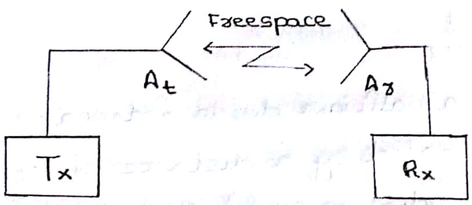
4-1-18

MODULE - I

Basic block diagram of freespace communication.



Antenna is a metallic conductor system capable of radiating and capturing electromagnetic energy. It is a transducer that converts electrical energy to electromagnetic waves and viceversa.

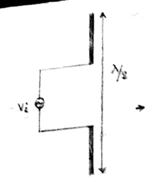
Antennas obey reciprocity principle.

Q) A transmission line carries a signal of 50Hz. Can this transmission line provide effective radiation ?

☆ First find wavelength of the signal

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{50} = 6\times10^6 m = 6000 Km$$

For effective radiation to take place the seperation between conductors of the txion line should be comparable to the wavelength of the radiating signal. The seperation of conductors of txion line is very less compared to wave length and hence will not act as an effective radiator. Thus a transmission line carries a signal of 50Hz does not provide effective radiation.



The above figure is a halfwave dipole antenna where the conductors are seperated by a distance of $\lambda/2$, where λ is the wavelength of radiating signal. Thus a half wave dipole antenna is an effective radiator.

* Antenna Parameters

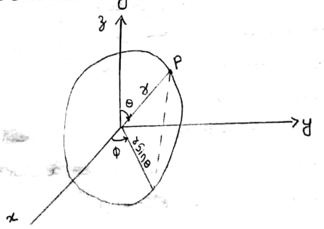
(i) Radiation Pattern

It is the graphical representation of radiation properties of an antenna as a function of space coordinates (θ and ϕ)

• Isotropic radiator

It is a hypothetical radiator that radiates equally in all directions. It is not used practically but used for reference purposes.

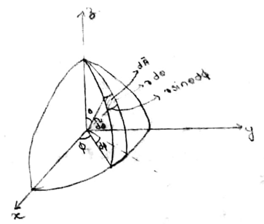
The radiation pattern of an antenna is based on spherical coordinate system.



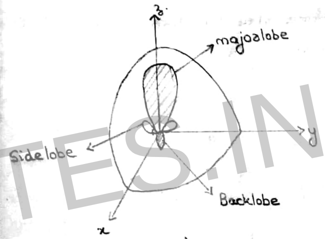
θ - elevation angle

ϕ - azimuthal angle

To obtain the incremental surface



$d\bar{A} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$



* Radiation Pattern lobes

Various parts of radiation patterns are called lobes.

Radiation lobes are :-

(i) Major lobe : It is defined as a radiation lobe containing the direction of maximum radiation. In the figure major lobe is in $\theta = 0$ direction.

(ii) Minor lobe : All the lobes in radiation pattern except major lobe are called minor lobe. They represent unnecessary or unwanted radiation. They are further divided into sidelobes and back lobe

- Sidelobes : The lobes which are adjacent to the major lobes and occupies hemisphere in the direction of major lobe
- Back lobe : It is the lobe that occupies hemisphere in a direction opposite to the major lobe.

Sidelobes are the largest of minor lobe and the level is expressed as side lobe level. It is the ratio of power of sidelobe to that of major lobe.

$$\boxed{\text{Sidelobe } \cancel{\text{ratio}} \text{ level (SLL)} = 10\log_{10} \frac{P_{sidelobe}}{P_{majorlobe}}}$$

It should be minimum

In radars to reduce false detection -30dB is used.

Normal communication = -20 dB

* Comparison

-20 dB	-30 dB
$\frac{P_s}{P_m} = \frac{1}{100}$	$\frac{P_s}{P_m} = \frac{1}{1000}$
Normal communication	used in radars

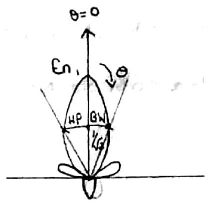
* Field Pattern and Power pattern

Radiation pattern is further classified into power pattern and Field pattern.

Power pattern - It is the graphical representation of power in space coordinates

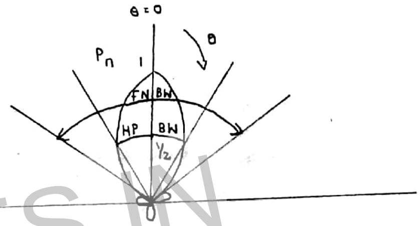
Field pattern - It is the graphical representation of field intensity / strength in

* Normalised Field Pattern



HPBW - Half power Beam Width

* Normalised Power Pattern



Beam width is the measure of directivity of beam. Smaller the beam width more directive the beam. Parabolic dish antennas which is a microwave antenna has a beam width of 1° or lesser. They are used in earth stations for satellite communications. Half power beamwidth also called -3 dB beamwidth is the angular measurement between the half power points.

Q) An antenna has the field pattern given by $E(\theta) = \cos^2\theta$ for $0 \le \theta \le 90°$. Find half power beamwidth?

A) Since it is a field pattern the half power point is $1/\sqrt{2}$.

Given $E(\theta) = \cos^2\theta$

$\frac{1}{\sqrt{2}}$ $\cos^2\theta = \frac{1}{\sqrt{2}}$ $\cos\theta = 0.841$ $\theta = \cos^{-1} 0.841$

$= 33°$

Half power Beamwidth $= 2\theta = \underline{66^\circ}$

Q) An antenna has a field pattern given by $E(\theta) = \cos\theta \cos 2\theta$ where $0 \leq \theta \leq 90$ find HPBW (ii) First Null Beam Width (FNBW)

* Since it is a field pattern the half powerpoint is $1/\sqrt{2}$

Given $E(\theta) = \cos\theta \cos 2\theta$

$$E(\theta) = \frac{1}{\sqrt{2}}$$

$$\cos\theta \cos 2\theta = \frac{1}{\sqrt{2}}$$

* Principal Patterns

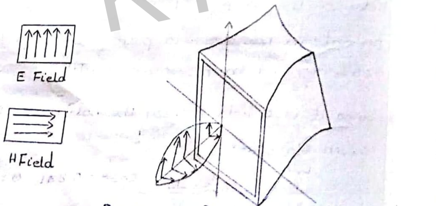
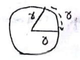


fig: $\bar{E}$ and $\bar{H}$ field pattern of horn antenna

There are 2 types of principal patterns. $\bar{E}$ plane pattern and $\bar{H}$ plane pattern. $\bar{E}$ plane pattern is the plane containing electric field vector in the direction of maximum radiation. $\bar{H}$ plane pattern is the plane containing magnetic field vector or $\bar{H}$ vector in the direction of maximum radiation. $\bar{E}$ and $\bar{H}$ plane pattern are obtained by cutting 3 dimensional radiation pattern using 2 orthogonal plane.

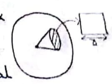
- Plane angle

The measure of plane angle is radian. one radian is defined as the angle with its vertex at the centre of a circle of radius 'r' by an arc of length 'r'. 2π radian is the ratio of circumference of circle to the radius



* Solid angle

The measure of solid angle is steradian. one steradian is defined as the solid angle with its vertex at the centre of a sphere of radius 'r' is subtended by a spherical surface area equal to that of a square of side length 'r'. The solid angle of a closed sphere is $\frac{4\pi r^2}{r^2} = 4\pi$ Sr. Differential area of a $|d\bar{A}|$ on the surface of a sphere of radius r is given as $|d\bar{A}| = r^2 \sin\theta \, d\theta \, d\phi$. Differential solid angle represented (element of solid angle) as $d\Omega$ is given as $\frac{|d\bar{A}|}{dr^2}$



$$\boxed{d\Omega = \sin\theta \, d\theta \, d\phi}$$

* Poynting Vector

$\mathbb{P} = E \times \bar{H}$

It gives instantaneous power density. The power density or radiation density of an antenna is given as

(ii) $\bar{W}_{rad} = \frac{1}{2} Re\left[\bar{E}_s \times H_s^*\right]$ unit w/m^2

The total power radiated is obtained as power radiator

(iii) $\boxed{P_{rad} = \int_S \bar{W}_{rad} \cdot d\bar{s}}$ unit W

(Q) The radiant component of radiated power density of antenna is given as $\bar{W}_{rad} = \frac{A_0 \sin\theta}{r^2} \hat{a_r}$ where A_0 is the peak value of power density determine the total radiated power?

☆ $P_{rad} = \int_S \bar{W}_{rad} \cdot d\bar{s}$

$$= \int_S \frac{A_0 \sin\theta}{r^2} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$$= \frac{A_0}{\cancel{r^4}} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin^2\theta \, d\theta \, d\phi$$

$$= \frac{A_0}{\cancel{r^4}} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1-\cos 2\theta}{2} \, d\theta \, d\phi \quad = \frac{A_0}{\cancel{r^4}} \times 2\pi \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4\theta}\right)_0^{\pi}$$

$$= \frac{2\pi A_0 \cancel{\pi_0}}{\cancel{2}} \left(\theta - \frac{\sin 2\theta}{2\theta}\right)_0^{\pi} \quad = \pi A_0 (\pi)$$

$$= \underline{\underline{A_0 \pi^2}}$$

Q) What is the power density of an isotropic antenna in terms of radiated power?

☆ An isotropic radiator radiates equally in all direction. Hence $\bar{W}_{rad}$ for an isotropic radiator is independent of θ and ϕ. $\bar{W}_{rad}$ is just a function of r. Therefore $\bar{W}_{rad}$ is equal to $W_{r0}(r)$

$\bar{W}_{rad} = W_{r0}(r)$ $\qquad \bar{W}_{rad} = W_{rad} \hat{a_r}$ here we take this

$P_{rad} = \oint_S \bar{W}_{r0}(r) \cdot d\bar{s}$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} W_{r0}(r) \hat{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$$= W_{r0}(r) r^2 \cdot 2\pi \int_{\theta=0}^{\pi} \sin\theta \, d\theta$$

$$= W_{r0}(r) \cdot r^2 \cdot 2\pi \cdot 2$$

$$= 4\pi r^2 W_{r0}(r)$$

$$\boxed{W_{r0}(r) = \frac{P_{rad}}{4\pi r^2}}$$

(iv) Radiation intensity

Radiation intensity 'u' is given as $u = r^2 W_{rad}$ where 'r' is the distance of the point of observation from the antenna. Unit is watts/unit solid angle

$u = r^2 W_{rad}$

$$P_{rad} = \oint_{\Omega} u \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} u \sin\theta \, d\theta \, d\phi$$

Q) The radial component of radiated power density is

$\overline{W}_{rad} = \frac{A_0 \sin\theta}{r^2} \hat{a}_r$. (i) Find radiation intensity

(ii) Find radiated power using radiation intensity?

