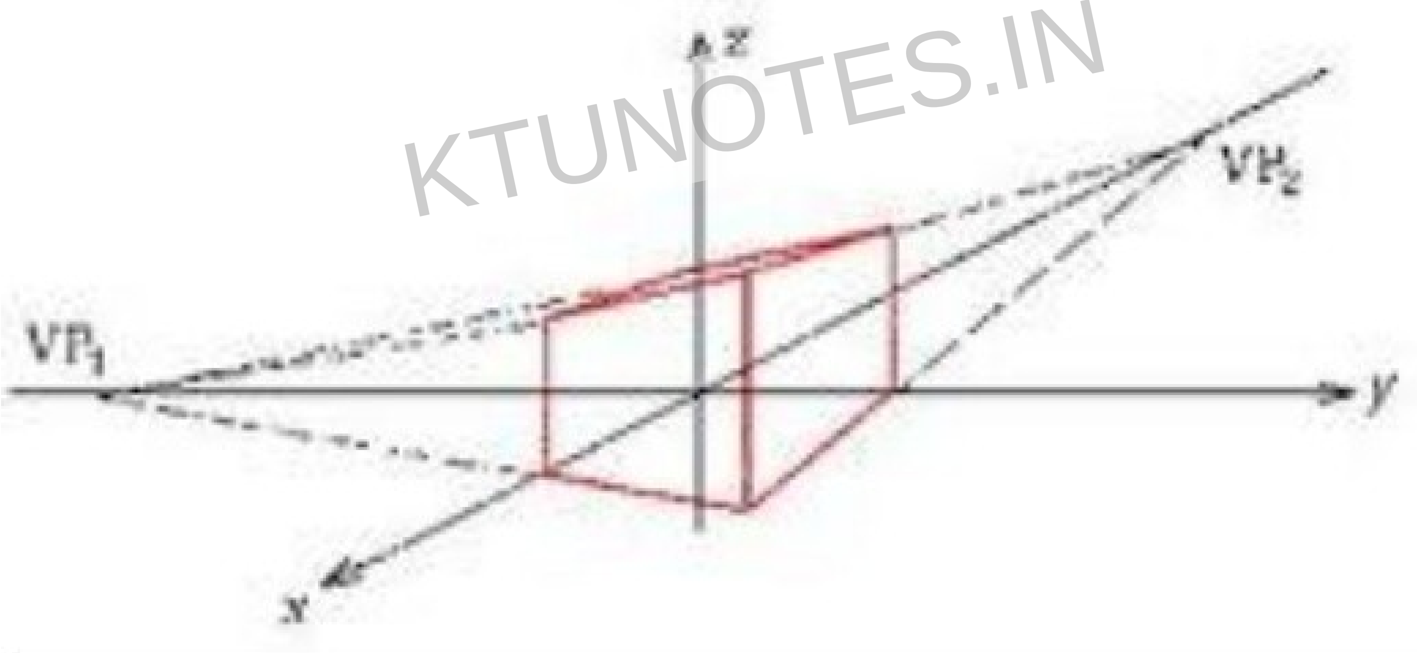
1. ONE POINT PERSPECTIVE PROJECTION



Two-point perspective projection:

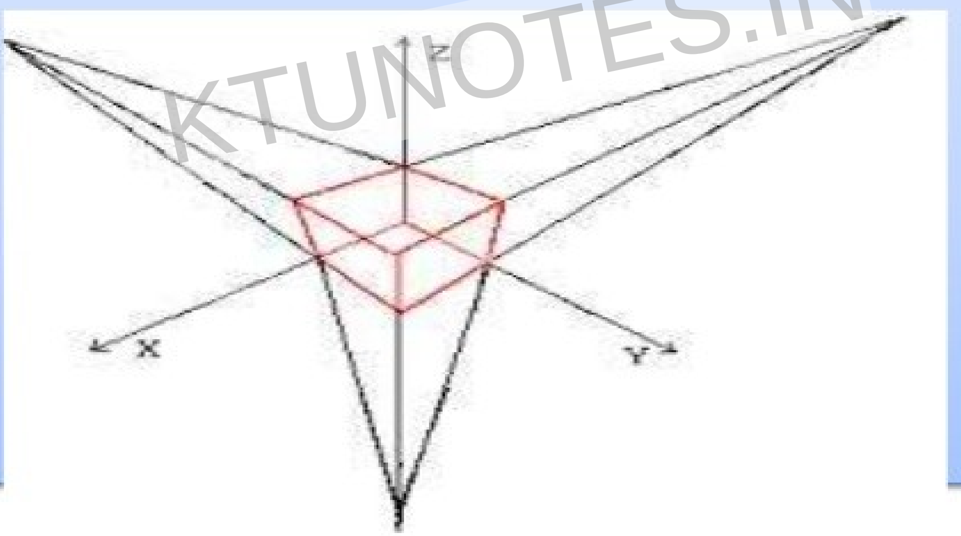
– This is often used in architectural, engineering and industrial design drawings.

–



Three-point perspective projection

- Three-point perspective projection is used less frequently as it adds little extra realism to that offered by two-point perspective projection



Perspective Projection

ADVANTAGE:

- Looks realistic
  - because size varies inversely with distance

DISADVANTAGE:

- We can not judge distance as parallel projection

Perspective v Parallel

- **Perspective:**
  - visual effect is similar to human visual system...
  - has 'perspective foreshortening'
    - size of object varies inversely with distance from the center of projection. Projection of a distant object are smaller than the projection of objects of the same size that are closer to the projection plane.
- **Parallel:**

  It preserves relative proportion of object.
  - less realistic view because of no foreshortening
  - however, parallel lines remain parallel.

Perspective Transformation

- Using 3D homogeneous coordinate representation, **perspective projection transformation** is shown.

Reconstruction of 3D object

Reconstruction of 3D object

- It is the process of capturing shape and appearance of real object.
- If the model is allowed to change its shape in time, this is referred to as non-rigid or spatio-temporal reconstruction.

Reconstruction of 3D object

- This process can be accomplished either by:
  - Active method
  - Passive methods

1. Active method

- Reconstruct the 3D profile by numerical approximation approach and build the object in scenario based on model using Range finders.

Reconstruction of 3D object

2. Passive method

– Passive methods of 3D reconstruction do not interfere with the reconstructed object;

– they only use a sensor to measure the radiance reflected or emitted by the object's surface to infer its 3D structure through image understanding

Application of Reconstruction of 3D object

3D reconstruction system finds its application in a variety of field they are:

- Medicine
- Film industry
- Robotics
- City planning
- Gaming
- Virtual environment
- Earth observation
- Archaeology
- Augmented reality
- Reverse engineering
- Animation
- Human computer interaction