★ $\overline{W}_{rad} = \frac{A_0 \sin\theta}{r^2} \hat{a}_r$

$W_{rad} = \frac{A_0 \sin\theta}{r^2}$

(i) $U = r^2 W_{rad} = \underline{\underline{A_0 \sin\theta}}$

(ii) $P_{rad} = \oiint_{\Omega} U \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U \sin\theta \, d\theta \, d\phi$

$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} A_0 \sin^2\theta \, d\theta \, d\phi = A_0 2\pi \int_{\theta=0}^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$

$= \frac{A_0 2\pi}{2} \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$

$= \underline{\underline{A_0 \pi^2}}$

Q) Find out the radiation intensity of an isotropic radiator in terms of radiated power

★ Since it is an isotropic radiator, radiation intensity 'u' will be independent of θ and ϕ.

$\therefore U = U_0$

$U_0 = r^2 W_{rad}$

$P_{rad} = \oiint_{\Omega} U_0 \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U_0 \sin\theta \, d\theta \, d\phi$

$= U_0 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$

$$\boxed{\therefore \; U_0 = \frac{P_{rad}}{4\pi}}$$

(v) Directivity

It is the measure of directional properties of antenna. It is defined as the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all direction. The average radiation intensity is equal to radiated power divided by 4π.

$D = \frac{\text{RI in a given dir}}{\text{RI avge. over all dir}}$ $\rightarrow \frac{P_{rad}}{4\pi}$

$D = \frac{U(\theta,\phi)}{\frac{P_{rad}}{4\pi}}$

$$\boxed{D = \frac{4\pi U(\theta,\phi)}{P_{rad}}}$$

D is also written as $D(\theta,\phi)$

Directivity is also defined as the radiation intensity in a given direction to the radiation intensity of an isotropic source (isotropic radiator)

$$\boxed{D = \frac{U(\theta,\phi)}{U_0} = \frac{U(\theta,\phi)}{\frac{P_{rad}}{4\pi}}}$$

$$\boxed{D_{max} = \frac{4\pi U(\theta,\phi)|_{max}}{P_{rad}} = \frac{4\pi U_{max}}{P_{rad}}}$$

For an isotropic source, directivity is equal to unity. Thus for all direction antennas directivity will be equal to greater than 1.

$$\boxed{1 < D(\theta,\phi) \leq D_{max}}$$

Q) The radial component of radiated power density of an antenna is $\overline{W}_{rad} = \frac{A_0 \sin\theta}{r^2}\hat{a}_r$. Find directivity? (choose D_{max})
Express directivity in terms of max: directivity and function of θ and ϕ.

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}}$$

$$U = r^2 W_{rad} \qquad U_{max} = A_0$$

$$= \cancel{r^2}\frac{A_0 \sin\theta}{\cancel{r^2}}$$

$$P_{rad} = \oiint_{\Omega} U\, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U \sin\theta\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} A_0 \sin^2\theta\, d\theta\, d\phi = \frac{A_0}{2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 1-\cos 2\theta\, d\theta\, d\phi$$

$$= \frac{A_0 \pi}{\cancel{2}}\left[\theta - \frac{\sin 2\theta}{2\theta}\right]_0^{\pi} = A_0\pi^2$$

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \cancel{A_0}}{\cancel{A_0}\pi^2} = \frac{4}{\pi} = 1.27$$

$$D(\theta) = D_{max}\sin\theta$$

$$D(\theta) = 1.27 \sin\theta \,\}\, U$$

Q) The radial component of radiated power density of an antenna is given as $\overline{W}_{rad} = \frac{A_0 \sin^2\theta}{r^2}\hat{a}_r$. Find the directivity? (ii) Express directivity as a function of θ and ϕ?

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}}$$

$$U = r^2 \overline{W}_{rad} = \cancel{r^2}\frac{A_0 \sin^2\theta}{\cancel{r^2}} \qquad U_{max} = A_0$$

$$P_{rad} = \oiint_{\Omega} U\, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U \sin\theta\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} A_0 \sin^3\theta\, d\theta\, d\phi$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$= A_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{3\sin\theta - \sin 3\theta}{4}\, d\theta\, d\phi$$

$$= \frac{A_0 \cancel{2}\pi}{\cancel{4}\,2}\left[\left(-3\cos\theta\right)_0^{\pi} + \left(\frac{\cos 3\theta}{3}\right)_0^{\pi}\right]$$

$$= \frac{A_0\pi}{2}\left[\frac{3}{3} + \frac{-1}{3}\left(\frac{-1}{3}\right)\right]$$

$$= \frac{A_0\pi}{\cancel{2}} \times \frac{\cancel{16}\,8}{3} = \frac{8A_0\pi}{3} = A_0\frac{8\pi}{3}$$

$-1(\cos\pi - \cos 0)$

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \cancel{A_0}}{\cancel{A_0}\frac{8\pi}{3}} = 3/2 = 1.5$$

$$D(\theta) = D_{max}\sin^2\theta$$

$D(\theta) = 1.5 \sin^2\theta$ This is an omnidirectional pattern where radiation intensity is a fn: of only θ and ϕ = Const

Q) Compare isotropic, directional and omnidirectional patterns

(vi) **Beam Solid angle**

This is calculated from radiation pattern of the antenna

Proof:

Let the radiation intensity 'u' of an antenna be $U = B_0 F(\theta,\phi)$

$U_{max} = B_0 F(\theta,\phi)\big|_{max}$

Power radiated (P_{rad}) $= \oint\!\!\oint_{\Omega} U\,d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} B_0 F(\theta,\phi)\sin\theta\,d\theta\,d\phi$

Directivity of the antenna,

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}}$$

$$D_{max} = \frac{4\pi\, B_0 F(\theta,\phi)\big|_{max}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} B_0 F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

Divide numerator $(B_0 F(\theta,\phi)|_{max})$ and deno: by this.

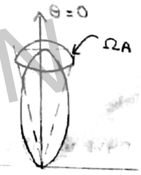
$$D_{max} = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{F(\theta,\phi)}{F(\theta,\phi)|_{max}}\sin\theta\,d\theta\,d\phi}$$

$$D_{max} = \frac{4\pi}{\underbrace{\int_{\phi}\int_{\theta} F_n(\theta,\phi)\sin\theta\,d\theta\,d\phi}_{\Omega_A}}$$

$$\boxed{D_{max} = \frac{4\pi}{\Omega_A}}$$ where 'Ω_A' $= \int_{\phi}\int_{\theta} F_n(\theta,\phi)\sin\theta\,d\theta\,d\phi$

This Ω_A is called Beam solid angle.

Beam Solid angle is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant and equal to the max: value for all the angles within Ω_A.



$\Omega_A = \theta_1\theta_2$

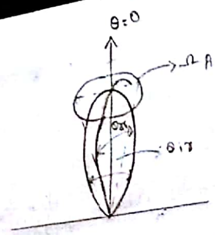
Beam solid angle is approximately equal to the product of HPBW in the two perpendicular planes. If the pattern is rotationally symmetric, the two HPBW angles are the same.

ie $$\boxed{D_{max} = \frac{4\pi}{\theta_{1r}\theta_{2r}}}$$

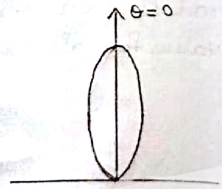
where θ_{1r} and θ_{2r} are HPBW in the 2 ⊥r planes expressed in radians.

$$\boxed{D_{max} = \frac{4\pi(180/\pi)^2}{\theta_{1d}\,\theta_{2d}} = \frac{41253}{\theta_{1d}\theta_{2d}}}$$

where θ_{1d} and θ_{2d} are HPBW in the 2 ⊥r planes expressed in degree...



Q) The radiation intensity of major lobe of the antenna is represented as $U = B_0 \cos\theta$. The radiation intensity exist only in the upper hemisphere. Find directivity using the formula as well as from the pattern and compare the value.



* $D_{max} = \dfrac{4\pi U_{max}}{P_{rad}}$

$D_{max} = \dfrac{4\pi}{\Omega_A}$

$\Omega_A = \theta_{1r}\theta_{2r}$ — θ_{1r} - HPBW in one plane, θ_{2r} - HPBW in ⊥r plane

It is a symmetrical pattern, hence $\theta_{1r} = \theta_{2r}$

Half power points are at $\theta = 60°$ ($\cos 60° = 1/2$)

θ_{1r} is 2 times θ $\theta_{1r} = 2 \times 60° = \underline{120°}$

$\theta_{1r} = \theta_{2r} = 120°$

$D_{max} = \dfrac{41253}{\theta_{1d}\,\theta_{2d}} = \dfrac{41253}{120 \times 120} = \underline{\underline{2.8647}}$ $120 \times \dfrac{\pi}{180} \rightarrow$ radian

This is an approximate value. Exact value is obtained using the formula $D_{max} = \dfrac{4\pi U_{max}}{P_{rad}}$

$P_{rad} = \oint_\Omega U d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} U \sin\theta\, d\theta\, d\phi$

$= B_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \cos\theta \sin\theta\, d\theta\, d\phi$ $2\sin\theta\cos\theta = \sin 2\theta$, $\cos\theta\sin\theta = \dfrac{\sin 2\theta}{2}$

θ varies from 0 to π/2 (radiation intensity exist only in upper hemisphere).

$= B_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \dfrac{\sin 2\theta}{2}\, d\theta\, d\phi$

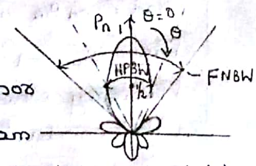
$= \dfrac{B_0 \cdot 2\pi}{2}\left(\dfrac{-\cos 2\theta}{2}\right)_0^{\pi/2} = \dfrac{B_0\pi}{2}\left[-\left(\cos 2\pi/2 - \cos 0\right)\right]$

$= \dfrac{B_0\pi}{\cancel{2}}\left[-(\cancel{2}\,{-1})\right] = \underline{\underline{B_0\pi}}$ $U_{max} = B_0$

$D_{max} = \dfrac{4\pi U_{max}}{P_{rad}} = \dfrac{4\pi \cancel{B_0}}{\cancel{B_0\pi}}$

$\therefore \underline{D_{max} = 4}$

(vii) Beamwidth



As Beamwidth increases no: of minor lobes decreases and viceversa. Beam width is used for describing the resolution capability of radars. Two targets will be distinguished as two different targets by a radar only if angular seperation between them is greater than $\dfrac{FNBW}{2}$ where FNBW is the First Null Beamwidth of radar antenna.

$\theta > \dfrac{FNBW}{2} \cong HPBW$

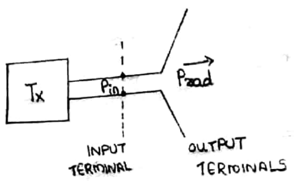
BW ↑ no: of minorlobes ↓

(viii) Gain of the antenna

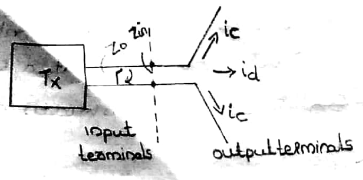
Gain of an antenna considers the efficiency of the antenna along with the directional properties whereas directivity considers only the directional properties.

$$\text{Gain} = 4\pi \cdot \frac{\text{Radiation Intensity}}{\text{Input Power}}$$

$$\text{Gain} = \frac{4\pi \cdot u(\theta,\phi)}{P_{in}} \Rightarrow \boxed{G(\theta,\phi) = \frac{4\pi \cdot u(\theta,\phi)}{P_{in}}}$$



$P_{rad} = e_{cd} P_{in}$ where e_{cd} is called antenna radiation efficiency. $e_{cd} = e_c \cdot e_d$ where 'e_c' is the conductor efficiency which takes into account the losses incurred in the conductor of an antenna. 'e_d' accounts for losses in the dielectric medium and is called dielectric efficiency.



Antenna losses

$$G(\theta,\phi) = \frac{4\pi \cdot u(\theta,\phi)}{\frac{P_{rad}}{e_{cd}}} \Rightarrow G(\theta,\phi) = e_{cd}\left(\frac{4\pi\, u(\theta,\phi)}{P_{rad}}\right)$$

$$\boxed{G(\theta,\phi) = e_{cd}\, D(\theta,\phi)}$$

D - Directivity } Relationship
G - Gain }

$$G(\theta,\phi)|_{max} = e_{cd}\, D(\theta,\phi)|_{max}$$

$$\boxed{G_{max} = e_{cd}\, D_{max}}$$

* Absolute Gain

Absolute Gain takes into account the reflection losses in addition to conductor and dielectric losses.

$$\boxed{G_{abs}(\theta,\phi) = e_r(\theta,\phi)}$$

where 'e_r' is reflection efficiency and $e_r = 1 - |\Gamma|^2$

Γ - reflection coefficient

$$\boxed{\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}}$$

Z_{in} - input impedance of antenna

Z_0 - characteristic impedance of txion line connecting transmitter to antenna

$$G_{abs}(\theta,\phi) = e_r\, G(\theta,\phi)$$
$$= \underbrace{e_r\, e_{cd}}_{e_0}\, D(\theta,\phi)$$

$$\boxed{G_{abs}(\theta,\phi) = e_0\, D(\theta,\phi)} \qquad e_0 = e_r\, e_{cd}$$

where e_0 is called antenna efficiency. It takes into account reflection losses and the conduction losses.

Absolute gain is equal to gain, when $e_r = 1$

$$G_{abs}(\theta,\phi) = e_r\, G(\theta,\phi)$$

$e_r = 1$ $(|\Gamma| = 0)$ $(Z_{in} = Z_0)$

$G_{abs}(\theta,\phi) = e_r e_{cd} D(\theta,\phi)$

$= e_0 D(\theta,\phi)$

$G_{abs}(\theta,\phi)|_{max} = e_0 D(\theta,\phi)_{max}$

IMP Q) The lossless resonant halfwavelength dipole antenna with input impedance of 73 Ω is connected to a txion line whose characteristic impedance is 50 Ω. The radiation pattern is $U = B_0 \sin^3\theta$. Find the max: absolute gain of this antenna?

★ $G_{abs}(\theta,\phi)|_{max} = e_r e_{cd} D_{max}$

$D_{max} = \frac{4\pi U_{max}}{P_{rad}}$ $\quad U = B_0 \sin^3\theta \quad U_{max} = B_0$

$$P_{rad} = \iint_{\Omega} U \, d\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} B_0 \sin^3\theta \sin\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} B_0 \sin^4\theta \, d\theta \, d\phi$$

$\sin^4\theta = (\sin^2\theta)^2 = \left(\frac{1-\cos 2\theta}{2}\right)^2$

$$= \frac{B_0}{4}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} (1-\cos 2\theta)^2 \, d\theta \, d\phi$$

$(a-b)^2$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\cos^2 2\theta = \frac{1+\cos 4\theta}{2}$

$$= \frac{B_0 \cdot 2\pi}{4}\int_{\theta=0}^{\pi} 1 - 2\cos 2\theta + \cos^2 2\theta \, d\theta$$

$$= \frac{B_0\pi}{2}\int_{\theta=0}^{\pi}\left[1 - 2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)\right] d\theta$$

$$= \frac{B_0\pi}{2}\left[\theta - \frac{2\sin 2\theta}{2} + \frac{\theta}{2} + \frac{\sin 4\theta}{8}\right]_0^{\pi}$$

$$= \frac{B_0\pi}{2}\left[\pi + \frac{\pi}{2}\right] = \frac{B_0\pi}{2}\left[\frac{3\pi}{2}\right]$$

$P_{rad} = B_0\left(\frac{3\pi^2}{4}\right)$

$D_{max} = \frac{4\pi \cdot B_0}{B_0\left(\frac{3\pi^2}{4}\right)} = \frac{16}{3\pi} = \underline{\underline{1.69}}$

To express it in dB $\quad D_{max} = 10\log_{10}(1.69)$

$= \underline{\underline{2.278 \text{ dB}}}$

$G_{abs\,max} = e_r e_{cd} D_{max}$

For lossless antenna $e_{cd} = 1$

$e_r = 1 - |\Gamma|^2$

$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{73-50}{73+50} = \underline{\underline{0.186}}$

$e_r = 1 - |0.186|^2 = \underline{\underline{0.965}}$

$G_{abs\,max} = 0.965 \times 1.69 = \underline{\underline{1.631}}$

To express it in dB ~~Dm~~ $G_{abs\,max} = 10\log_{10}(1.631)$

$= \underline{\underline{2.12 \text{ dB}}}$

(ix) <u>Antenna radiation efficiency</u> (Conductor dielectric efficiency) (e_{cd})

Let the antenna impedance be

$Z_A = R_A + jX_A$

$= R_r + R_L + jX_A$

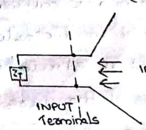


Fig: Antenna in Recieving mode

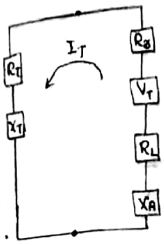
R_r = radiation resistance of antenna

R_L = Loss resistance of antenna

Impedance of reciever circuitry $Z_T = R_T + jX_T$

V_T - induced voltage by the incident electromagnetic wave

The conductor dielectric efficiency or antenna radiation efficiency is the ratio of power delivered to the radiation resistance R_r to the power delivered to the resistance $R_r + R_L$. R_r called the radiation resistance which is the fictious resistance across which radiated power P_{rad} is assumed to be dissipated.



Thevenin equivalent circuit of antenna in recieving mode

Loss resistance R_L takes into account conduction and dielectric losses. For a copper wire of length 'l' conductivity 'σ' and $\boxed{R_L = R_{hf} = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}}}$

where l is the length of the wire/antenna used

P is the perimeter of crosssection of wire

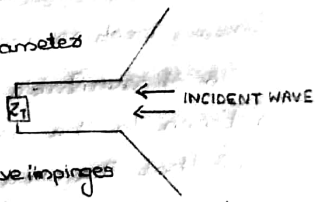
σ is the conductivity of wire

μ_0 - permeability of free space

Q) A Resonant halfwave dipole antenna is made of copper wire of conductivity $\sigma = 5.7 \times 10^7$ Siemens/m. Determine the radiation efficiency of antenna at freq: 100MHz. If radius of the wire is $3\times10^{-4}\lambda$ and the radiation resistance is 73Ω.

(X) * Effective length of the antenna ($\bar{l}_e$)

Effective length is an antenna parameter which is used to determine the voltage induced on the open circuit terminal of antenna when the wave impinges on it. It is computed for both linear (dipole antenna) and aperture antenna (eg: horn antenna).



$\bar{l}_e$ is a complex vector quantity and is expressed as; $\bar{l}_e = l(\theta,\phi)\hat{a_\theta} + l(\theta,\phi)\hat{a_\phi}$

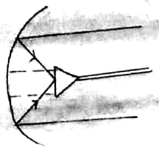
It is a far field parameter. The open circuit voltage developed at terminals of a receiving antenna is given by

$V_{oc} = \bar{E_i} \cdot \bar{l}_e$

E_i - incident electric field having the same polarization as that of the antenna.

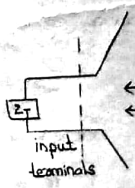
"The effective length of antenna is thus defined as the ratio of open ckt voltage developed at the terminals of the antenna to the electric field strength in the direction of antenna polarization". Only these electric fields having the same polarization as that of the antenna will be absorbed by it.

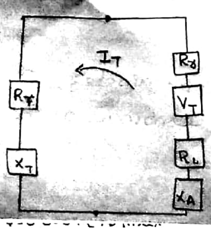
(xi) Aperture area of the antenna (Effective area)



Aperture area / effective area is defined as the ratio of available power at the terminals of a receiving antenna to the power density of the wave incident on it such that the wave is polarization-match to the antenna.

$$A_e = \frac{P_T\,(W)}{W_i\,(W/m^2)} = \frac{|I_T|^2 R_T}{2W_i}$$





$$|I_T|^2 = \frac{|V_T|^2}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2} \qquad I_T = \frac{V_T}{(R_r + R_L + R_T) + j\,(X_A + X_T)}$$

$$A_e = \frac{|V_T|^2 R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2 \cdot 2W_i} \qquad |I_T| = \frac{|V_T|}{\sqrt{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}}$$

$$\boxed{A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_T + R_r + R_L)^2 + (X_A + X_T)^2}\right]}$$

* Expression for maximum effective aperture area

Applying conjugate matching to the above expression of A_e

$R_T = R_r + R_L \quad X_A = -X_T$

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_r + R_L}{(2R_r + 2R_L)^2}\right]$$

$$\boxed{A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right]}$$

Only half of the energy absorbed by the antenna reaches the load and the rest is scattered or lost. We need to account these lost radiations also.

(i) Scattering Area

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

It is defined as the area when multiplied with incident power density is equal to the scattered power

(ii) Loss area

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_r+R_L)^2}\right]$$

Loss Area is equivalent area which when multiplied with the incident power density gives the power dissipated as heat through R_L

(iii) Capture Area

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_r+R_L+R_T}{(R_r+R_L)^2}\right]$$

$$\boxed{A_c = \text{Scatter area} + \text{loss area} + \text{effective aperture area}}$$

Capture Area is defined as the equivalent area which when multiplied with incident power density gives total power captured or collected by the antenna i.e. sum of all other areas.

* Aperture efficiency

It is defined as the ratio of maximum aperture area to physical area of the antenna.

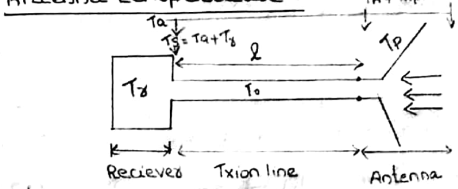
$$\epsilon_{ap} = \frac{\text{max: aperture area}}{\text{physical area}}$$

$\epsilon_{ap} \le 1$; for Aperture Antenna

For parabolic dish antenna, physical area is greater than max: aperture area.

Aperture area is greater than physical area for wire antennas. The aperture area of wire antenna is the area of the crosssection of the wire when it is split lengthwise along its diameter

(xii) Antenna temperature



All objects with temperature above 0K will radiate energy. The amount of energy radiated is represented by $T_B(\theta,\phi)$ which is called the brightness temperature. The brightness temperature emitted by different sources is intersected by the antenna and it appears as antenna temperature at the terminals. The expression for antenna temp: is

$$T_A = \frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} T_B(\theta,\phi)\, G(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

where $G(\theta,\phi)$ is the gain of antenna.

Assuming no losses or other contributions between the antenna and the reciever the noise power transfered to the recieve r is $P_r = kT_A\Delta f$, where Δf is the bandwidth in Hz, 'k' is the Boltzman constant (1.38×10^{-23} J/K)

If the antenna and the transmission line are maintained at a temperature ... and the transmission is tx line is lossy and T_A has to be modify to account for these losses. The txion line is of length 'l' and is having a uniform attenuation α Np/m

$$T_a = (T_A + T_{AP})e^{-2\alpha l} + T_o(1 - e^{-2\alpha l})$$

where $T_{AP} = \left(\frac{1}{e_A} - 1\right)T_p$

e_A - thermal efficiency of antenna

T_{AP} - antenna temp at the terminals due to the physical temp T_p

T_o - temp: of txion line

$$P_r = kT_a \Delta f$$

Thus the antenna noise power at the receiver terminal is modified as given above. ($P_r = kT_a\Delta f$)

The receiver itself has certain noise-temperature (T_r) (due to thermal noise). The system noise power at the terminals is $\boxed{P_s = k(T_a + T_r)\Delta f}$

Q) The effective antenna temperature of a target at the input terminals of antenna is 150K. Assuming antenna is maintained and at a thermal temperature of 300 K and has a thermal efficiency of 99% and is connected to a receiver through an X-band (8-12GHz) rectangular waveguide of 10m. of attenuation α is 0.13 dB/m and at a temperature of 300K. Find effective antenna temperature at the receiver terminals.

Ans:- $T_a = (T_A + T_{AP})e^{-2\alpha l} + T_o(1 - e^{-2\alpha l})$

$T_A = 150K$

$$T_{AP} = \left(\frac{1}{e_A} - 1\right)T_p \qquad e_A = 0.99 \text{ dB}$$

$T_p = 300K$

$T_{AP} = \underline{3.03K}$

1 Nepper = 8.686 dB

$\alpha = 0.13$ dB/m

$1\text{dB} = \frac{1}{8.686}$

$0.13\text{dB} = \frac{1}{8.686} \times 0.13 = \underline{0.0149 \text{ Np/m}}$

$T_o = 300K$

$$T_a = (150 + 3.03)e^{-2\times 0.0149\times 10} + 300\left(1 - e^{-2\times 0.0149\times 10}\right)$$

$= 113.59 + 77.309$

$= \underline{190.90K}$

Q) A uniform plane wave is incident upon a very short lossless dipole ($l << \lambda$). Find the max: effective area - assuming that radiation resistance $R_r = 80\left(\frac{\pi l}{\lambda}\right)^2$, the incident field is linearly polarized along its axis.

★ max: Aperture area $A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right]$

Since the antenna is lossless $R_L = 0$

There are ③ radiating elements

(1) Hertzian dipole (very short dipole) $l << \lambda$

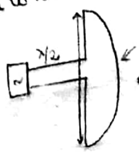
For this dipole the current distribution is assumed to be constant throughout its length.

(ii) Short dipole ($l < \lambda$)

Here the current distribution is triangular

(iii) Dipole antenna ($\lambda/2$)

It is assumed to have sinusoidal current distribution



$V_T = El$

$W_i = \frac{E^2}{2\eta_0}$

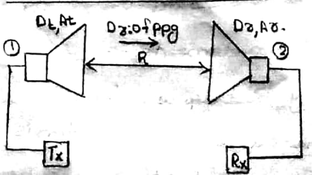
$R_r = 80\left(\frac{\pi l}{\lambda}\right)^2$

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right] = \frac{E^2 l^2}{8\frac{E^2}{2\eta_0}}\left[\frac{1}{80\left(\frac{\pi l}{\lambda}\right)^2}\right]$$

$$= \frac{1}{\left(8 \times 2 \times 120\pi\right)\left(\frac{80\pi^2}{\lambda^2}\right)}$$

$$= \frac{240\pi\lambda^2}{8 \times 80\pi^2} = \underline{\underline{0.119\lambda^2}}$$

* **Relationship between maximum directivity and maximum aperture area:**



If antenna ① were isotropic ... density at a distance 'R' is

$$W_0 = \frac{P_t}{4\pi R^2}$$

But antenna ① is having directional properties. So the actual power density transmitted is

$$W_t = \frac{P_t D_t}{4\pi R^2}$$

The amount of power collected by reciever antenna

$$P_r = W_t A_r$$

$$P_r = \frac{P_t D_t}{4\pi R^2} A_r$$

$$D_t A_r = \frac{P_r}{P_t}(4\pi R^2) \rightarrow ①$$

Assume that antenna ② is transmitting and antenna ① is receiving

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \rightarrow ②$$

From eq: ① and ② we get

$$D_t A_r = D_r A_t$$

$$\boxed{\frac{D_t}{A_t} = \frac{D_r}{A_r}}$$

The above relationship shows that increasing directivity of an antenna increases its aperture area since they are directly proportional.

Assume directivity and aperture areas are maximum, then $\frac{D_{tm}}{A_{tm}} = \frac{D_{rm}}{A_{rm}}$

$$A_{tm} = \frac{A_{em}}{D_{tm}} = \frac{A_{em}}{D_{em}}$$

Assuming that the transmitting antenna is isotropic $D_{tm} = 1$, Then

$$A_{tm} = \frac{A_{em}}{D_{em}}$$

For a very short dipole $A_{em} = 0.119\lambda^2$

$D_{em} = 1.5$

$$A_{tm} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}$$

$$\boxed{A_{em} \cong \frac{\lambda^2}{4\pi} D_{max}}$$

maximum aperture area of any antenna is related to its maximum directivity through above relation

If losses are considered,

$$\boxed{A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_{max}}$$

* ~~Polarization~~

$$A_{em} = (1-|\Gamma|^2)\, e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_{max} \Rightarrow A_{em} = (1-|\Gamma|^2)\, e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_{max}$$

* **Polarization loss factor (PLF)**

Polarization of the receiving antenna will not be the same as that the polarization of incident wave. This is called polarization mismatch. When polarization mismatch occurs

Assume the electric field of the incoming wave is

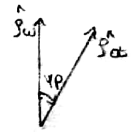
$$\bar{E}_i = \hat{\rho}_w E_i$$

$\hat{\rho}_w \rightarrow$ unit vector in the direction of E_i

The electric field of the receiving wave is expressed as $\bar{E}_A = \hat{\rho}_a E_a$

$\hat{\rho}_a \rightarrow$ unit vector in the direction of electric field of the receiving antenna.

Then $\boxed{PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 \approx \cos^2 \psi_p}$



Q) Electric field of a linearly polarized electromagnetic wave is given as $\bar{E}_i = \hat{a}_x E_0(x,y) e^{-jkz}$. This is incident on a linearly polarized antenna, whose electric field polarization is given as $\bar{E}_a = (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi)$. Find polarisation loss factor (PLF)?

* $\hat{\rho}_w = \hat{a}_x$

$$\hat{\rho}_a = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$$

$$\hat{\rho}_w \cdot \hat{\rho}_a = \left| \hat{a}_x \cdot \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

$$PLF(dB) = 10 \log_{10}\left(\frac{1}{2}\right) = -3dB$$

Title

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ANTENNA & WAVE PROPAGATION

Antenna

Antennas are basic components of an electric system, also they are the connecting link b/w transmitter and receiver. Different antenna parameters are gain, directivity, beam width, effective aperture, effective height, radiation resistance, radiation temperature etc.

Isotropic radiator (sphere)

It is a fictitious radiator which radiates uniformly in all directions. It is a hypothetical lossless radiator with which the practical antennas are compared. It is used as a reference antenna.

Radiation pattern

It is the 3-D graph which shows the variation of electromagnetic field which are at equal distance from the antenna. It is the graphical representation of radiation properties of an antenna as a function of space coordinates. Radiation pattern of isotropic radiator is sphere.

Radiation pattern lobe

It is a portion of the radiation pattern bounded by the region of relatively weak radiation intensity. It includes major lobe, minor lobe, side lobe and back lobe.

⇒ Major lobe

Radiation pattern in the direction of maximum radiation intensity. For an antenna there may exists more than one major lobe.

→ Minor lobe

All lobes except major lobe are called minor lobes.

→ Side lobe

It is the lobe in any direction other than major lobe. It is adjacent to the main lobe and occupies the hemisphere in the direction of main lobe. It is the largest of the minor lobes.

→ Back lobe

The minor lobe that occupies the hemisphere in a direction opposite to that of the major lobe.

* Radiation pattern narrows then directivity ↑

* Directivity

It is the dimensionless quantity which indicates the effectiveness of concentrating power into a desired direction. The narrower the solid angle the higher the directivity.

$G = KD$; $K \to$ efficiency factor
{when there is no losses or 100% efficient or isotropic antenna then $G = D$} $K=1$

* Beam solid angle (Ω_A)

This is solid angle through which all the power radiated would stream if the power per unit solid angle equal to the maximum value of radiation intensity.

OR

It is the solid angle through which all the power of antenna would flow if its radiation intensity is constant and equal to the maximum radiation intensity for all angles within the solid angle.

unit measurement is equal: unit solid angle

* Radiation intensity

It is the power radiated from an antenna per unit solid angle. Unit Watt/steradian

* Antenna efficiency / Radiation efficiency

It is the ratio of power radiated to the total i/p power supplied to the antenna.

* Radiation resistance (R_r)

It is defined as a fictitious resistance when connected in series with the antenna will consume the same power as it is actually radiated. The value of radiation resistance depends on

* Configuration of antenna
* Point where radiation resistance is considered.
* location of antenna w.r.t ground.
* due to coronal discharge.

5/1/2017
Tuesday

* Gain (G)

1) It is the ratio of maximum radiation intensity in a given direction from the test antenna to the maximum radiation intensity from a reference antenna produced in the same direction with same power i/p.

2) It is the ratio of the maximum received power from the test antenna to the maximum received power from the reference antenna for the same i/p power.

3) It is the ratio of maximum field strength from test antenna to the maximum field strength from reference antenna.

* Directive Gain (G_d)

1) It is the ratio of radiation intensity in a particular direction to the average radiated power.

2) It is the ratio of power density in a particular direction by test antenna to the power density in that direction by an isotropic antenna for the same radiated power.

* Power Gain (G_p)

1) It is the ratio of power density in a particular direction from a test antenna to the power density

in that direction from an isotropic antenna for the same total o/p power.

for difference

G_p & G_d are identical except that power gain takes into account the antenna losses.

$G_p = \eta G_d$

$\eta \rightarrow$ Efficiency factor

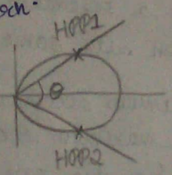
When there is no losses $\eta = 1$

$\therefore G_p = G_d$

2) It is the ratio of radiation Intensity in a particular direction to the average total i/p power.

* Antenna beam width

It is defined as the angular separation b/w two half power points of the major lobe in the radiation pattern.



Half power point / 3dB point i.e; $\frac{1}{\sqrt{2}}$

Directivity $\propto \frac{1}{\text{Beam width}}$

Factors affecting the beam width are:-

* Shape of the radiation pattern (narrow/wide)
* Wavelength
* Dimensions of antenna

* Beam efficiency

It is the ratio of main beam area to total beam area.

$$BE = \frac{\Omega_M}{\Omega_M + \Omega_m} = \frac{\Omega_M}{\Omega_A}$$

$\Omega_M \rightarrow$ Beam area of major lobe

$\Omega_m \rightarrow$ Beam area of minor lobe

$\Omega_A = \Omega_M + \Omega_m$

$\frac{\Omega_M}{\Omega_A} = BE$

$\frac{\Omega_m}{\Omega_A} =$ stray factor

* Effective aperture / Effective area

1) It is the area over which it extracts electromagnetic energy from the travelling electromagnetic waves.

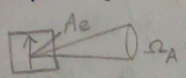
2) It is defined as the ratio of power received at the antenna load to the power density of the incident wave.

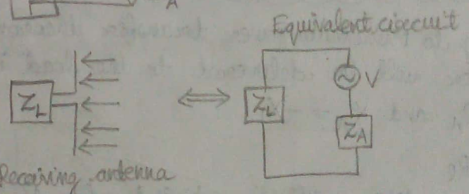
$A_e = \frac{W}{P}$ ($W \rightarrow$ received power (losses also there); $P \rightarrow$ i/p power)

$A_p > A_e$

Aperture efficiency

$\varepsilon_{ap} = \frac{A_e}{A_p}$ $\rightarrow$ Physical aperture





Let a receiving antenna be placed in the field of a plane polarized electromagnetic waves having an effective area A_e. The receiving antenna is terminated in a load impedance Z_L.

$Z_A \rightarrow$ Antenna Impedance

$Z_A = R_A + jX_A$

$Z_L = R_L + jX_L$

$R_A = R_r + R_l$ (Radiation resistance + loss resistance)

$w = I_{ame}^2 R_L$

$$I_{ame} = \frac{V}{Z_A + Z_L} = \frac{V}{(R_A + jX_A) + (R_L + jX_L)}$$

$$= \frac{V}{(R_r + R_l + jX_A) + (R_L + jX_L)}$$

$$I_{ame} = \frac{V}{(R_r + R_l + R_L) + j(X_A + X_L)}$$

$$|I_{ame}| = \frac{V}{\sqrt{(R_r + R_l + R_L)^2 + (X_A + X_L)^2}}$$

$$w = I_{ame}^2 R_L$$

$$= \frac{V^2 R_L}{(R_r + R_l + R_L)^2 + (X_A + X_L)^2}$$

$$A_e = \frac{w}{P}$$

$$A_e = \frac{V^2 R_L}{(R_r + R_l + R_L)^2 + (X_A + X_L)^2} \times \frac{1}{P}$$

According to Maximum power transfer theorem, maximum power will be delivered to the load if & only if $R_L = R_A$ and $X_A = -X_L$

$R_A = R_r + R_l$

The above condition will be satisfied when $R_l = 0$.

Therefore, $R_L = R_A = R_r$

$$A_{e(max)} = \frac{V^2 R_r}{4 R_r^2 P}$$

$$\alpha = \frac{A_e}{A_{e(max)}} \rightarrow \text{Effectiveness ratio.}$$

$$A_{e(max)} = \frac{V^2}{4 R_r P}$$

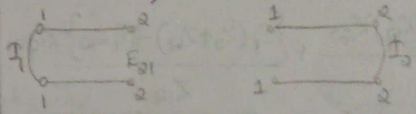
$$\frac{A_{e(max)}}{A_p} \rightarrow \text{Absorption ratio}$$

9/1/2018 Tuesday

Reciprocity theorem

1) If an EMF is applied to the terminals of an antenna no: 1 and the current measured at the terminals of another antenna no: 2, then an equal current both in amplitude and phase will be obtained at the terminals of antenna no: 1 if the same EMF is applied to the terminals of antenna no: 2.

2) If a current I_1 at the terminals of antenna no: 1 induces an EMF, E_{21} at the open terminals of antenna no: 2 and a current I_2 at the terminals of antenna no: 2 induces an EMF, E_{12} at the open terminals of antenna no: 1, then $E_{12} = E_{21}$ provided $I_1 = I_2$.



Proof

Assumptions

⇒ EMF are of same frequency

⇒ Medium b/w the two antennas are linear, passive & isotropic

⇒ Generator producing EMF and the ammeter for measuring the current has zero impedance.

Figure 1

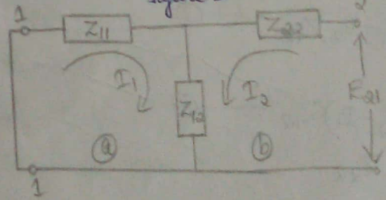
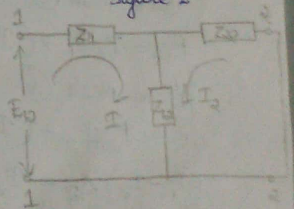


Figure 2



Z_{12} or Z_M → Mutual Impedance

Z_{11} → Impedance due to antenna no: 1

Z_{22} → Impedance due to antenna no: 2

From mesh ⓐ

$$0 = I_1 Z_{11} + (I_1 - I_2) Z_{12} \quad \text{---} (1)$$

From mesh (b)

$$E_{21} = I_2 Z_{22} + (I_2 - I_1) Z_{12} \quad \text{—— (2)}$$

From (1)

$$(I_2 - I_1) Z_{12} = I_1 Z_{11}$$

$$I_2 = \frac{I_1 Z_{11} + I_1 Z_{12}}{Z_{12}}$$

$$I_2 = \frac{I_1 (Z_{11} + Z_{12})}{Z_{12}} \quad \text{—— (1a)}$$

Substitute (1a) in (2)

$$E_{21} = \frac{I_1 (Z_{11} + Z_{12})}{Z_{12}} Z_{22} + \left(\frac{I_1 (Z_{11} + Z_{12})}{Z_{12}} - I_1 \right) Z_{12}$$

$$= \frac{I_1 (Z_{11} + Z_{12}) Z_{22}}{Z_{12}} + \frac{\{ I_1 (Z_{11} + Z_{12}) - I_1 Z_{12} \} Z_{12}}{Z_{12}}$$

$$= \frac{I_1 (Z_{11} + Z_{12})(Z_{22} + Z_{12}) - I_1 Z_{12}^2}{Z_{12}}$$

$$I_1 = \frac{E_{21} Z_{12}}{(Z_{11} + Z_{12})(Z_{22} + Z_{12}) - Z_{12}^2} \quad \text{—— (I)}$$

From 2nd figure

From mesh (c)

$$E_{12} = Z_{11} I_1 + (I_1 - I_2) Z_{12} \quad \text{—— (3)}$$

From mesh (d)

$$0 = Z_{22} I_2 + (I_2 - I_1) Z_{12} \quad \text{—— (4)}$$

$$(I_1 - I_2) Z_{12} = I_2 Z_{22}$$

$$I_1 = \frac{I_2 Z_{22} + I_2 Z_{12}}{Z_{12}}$$

$$I_1 = \frac{I_2 (Z_{22} + Z_{12})}{Z_{12}} \quad \text{—— (4a)}$$

Substitute (4a) in (3)

$$E_{12} = Z_{11} \frac{I_2 (Z_{22} + Z_{12})}{Z_{12}} + \left(\frac{I_2 (Z_{22} + Z_{12})}{Z_{12}} - I_2 \right) Z_{12}$$

$$= Z_{11} \frac{I_2 (Z_{22} + Z_{12})}{Z_{12}} + \frac{\{ I_2 (Z_{22} + Z_{12}) - I_2 Z_{12} \} Z_{12}}{Z_{12}}$$

$$E_{12} = \frac{I_2 (Z_{22} + Z_{12})(Z_{11} + Z_{12}) - I_2 Z_{12}^2}{Z_{12}}$$

$$I_2 = \frac{E_{12} Z_{12}}{(Z_{22} + Z_{12})(Z_{11} + Z_{12}) - Z_{12}^2} \quad \text{—— (II)}$$

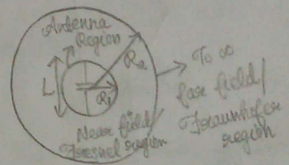
(I) = (II) according to reciprocity theorem

$I_1 = I_2$

ie; (I) = (II)

$$\frac{E_{21} Z_{12}}{(Z_{11} + Z_{12})(Z_{22} + Z_{12}) - Z_{12}^2} = \frac{E_{12} Z_{12}}{(Z_{22} + Z_{12})(Z_{11} + Z_{12}) - Z_{12}^2}$$

* Antenna field zones



$$R_1 = 0.62 \sqrt{L^3/\lambda}$$

$$R_2 = \frac{2L^2}{\lambda}$$

Fraunhofer region $> \frac{2L^2}{\lambda}$

* Effective height

It is the ratio of induced voltage at the terminals of the receiving antenna to the incident E.F intensity.

$$h_e = \frac{V}{E}$$

$$h_e = \sqrt{\frac{A_e(R_A+R_L)^2+(X_A+X_L)^2}{ZR_L}}$$

$R_A = R_r + R_l$ — By maximum power transfer theorem.

$R_r = R_A = R_L$, $R_l = 0$

$$(h_e)_{max} = \sqrt{\frac{(A_e)_{max}(2R_r)^2}{ZR_r}}$$

$$= \sqrt{\frac{(A_e)_{max}\,4R_r}{Z}}$$

$$= 2\sqrt{\frac{(A_e)_{max}R_r}{Z}} \quad \text{OR} \quad 2\sqrt{\frac{(A_e)_{max}R_L}{Z}}$$

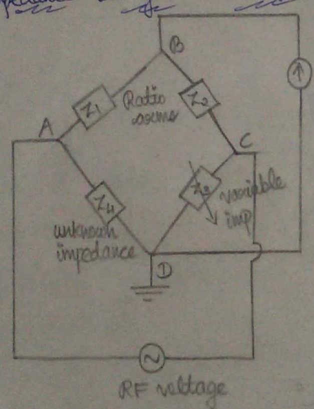
Module II

Antenna Measurement

11/1/2018 Thursday

Impedance Measurement

⇒ Impedance Bridge Method for low frequency



$$\frac{Z_1}{Z_2} = \frac{Z_4}{Z_3} \quad \text{(under balanced condition)}$$

$$Z_4 = \frac{Z_1 Z_3}{Z_2}$$

Wheatstone's bridge is used to measure unknown impedance by comparison with known impedance. It consists of 4 arms to which four impedances are connected. Z_1, Z_2 are called ratio arms and Z_3 is variable arm impedance which varies to get null in the detector and Z_4 is the unknown impedance/antenna under test. When the bridge is balanced by varying impedance Z_3 no potential difference exist b/w point B & D and the meter in the detector circuit will give a null. Bridge is balanced not only for the magnitude but also for phase. Under this condition

$$\frac{Z_1\angle\theta_1}{Z_2\angle\theta_2} = \frac{Z_4\angle\theta_4}{Z_3\angle\theta_3}$$

Two balanced conditions are $Z_1Z_3 = Z_2Z_4$ (magnitude)

$\angle\theta_1 + \angle\theta_3 = \angle\theta_2 + \angle\theta_4$ (phase)

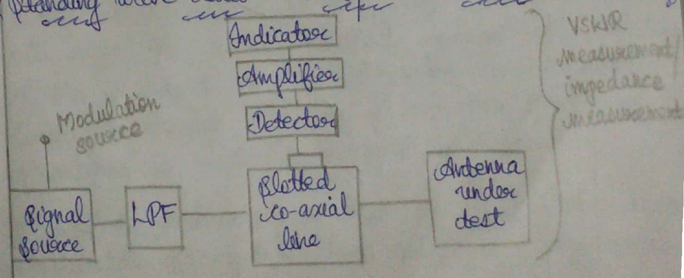
Procedure

The bridge is balanced with unknown impedance terminal by a short or open circuit. The short or open circuit is removed and the unknown impedance is inserted b/w point A & D and the bridge is rebalanced. The unknown impedance is now determined by balanced condition.

Note:-

Upto 30MHz this method is good choice for impedance measurement and may be used up to 1000MHz also.

⇒ Standing wave ratio method / Slotted line method (>1000MHz) — VSWR measurement/ impedance measurement



16/1/2018 Tuesday

$$S = \frac{V_{max}}{V_{min}} = \frac{V_i + V_r}{V_i - V_r}$$

$$S = \frac{\frac{V_i}{V_i} + \frac{V_r}{V_i}}{\frac{V_i}{V_i} - \frac{V_r}{V_i}}$$

where $\frac{V_r}{V_i} = K$

$$S = \frac{1+K}{1-K}$$

$$Z_L = Z_0\left(\frac{1+Ke^{-2Pl}}{1-Ke^{-2Pl}}\right)$$

Page No: 848

$K \rightarrow$ Reflection coefficient $\left(\frac{V_r}{V_i}\right)$

$S \rightarrow$ VSWR

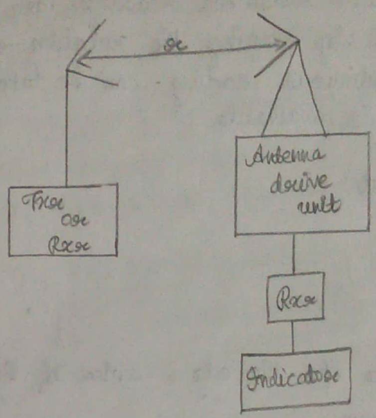
$Z_0 \rightarrow$ characteristic Impedance

$Z_L \rightarrow$ Unknown Impedance / Antenna Impedance

$P \rightarrow$ primary constant

$l \rightarrow$ length of the transmission line

Radiation pattern Measurement



Radiation pattern is a 3D figure and is obtained by measuring the field intensity all over the spatial angles.

It is necessary to have two antennas for this measurement. One is antenna under test, also called as primary antenna, and the other at some distance away for illuminating the former, also called secondary antenna.

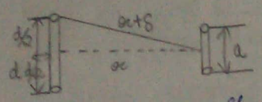
Procedure

There are two procedures:-

i) The 1° antenna is kept stationary whereas the 2° antenna is transposed around along the circular path at constant distance. The 2° antenna is kept aimed at 1° antenna so that only 1° antenna pattern will affect the result. The field strength and direction of the 2° antenna w.r.t 1° antenna are recorded. From this reading the plot of radiation pattern of 1° antenna is made (by reciprocity theorem)

ii) Both the antennas are kept in fixed positions having a suitable spacing b/w them and 2° antenna beam is aimed at 1° antenna. Now the 1° antenna is rotated about a vertical axis. The readings are taken at a no: of points by stopping the rotation of 1° antenna or a continuous reading can be taken if pattern recorder is available

vii) * Distance Requirement



$\delta \rightarrow$ phase difference b/w the edge & center of the antenna

By pythagoras theorem,

$$(x+\delta)^2 = x^2 + \left(\frac{d}{2}\right)^2$$

$$x^2+\delta^2+2x\delta = x^2 + \frac{d^2}{4}$$

δ is very small so δ^2 can be neglected.

$$x^2+2\delta x = x^2 + \frac{d^2}{4}$$

$$x = \frac{d^2}{8\delta}$$

The limit specified for the phase difference should not exceed $\lambda/16$.

ie; $\delta \leq \lambda/16$

$$\therefore x = \frac{d^2 \times 16}{8 \times \lambda} = \frac{2d^2}{\lambda}$$

$$x = \frac{d^2}{8\delta} = \frac{2d^2}{\lambda}$$

In order to obtain accurate Fraunhofer radiation pattern (far field), the distance b/w 1° & 2° antenna must be large whereas the near field or Fresnel field is obtained when the distance b/w the antennas are very much small. So there should be a limit for the distance b/w the antennas in order to obtain the correct radiation pattern.

* Measurement of Directivity

Black diagram, explanation & procedure is as same as radiation pattern measurement.

$$D = \frac{4\pi \times \text{Maximum Radiation Intensity}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \text{Radiation Intensity } \sin\theta \, d\phi \, d\theta}$$

Two methods :-

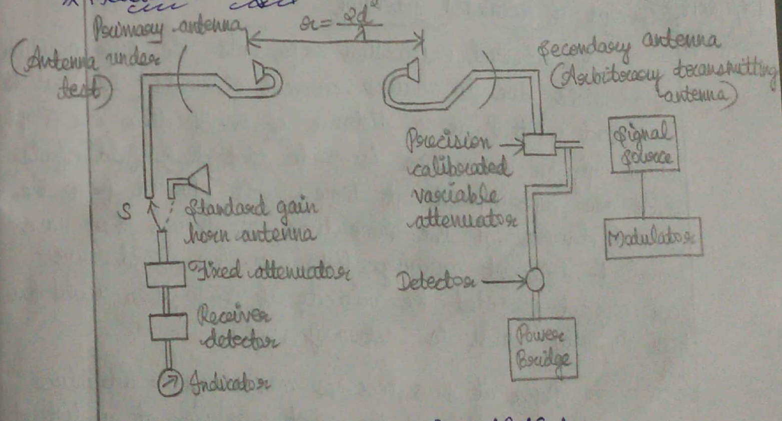
i) Orange slice method

Each pattern is multiplied by $\sin\theta$ and then integrated.

ii) Conical cut method

Each pattern is integrated & the integrated values are multiplied by weighing factor $\sin\theta$.

* Measurement of Gain



Direct Comparison Method

Procedure

At first the standard antenna is connected to the receiver with the help of a switch, S. The o/p to the transmitting antenna is adjusted to our convenient level & the corresponding reading at the receiver is recorded. The attenuated dial setting is recorded as w_1 and the power bridge reading is recorded as P_1.

Now connect the subject antenna whose gain is to be measured in place of standard gain antenna. The variable attenuator dial is adjusted such that receiver indicates the same previous reading as was with standard gain antenna. Let the attenuator dial setting be w_2 and power bridge reading as P_2.

$$G = \frac{P_1}{P_2} \times \frac{w_2}{w_1}$$

$$\boxed{G = G_P \frac{P_1}{P_2}}$$

where G_P is the power gain $= \frac{w_2}{w_1}$

17/1/2018 Wednesday

VIP

Concept of retarded potential

The potential expression represents the superposition of potentials due to various current elements (Idl) at a distant point P at a distance of r. If these are simply added up, an assumption is made that these field effects which are superimposed at time t, all started from the current element at the same time, eventhough they have travelled different varying distances. This would have been correct provided the velocity of propagation would have been infinity which is actually not.

Hence it now becomes necessary to introduce concept of retardation ie; the effect reaching at a distant point P from a given element at an incident t is due to the current value which is followed at an earlier time or the current effective introducing the field at earlier time. This concept was introduced by Lorentz. Without considering the finite time of propagation, Instantaneous current, $\boxed{I = I_m \sin \omega t}$ whereas when finite time propagation is taken into account, $[I] = I_m \sin \omega \left(t - \frac{r}{c}\right)$

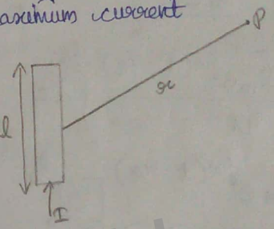
$\boxed{\text{OR } I = I_m e^{j\omega t}}$

$[I] \rightarrow$ retarded current

$r \rightarrow$ distance travelled

$c \rightarrow$ velocity of propagation

$I_m \rightarrow$ maximum current



[ ] is used to indicate retarded value.

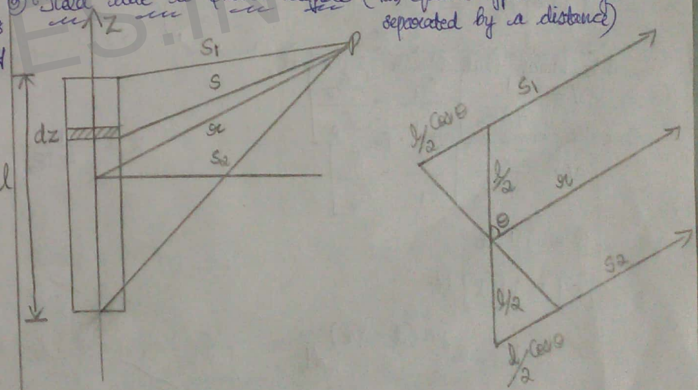
$[A] \rightarrow$ Retarded vector potential

$[V] \rightarrow$ Retarded scalar potential

18/1/2018 Thursday

VIP

Field due to short dipole (Two equal & opposite charges separated by a distance)



We know that E.F intensity, $E = -\nabla V - \frac{\partial}{\partial t} A$

By considering the propagation time delay $E = -\nabla[V] - \frac{\partial}{\partial t}[A]$

$$= -\nabla[V] - j\omega[A]$$

$$A = A_z = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{[I]}{z}\, dz$$

$$= \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{I_m e^{j\omega(t - \frac{x}{c})}}{z} dz$$

$z \rightarrow$ distance

If the distance from the dipole is large compared to its length ie; $x >> l$ and if the wavelength is very large compared to the length ie; $\lambda >> l$. Then $z = x$

$$A_z = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{I_m e^{j\omega(t - \frac{x}{c})}}{x} dz$$

$$= \frac{\mu_0}{4\pi} \times 2 \int_0^{l/2} \frac{I_m e^{j\omega(t - x/c)}}{x} dz$$

$$= \frac{\mu_0 \times 2 I_m e^{j\omega(t - x/c)}}{4\pi} \int_0^{l/2} dz$$

$$= \frac{\mu_0}{2\pi x} I_m e^{j\omega(t - x/c)} [z]_0^{l/2}$$

$$[A] = [A_z] = \frac{\mu_0 l I_m e^{j\omega(t - x/c)}}{4\pi x}$$

We know that scalar potential,

$$V = \frac{1}{4\pi\varepsilon_0} \left[\frac{Q_1}{S_1} - \frac{Q_2}{S_2}\right]$$

Here the charges are equal. So

$$V = \frac{1}{4\pi\varepsilon_0} \left[\frac{Q}{S_1} - \frac{Q}{S_2}\right] \qquad K = \frac{1}{4\pi\varepsilon_0}$$

$$Q = \int I dt$$

$$[Q] = \int [I] dt$$

$$= \int I_m e^{j\omega(t - x/c)} dt$$

$$[Q] = \frac{I_m e^{j\omega(t - x/c)}}{j\omega}$$

By substituting $x = S_1$ for Q_1 & $x = S_2$ for Q_2

$$[V] = \frac{1}{j\omega 4\pi\varepsilon_0} \left[\frac{I_m e^{j\omega(t - S_1/c)}}{S_1} - \frac{I_m e^{j\omega(t - S_2/c)}}{S_2}\right]$$

From the figure,

$S_1 = x - \frac{l}{2}\cos\theta$ &

$S_2 = x + \frac{l}{2}\cos\theta$

Substituting these values of S_1 & S_2 in the above equation we get

$$[V] = \frac{1}{j\omega 4\pi\varepsilon_0}\left[\frac{I_m e^{j\omega(t - \{x - \frac{l}{2}\cos\theta/c\})}}{x - \frac{l}{2}\cos\theta} - \frac{I_m e^{j\omega(t - \{x + \frac{l}{2}\cos\theta/c\})}}{x + \frac{l}{2}\cos\theta}\right]$$

$$= \frac{1}{j\omega 4\pi\varepsilon_0} I_m e^{j\omega(t - x/c)} \left[\frac{e^{j\omega \frac{l\cos\theta}{2c}}}{x - \frac{l}{2}\cos\theta} - \frac{e^{-j\omega \frac{l\cos\theta}{2c}}}{x + \frac{l}{2}\cos\theta}\right]$$

substitution

$$[V] = \frac{I_m e^{j\omega(t - x/c)}}{j\omega 4\pi\varepsilon_0}\left[\frac{e^{\frac{j\omega l\cos\theta}{2c}} \times \left[x + \frac{l}{2}\cos\theta\right] - e^{-\frac{j\omega l\cos\theta}{2c}} \times \left[x - \frac{l}{2}\cos\theta\right]}{x^2 - \left(\frac{l}{2}\cos\theta\right)^2}\right]$$

Since $x >> l$; $\left(\frac{l}{2}\cos\theta\right)^2$ can be neglected when compared with x^2.

We know that $e^{i\theta} = \cos\theta + i\sin\theta$.

So the above equation of $[V]$ becomes

$$[V] = \frac{I_m e^{j\omega(t - x/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{\left[\cos\frac{\omega l\cos\theta}{2c} + j\sin\frac{\omega l\cos\theta}{2c}\right]\left(x + \frac{l}{2}\cos\theta\right) - \left[\cos\frac{\omega l\cos\theta}{2c} - j\sin\frac{\omega l\cos\theta}{2c}\right]\left(x - \frac{l}{2}\cos\theta\right)}{x^2}\right\}$$

19/1/2018 Friday

Since $\theta = \frac{\omega l \cos\theta}{2c} \approx \frac{\pi l \cos\theta}{\lambda}$

$\lambda >> l$

$\therefore \cos\theta \approx 1$ when θ is very small & $\sin\theta \approx \theta$ when θ is very small.

On substituting the above condition in the equation, we get

$$[V] = \frac{I_m e^{j\omega(t - r/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{\left(1+\frac{j\omega l\cos\theta}{2c}\right)\left(r+\frac{l}{2}\cos\theta\right) - \left(1-\frac{j\omega l\cos\theta}{2c}\right)\left(r-\frac{l}{2}\cos\theta\right)}{r^2}\right\}$$

$$= \frac{I_m e^{j\omega(t-r/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{r+\frac{l}{2}\cos\theta + \frac{j\omega l r\cos\theta}{2c} + j\frac{\omega l^2\cos^2\theta}{4c} - \left(r - \frac{l}{2}\cos\theta - j\frac{\omega l r\cos\theta}{2c} + \frac{j\omega l^2\cos^2\theta}{4c}\right)}{r^2}\right\}$$

$$= \frac{I_m e^{j\omega(t-r/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{\cancel{r} + \frac{l}{2}\cos\theta + \frac{j\omega l r\cos\theta}{2c} + \cancel{j\frac{\omega l^2\cos^2\theta}{4c}} - \cancel{r} + \frac{l}{2}\cos\theta + \frac{j\omega l r\cos\theta}{2c} - \cancel{j\frac{\omega l^2\cos^2\theta}{4c}}}{r^2}\right\}$$

$$= \frac{I_m e^{j\omega(t-r/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{\cancel{2}\times\frac{l}{\cancel{2}}\cos\theta + j\frac{\cancel{2}\omega l r\cos\theta}{\cancel{2}c}}{r^2}\right\}$$

$$[V] = \frac{I_m e^{j\omega(t-r/c)}}{j\omega 4\pi\varepsilon_0}\left\{\frac{l\cos\theta + \frac{j\omega l r\cos\theta}{c}}{r^2}\right\}$$

$$= \frac{I_m e^{j\omega(t-r/c)}}{4\pi\varepsilon_0 c}\left\{\frac{l\cos\theta}{j\omega r^2}\left(j\omega r + c\right)\right\}$$

$$[V] = \frac{I_m e^{j\omega(t-r/c)}}{4\pi\varepsilon_0 c}\, l\cos\theta\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right]$$

In polar coordinate system, the vector potential, A

$A = A_r r + A_\theta \theta + A_\phi \phi$

Since the dipole is placed in the z axis vector potential has only z component. Hence $A_\phi = 0$

$A_r = A_z\cos\theta$

$A_\theta = -A_z\sin\theta$

In polar coordinate system the gradient of scalar potential $\nabla V = \frac{\partial V}{\partial r} + \frac{1}{r}\frac{\partial V}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}$

We know that $E = -\nabla V - \frac{\partial}{\partial t}A$

By Representing the above equation in polar coordinate system, we get

$$(E_r + E_\theta + E_\phi) = -\left[\frac{\partial V}{\partial r} + \frac{1}{r}\frac{\partial V}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\right] - \frac{\partial}{\partial t}\left[A_z\cos\theta - A_z\sin\theta\right]$$

$$= -\left[\frac{\partial V}{\partial r} + \frac{1}{r}\frac{\partial V}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\right] - j\omega\left[A_z\cos\theta - A_z\sin\theta\right]$$

$$E_r = \frac{-\partial V}{\partial r} - j\omega A_z\cos\theta$$

$$E_\theta = \frac{-1}{r}\frac{\partial V}{\partial\theta} + j\omega A_z\sin\theta$$

$$E_\phi = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial\phi}$$

Since V is independent of ϕ $\frac{\partial V}{\partial\phi} = 0$

$\therefore E_\phi = 0$

$$\frac{\partial V}{\partial r} = \frac{I_m e^{j\omega t} l\cos\theta}{4\pi\varepsilon_0 c}\left[e^{-\frac{j\omega r}{c}} \times \frac{1}{r} + e^{-\frac{j\omega r}{c}} \times \frac{c}{j\omega r^2}\right]$$

$$= \frac{I_m e^{j\omega t} l\cos\theta}{4\pi\varepsilon_0 c}\left[e^{-\frac{j\omega r}{c}}\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right]\right]$$

$$= \frac{I_m e^{j\omega t} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[e^{\frac{-j\omega r}{c}} \times \frac{\partial}{\partial r}\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right] + \frac{\partial}{\partial r} e^{\frac{-j\omega r}{c}} \times \left[\frac{1}{r} + \frac{c}{j\omega r^2}\right]\right]$$

$$= \frac{I_m e^{j\omega t} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[e^{\frac{-j\omega r}{c}} \times \left[\frac{-1}{r^2} + \frac{-2c}{j\omega r^3}\right] + \left(e^{\frac{-j\omega r}{c}} \times \frac{-j\omega}{c}\right) \times \left[\frac{1}{r} + \frac{c}{j\omega r^2}\right]\right]$$

$$\frac{1}{r} = r^{-1} = -1 \times r^{-1-1} = -r^{-2} = \frac{-1}{r^2}$$

$$-2 \times r^{-2-1} = -2r^{-3} = \frac{-2}{r^3}$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left\{\frac{-1}{r^2} \; \frac{-2c}{j\omega r^3} + \left(\frac{1}{r} + \frac{c}{j\omega r^2}\right) \times \frac{-j\omega}{c}\right\}$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left\{\frac{-1}{r^2} - \frac{2c}{j\omega r^3} - \frac{j\omega}{cr} - \frac{1}{r^2}\right\}$$

$$\frac{\partial V}{\partial r} = \frac{-I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[\frac{2}{r^2} + \frac{2c}{j\omega r^3} + \frac{j\omega}{cr}\right]$$

$$\therefore E_r = \frac{-\partial V}{\partial r} - j\omega A_z \cos\theta$$

$$= \frac{+I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[\frac{2}{r^2} + \frac{2c}{j\omega r^3} + \frac{j\omega}{cr}\right] - j\omega \times \frac{\mu_0 l}{4\pi r} I_m e^{j\omega(t - r/c)} \cos\theta$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi}\left[\frac{1}{\varepsilon_0 c}\left(\frac{2}{r^2} + \frac{2c}{j\omega r^3} + \frac{j\omega}{cr}\right) - \frac{j\omega\mu_0}{r}\right]$$

We know that $c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\therefore \mu_0 = \frac{1}{c^2 \varepsilon_0}$$

$$\therefore E_r = \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi}\left[\frac{1}{\varepsilon_0 c}\left(\frac{2}{r^2} + \frac{2c}{j\omega r^3} + \frac{j\omega}{cr}\right) - \frac{j\omega}{c^2 \varepsilon_0 r}\right]$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[\frac{2}{r^2} + \frac{2c}{j\omega r^3} + \cancel{\frac{j\omega}{cr}} - \cancel{\frac{j\omega}{cr}}\right]$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{4\pi\varepsilon_0 c}\left[\frac{2}{r^2} + \frac{2c}{j\omega r^3}\right]$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta \times 2\cancel{c}}{4\pi\varepsilon_0 \cancel{c}}\left[\frac{1}{r^2 c} + \frac{1}{j\omega r^3}\right]$$

$$E_r = \frac{I_m e^{j\omega(t - r/c)} \, l \cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{r^2 c} + \frac{1}{j\omega r^3}\right]$$

$$\frac{\partial V}{\partial \theta} = \frac{I_m e^{j\omega(t - r/c)}}{4\pi\varepsilon_0 c}\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right] \times \frac{\partial}{\partial\theta} l \cos\theta$$

$$= \frac{-I_m e^{j\omega(t - r/c)} \, l \sin\theta}{4\pi\varepsilon_0 c}\left[\frac{1}{r} + \frac{c}{j\omega r^2}\right]$$

$$E_\theta = \frac{I_m e^{j\omega(t - r/c)} \, l \sin\theta}{4\pi\varepsilon_0 c}\left[\frac{1}{r^2} + \frac{c}{j\omega r^3}\right] + j\omega \frac{\mu_0 l}{4\pi r} I_m e^{j\omega(t - r/c)} \sin\theta$$

$$= \frac{I_m e^{j\omega(t - r/c)} \, l \sin\theta}{4\pi}\left[\frac{1}{\varepsilon_0 c}\left(\frac{1}{r^2} + \frac{c}{j\omega r^3}\right) + \frac{j\omega\mu_0}{r}\right]$$

Substitute $\mu_0 = \frac{1}{c^2 \varepsilon_0}$

$$= \frac{I_m e^{j\omega(t-r/c)}\, l\sin\theta}{4\pi}\left[\frac{1}{\epsilon_0 c}\left(\frac{1}{r^2}+\frac{c}{j\omega r^3}\right)+\frac{j\omega}{c^2\epsilon_0 r}\right]$$

$$E_\theta = \frac{I_m e^{j\omega(t-r/c)}\, l\sin\theta}{4\pi\epsilon_0 c}\left[\frac{1}{r^2}+\frac{c}{j\omega r^3}+\frac{j\omega}{rc}\right]$$

$$E_r = \frac{I_m e^{j\omega(t-r/c)}\, l\cos\theta}{2\pi\epsilon_0}\left[\frac{1}{r^2 c}+\frac{1}{j\omega r^3}\right]$$

$$E_\phi = 0$$

M.F at any point is given by $\overset{B}{dH} = \nabla\times A$

In polar coordinate system $\nabla\times A$ is

$$a_r dH_r + a_\theta dH_\theta + a_\phi dH_\phi = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

We know that $A_\phi = 0$ & A is independent of ϕ so $\frac{\partial}{\partial\phi}=0$

$$a_r dH_r + a_\theta dH_\theta + a_\phi dH_\phi = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

By taking determinant

$a_r dH_r = 0$

$\therefore H_r = 0$

$a_\theta dH_\theta = 0$

$\therefore H_\theta = 0$

$$a_\phi dH_\phi = \frac{r\sin\theta\, a_\phi}{r^2\sin\theta}\left[\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right]$$

$$\frac{\partial}{\partial r} rA_\theta = \frac{\partial}{\partial r} r\times -A_z\sin\theta$$

$$= -\frac{\partial}{\partial r}\cancel{r}\times\left(\frac{\mu_0 l}{4\pi \cancel{r}} I_m e^{j\omega(t-r/c)}\sin\theta\right)$$

$$= \frac{-I_m e^{j\omega t}\sin\theta\,\mu_0 l}{4\pi}\times\frac{\partial}{\partial r}e^{-\frac{j\omega r}{c}}$$

$$= \frac{-I_m e^{j\omega(t-r/c)}\sin\theta\,\mu_0 l}{4\pi}\times -\frac{j\omega}{c}$$

$$\frac{\partial}{\partial r} rA_\theta = \frac{I_m e^{j\omega(t-r/c)}\sin\theta\,\mu_0 l\, j\omega}{4\pi c}$$

$$\frac{\partial}{\partial\theta}A_r = \frac{\partial}{\partial\theta}A_z\cos\theta$$

$$= \frac{\partial}{\partial\theta}\frac{\mu_0 l}{4\pi r} I_m e^{j\omega(t-r/c)}\cos\theta$$

$$\frac{\partial}{\partial\theta}A_r = \frac{-I_m e^{j\omega(t-r/c)}\mu_0 l\sin\theta}{4\pi r}$$

$$a_\phi dH_\phi = \frac{\cancel{r}\sin\theta\, a_\phi}{r^2\sin\theta}\left[\frac{I_m e^{j\omega(t-r/c)}\mu_0 l\, j\omega\sin\theta}{4\pi c} + \frac{I_m e^{j\omega(t-r/c)}\mu_0 l\sin\theta}{4\pi r}\right]$$

$$a_\phi dH_\phi = \frac{a_\phi}{r}\left[\frac{I_m e^{j\omega(t-r/c)}\mu_0 l\sin\theta}{4\pi}\left(\frac{j\omega}{c}+\frac{1}{r}\right)\right]$$

$$H_\phi = \frac{1}{r}\left[\frac{I_m e^{j\omega(t-r/c)}\cancel{\mu_0} l\sin\theta}{4\pi}\left(\frac{j\omega}{c}+\frac{1}{r}\right)\right]$$

μ

$$H_\phi = \frac{I_m e^{j\omega(t-r/c)}\, l\sin\theta}{4\pi}\left(\frac{j\omega}{rc}+\frac{1}{r^2}\right)$$